Chapter 1

Introduction and Literature Survey

In this chapter, the introduction of mathematical biology has been discussed. Also the different relationships among the species in the environment has been presented mathematically. The literature review related to the prey-predator relationship has been discussed with objective, scope and organization of the thesis.

1.1 Introduction

In real World, mathematical biology has a vast application. It is an interdisciplinary research field which combines theoretical biology with mathematics. By mathematical biology, the relationships among the living things are represented by some mathematical expressions. Using the tools of applied mathematics, different models can be developed. In theoretical biology, the focus is only on the development of the theoretical principles of the biological systems. So, mathematical biology has wide applications in biological, biomedical and biotechnology research. Therefore, the mathematical biology can have better simulated results and the properties which can be predicted to the experimenter that might not be evident.

Some mathematical models are developed to the study of the mathematical biology. It has many components of mathematics. Different new techniques were developed by this contribution. Since 19th century, the application of mathematics in biology has been introduced by different researchers.

The biotic community with the non-living environment forms an ecosystem. The organism and its environment are the basic functional units in it. Both are interrelated as they influenced each other for survivals. There exist some natural ecosystems like pond, lakes, oceans, grassland, forests, deserts, tundra and so on. Any ecosystem is mainly dependent on prey-predator relationship. The organism which eats another organism is known as predator. To whom the predator eats is known as prey. These prey-predator relationships exist between animals as well as between animal and plants. For example, consider an ecosystem of pond where phytoplankton is the prey and zooplankton is the predator. Again zooplankton plays the role of prey when fishes are the predator. Predators lives depend upon the prey. If they do not get sufficient prey they will die.

In mathematical biology, between prey and predator, these interrelation can be expressed quantitatively. Some effective models can improve the concept of understanding the natural World using the data analysis.

1.2 Basic information

1.2.1 Prey-predator relationship

When two different organisms interact between them i.e., one capture the other organism to feed then the first organism who is capturing called the predator and second one to whom first organism is capturing is called prey.

In any ecological system, the population control is determined by predation mechanism. Thus, the scarcity in predator population increases the number of prey population. When this situation happens to the predators then they enable to reproduce more predators and also they have possibility to change their habits of hunting. When the number of predators increases then prey populations decreases. This result indicates the scarcity of food to predators which can lead to the death of the predator populations.

1.2.2 Prey-predator model

Prey-predator model is the relationship among the organisms which are depicted by some mathematical expressions in an ecological system. To develop the model, first some assumptions are described as below:

1) The death of the preys is due to natural causes and being eaten by predators only.

2) The death of the predators causes due to natural causes.

3) The prey-predator interaction can be presented by a function.

Prey-predator relationship is mathematically represented by the systems of differential equations. Clearly, it is reasonable to expect that the two populations react in such a way as to influence each other's size. These system of differential equations are applied very much in different areas of science. But in real-life problem, it is very much interesting that there exist more than one unknown function. Therefore, system of differential equations is very much useful in mathematical biology.

1.2.3 Population

A collection of interbreeding individuals of the same species are known as population who live together in a region. In population ecology, the relationship of the population is studied with environment i.e., how they change with time, how environment influenced population density or distribution. The variations in population size and age structured are also studied in population ecology.

The size of population may be constant or increasing or decreasing. Mainly, two types of population model exist, one is continuous population model and other one is discrete population model.

• **Continuous population model**

If any population has continuous breeding season, the population growth model is known as continuous population growth model. In this model, the breeding of the population does not depend on food, season, climate etc. The population depends only on instantaneous per capita rate of growth.

Some preliminary definitions of continuous population model are discussed.

Exponential Growth

If a population has a constant birth rate through time and is never limited by food or disease, it has, what is known, as exponential growth. With exponential growth the birth rate alone controls how fast (or slow) the population grows.

Exponential Population Growth model

Exponential population growth occurs when a single species is not limited by other species (no predation, parasitism, competition), resources are not limited and environmental conditions are constant. In such conditions, population grows exponentially at constant percentage per time. Such a condition that permits exponential growth of a population is called an ecological vacuum. Ecological vacuum does not often occur in nature for a long period. In nature exponential population growth occurs commonly during a recovery of a population after a large scale disturbance (fire, epidemic, etc.).

The Logistic Population model

The differential equation of the logistic population model is denoted by

$$\dot{x} = rx\left(1 - \frac{x}{K}\right),$$

where $x(t)$ is the population size at time t, r denotes intrinsic growth rate, K is the carrying capacity and $\dot{x}(t)$ is the rate of change of the given population $x(t)$.

Harvesting in Population model

Let a population model is defined by the differential equation as follows:

$$\dot{x} = f(x).$$

Then if the population is removed or harvested in a specific rate $h(t)$ per unit time then the population model is changed to

$$\dot{x} = f(x) - h(t).$$

This differential equation is known as harvesting population model.

1. Constant-Yield Harvesting

If the harvesting rate $h(t)$ is constant i.e., $h(t) = H$, then the differential equation is changed to

$$\dot{x} = f(x) - H.$$

2. Constant-Effort Harvesting

If the harvesting rate $h(t)$ is a linear function of population size i.e., $h(t) = Ex(t)$, then the model can be expressed as:

$$\dot{x} = f(x) - Ex(t).$$

This type of harvesting is called proportional or constant-effort harvesting.

• Discrete population model

If any exponential population growth model follows the following criteria as discussed below may be considered as a discrete population growth model:

1. The populations have discrete breeding season.

2. They have overlapping generations or Non overlapping generation.

3. They have Semelparous life history or Iteroparous life history.

Discrete breeding season

Any population which is usually breed only at any specific time or any particular time of any year is known as discrete breeding population. So in regulative process, their breeding season have some delay. In the life cycle of these species where they lived for some number of years, they produce the new ones which are few young respectively in each year of time delay.

Overlapping generations or Non overlapping generation

In population of overlapping generation each generation lives for two periods like young and old age (two period life versions). At any time, period one generation of young coexists with one generation of elderly. At the beginning of the next period the elderly die off, the young themselves become elderly and a new generation of elderly is born. Thus there are two overlapping generations of people living at any one time. In non overlapping generation, every period a new generation arises and old one dies off. Generation precedes and follows each other but they do not overlap at any point.

Semelparous life history or Iteroparous life history

Semelparity and iteroparity refer to the reproductive strategy of an organism. A species is considered semelparous if it reproduces only a single time before it dies and Iteroparous if it can reproduce more than once in its lifetime. Some terminology are used in the research work.

1.2.4 Equilibrium point

If any dynamical system is stay in the same position that where it starts, then the point is called equilibrium point i.e., of a dynamical system equilibrium point is the solution of the system. So the equilibrium point is described the state of the dynamical system where it stays forever. Mathematically one can describe the system in a discrete time system as

$$x^*(n+1) = x^*(n)$$

where $n \in \mathbb{N}$ (set of natural numbers) and in a continuous time system as

$$\dot{x}^* = f(t, x^*) = 0$$

Let us consider a prey-predator two species model

$$\dot{x} = f(x, y)$$

$$\dot{y} = g(x, y)$$

In this system there are mainly four type equilibria exist:

- **Trivial equilibrium:** Trivial equilibrium is nothing but the solution at the origin i.e., (0,0). At this point there are neither prey population nor predator population. Then rate of change of any population is zero.
- **Prey free equilibrium:** Prey free population is the point $P(0, y^*)$ i.e., there does not exist any prey population but predator population has a non zero solution.
- **Predator free equilibrium:** Predator free equilibrium is the point $P(x^*, 0)$ i.e., there does not exist any predator population but prey population has a non zero solution.
- **Interior equilibrium:** Interior equilibrium is the point $P(x^*, y^*)$ i.e., both the populations have a non zero solution.

1.2.5 Stability

Any system is said to be stable if the final value is under control i.e., the solution is bounded if the initial values are bounded input values. If any system is not stable then it is called unstable. Stability are measured in two ways :

- **Local Stability:** Local stability of an equilibrium point is measured within a range of interval. i.e., if any value is taken within this interval then the system will converge to that equilibrium point.

- **Global Stability:** Global stability of an equilibrium point is measured within any range of the system. i.e., if any value is taken from anywhere of the system then the system will converge to that equilibrium point.

1.2.6 Bifurcation

In study of dynamical systems, if there is a small change in any parameter value which occurs a sudden qualitative change in the behavior of the system then the phenomenon is called a bifurcation and the parameter for which the change occurred is called bifurcation parameter. The bifurcation usually seen in both kind of systems (continuous and discrete). Henri Poincaré (79) first introduced the term 'bifurcation' in 1885. There exist different kind bifurcations out of which Hopf bifurcation is commonly used.

• Hopf bifurcation

An equilibrium point is called Hopf bifurcation where the stability of the system switches and arises to periodic solution. i.e., when there is a local change in the stability property causes the appear or disappear of a periodic solution.

• Migration

The word "Migration" means the movement of any population from one place to other place due to their live and food. Movement of species from their home to other area for some other reasons is called migration. Migration may be of different kinds depending on the species and their environment. For an example we can consider the species as fish. There are many kind of fishes who lived in different environment like pond, river, sea etc. In their life cycle some fishes moved one place to another due to reproduce or food habits. It is seen that they sometimes migrate in regular basis of time interval daily, monthly or yearly basis. Also sometimes it is seen that they migrated to a long distance. Depending on migration nature of the fishes they can be classified into different classes such as:

- **Anadromous fish:** Such fishes migrate from the sea to fresh water for their breeding, as for example Salmon.
- **Catadromous fish:** Such fishes migrate from fresh water to the sea for their breeding, as an example Eels.
- **Amphidromous fish:** Such fishes migrate from fresh water to the sea and sea to fresh water but not for breeding, as for example Torrentfish.
- **Potamodromous fish:** Such fishes migrate wholly within the fresh water, as for example Flathead catfish.
- **Oceanodromous fish:** Such fishes migrate wholly within the sea water, as for example Black grouper.

Like fishes different species use to migrate to one place to another place in seasonal basis in different ecological system. So migration is a very natural phenomenon in ecology.

On the other hand emigration is the process where the species leave their home for their more suitable home or suitable environment if their current habitat is not ideal to live. In this process they never back to that left place.

Therefore the main difference between migration and emigration is that in migration procedure the species come back to the previous place after a certain interval of time whereas in emigration the species does not come back to the same place after left the place.

1.2.7 Life cycle

Life cycle of a particular species is a process through which the species undergoes from a particular development stage to the same particular stage in the subsequential generation. As an example, the life cycle of frog has been shown in the Figure 1.1.

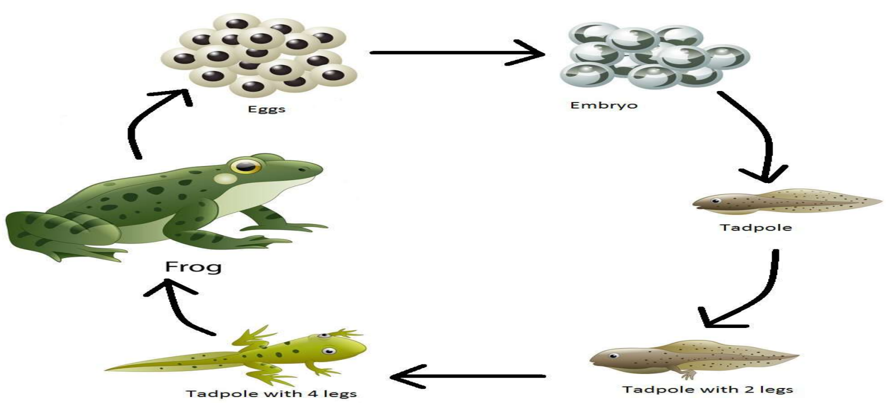

Figure 1.1: Life cycle of frog.

1.2.8 Reserve region

Reserve region indicates the natural reserve which is automatically reserved by the nature. It is sometimes known by different terminology like nature conservation area, bio reserve, wildlife refuge or refuge region, sanctuary. The reserve region for any species is the area where they are protected from predation. Nature reserved is sometimes designed and managed by government of any country or by some landowners or by some research institutions to keep the species safe, study and research. Natural reserved is maintained by local law and order. They come into different categories of IUCN and they are categorized by the level of protection. Depending on this category these regions have different official name in legislation.

1.2.9 Functional response

In ecology, functional response refers the intake capacity of a consumer which is a function of food density. There are mainly three types of functional response exists. They are Holling type I, Holling type II and Holling type III.

- **Holling type I:** When a functional response has a linear increasing in their intake rates with density of food then such kind of functional response is known as Holling type I functional response. Here it is assumed that the time is negligibly small for processing the food of any species or searching the food. This is the simplest functional response among three responses.

- **Holling type II:** When a functional response is distinguished by decelerating the intake rate then it is known as type II functional response. Here it is assumed that the consumer has a limited capacity for process food. Type II functional response is defined by the equation

$$f(x) = \frac{rx}{1 + rhx},$$

  where f denotes the intake rate and x denotes the density of the food. r is called the attack rate i.e., the rate at which the consumer encounters their food. h is the handling time i.e., the time spent to process the food. In this response, the mortality of prey population declines with increase in prey density. Predators in this type have maximum mortality rate when there is a low density of prey.

- **Holling type III:** Type III functional response usually seen in predator populations who increase their search activities when prey density increases. For example, some predators responds by the effect of kairomones (a chemical emitted by some prey) and causes the increase the activities of predator. In this response, mortality rate first increases with the increase of prey density then start to decline.

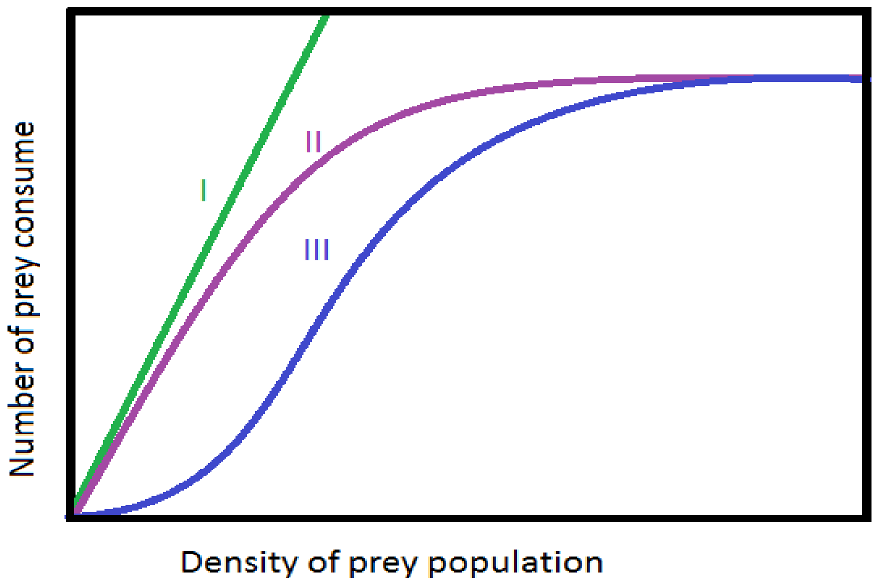

Figure 1.2: Representation of Holling Type I, II & III functional responses.

- **Time delay**

In population ecology, it is sometimes considered that the birth rate is instantaneous. But usually it is observed that there is a time gap between birth and their maturity or there is a gestation period. This time gap is known as time delay. Incorporating the time delay to the population model for some species give the realistic scenario. If T be the delay time then the differential equation for any population will be

$$\dot{x}(t) = f(x(t), x(t-T)).$$

1.2.10 Routh-Hurwitz conditions

In the interacting population models, the stability of the system is measured by solving the ordinary differential equation associated with the system i.e., by the roots of the characteristic polynomial. Let us consider the linear system of differential equations is

$$\frac{d\mathbf{X}}{dt} = \mathbf{AX}$$

where A denotes the matrix corresponding to linearized system. The solution of the system can be determined by putting

$$\mathbf{x} = \mathbf{x}_0 e^{\lambda t},$$

where $\mathbf{x}_0$ denotes the constant vector and λ is the eigen value of the characteristic polynomial

$$|A - \lambda I| = 0.$$

If all the roots of the characteristic polynomial is situated in the left hand side of the complex plane i.e., $re(\lambda) < 0$ then the solution $\mathbf{x} = 0$ is a stable solution. Let the characteristic polynomial is of the form

$$P(\lambda) = \lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + ... + a_n = 0$$

with $a_n \neq 0$ (otherwise $\lambda = 0$ will be a solution). Now our aim is to find the root λ such that $re(\lambda) < 0$. The necessary and sufficient conditions for this to hold are known as Routh-Hurwitz conditions. This can be represented in different form. One form can be described with $a_n > 0$ as follows:

$$D_1 = a_1 > 0, D_2 = \begin{vmatrix} a_1 & a_3 \\ 1 & a_2 \end{vmatrix} > 0, D_3 = \begin{vmatrix} a_1 & a_3 & a_5 \\ 1 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix} > 0,$$

$$D_k = \begin{vmatrix} a_1 & a_3 & a_5 & \dots & & \\ 1 & a_2 & a_4 & . & . & . \\ 0 & a_1 & a_3 & . & . & . \\ 0 & 1 & a_2 & . & . & . \\ . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & a_k \end{vmatrix} > 0.$$

1.3 Literature survey

An ecosystem is the interaction among the living thing which includes plants, organisms and animals. The interaction among these living organisms can be represented by some mathematical relationships. The mathematical representation of the ecosystem is the ecological model. In 1925, first ecological model was developed by Lotka (63). In this model a prey-predator relationship was discussed. Then after in 1926 Volterra (105) proposed a prey-predator model. These models were discussed by simply two ordinary differential equations. One of which represents the prey population and other represents the predator populations. In the last 90 years, various prey-predator models have been developed and rectified. Till now development and rectification of these models are going on and also burning topics. In 1934, Gause (32) studied different types of mathematical model in mathematical biology and ecology in his book "the struggle for existence" and again Gause *et al.* (33) discussed about futuristic studies of interaction between prey and predators in 1936. In 1956, Lotka (64) published a book on mathematical modelling on ecological system. In this book, different prey-predator mathematical models were discussed. Graphical representation with stability conditions of a prey-predator system has been studied by Rosenzweig and MacArthur (84) in 1963. In 1967, the interaction between different species in an island has been investigated by MacArthur and Wilson (66). Freedman studied different type prey-predator models. He studied deterministic mathematical models in population ecology (25) in 1980, after that he studied persistence in prey-predator systems with ratio-dependent predator influence (28) in 1984, global stability persistence of simple food chains (27) in 1985, persistence in models of three interacting prey-predator populations (26) in 1993. The book "Theory and Application of Hopf-Bifurcation" written by Hassard et al. (40) in 1981. In 1982, Birkhoff and Rota (9) wrote the book “Ordinary differential equations". Ives and Dobson (45) introduced a simple prey-predator population dynamics with anti-predator behavior. In 1989, Arditi and Ginzburg (3) studied ratio dependent coupling in prey-predator dynamics. Mathematical Biology was introduced by Murray (70). In 1992, Berryman (8) evaluated prey-predator theory. Abrams and Matsuda (1) discussed about effects of adaptive predator and anti-predator behaviour in 1993. Venkatsubramanian et al. (103) discussed about local bifurcation and also in this work, the feasibility regions in differential-algebraic systems has been discussed in 1995. Effects of toxic substance has been discussed by Chattopadhyay (13). In 1996, Li and Muldowney (58) described geometric approach to global stability problem. In 1996, Perko (78) published a book “Differential Equations and Dynamical System". Nonlinear dynamics for interacting populations were introduced by Bazykin and Khibnik (6) in 1998. In 2003, Gakkhar and Naji (30) discussed order and chaos in predator to prey. After that, Zhao (111) gave a concept about dynamical systems in population biology. In 2015, Roy and Roy (85) analyzed a prey-predator model considering different types of predators. After that, Kuang and Beretta (55) discussed global quantitative analysis of a prey-predator system which is ratio dependent. In 2015, Tang and Xiao (102) studied the impact of anti-predator behaviour with bifurcation analysis of the prey-predator dynamics. Jana and Roy (47) has been discussed Behavioural analysis of two prey-two predator model. In 2016, the effects of prey predation, competition with commensalism has been investigated by Gakkhar and Gupta (29).

In biology, growth rate of a population means the number of increasing of an individual associated in a population. In 1838, Verhust (104) formulated logistic growth rate. Pearl and Reed (76) studied again the growth rate of the population in 1920. In 1825, Gompertz (35) constructed a growth rate which is known as gompertz growth rate. Ayala *et al.* (4) and Nisbet (71) also studied different types of growth rate in their model.

In ecology, the intake rate of a consumer which is a function of food density is known as a functional response. Holling (41) gave a concept about functional response in 1959. A more functional response to prey-predator stability has been discussed by Levin (57) in 1977. In 1975, DeAngelis *et al.* (21) studied a different type of functional response. In 2003, Aziz-Alaoui and Okiye (5) discussed about global stability and boundedness of a prey-predator system with Holling type II functional response. The effects of Holling type I functional response on predator prey dynamics has been studied by Gunog and DeAngelis (37) in 2011. In 2015, Madhusudanan and Vijaya (67) studied the effects of Holling type-II functional response function on prey-predator dynamics. Morozov (69) reported the role of Holling type III functional response on predator prey interaction in 2010. A modified leslie-gower model with Beddington-DeAngelis functional response studied by Yu (109) in 2014. Holling-Tanner prey-predator model with Beddington-DeAngelis functional response including delay has been analyzed by Jana and Roy (46). In 2017, Roy et al. (86) studied Holling-Tanner model with Beddington-DeAngelis functional response.

A natural reserve or bio-reserve is a protected area which is important for a ecosystem. For this reservation system, all species cannot enter that reserve spaces. Predation process does not work properly. Due to the prey refuge behavior, predator population cannot consume always sufficient amount of food for their growth. In 1987, the impact of prey refuge on the stability of a predator-prey system has been studied by Sih (97). The effects of use of refuge in a short time duration has been reported by Ruxton (91) in 1995. Effects of immigration has been discussed by Stone and Hart (101) in 1999. In 2006, the impact of prey refuge on a prey-predator system with Holling type III functional response was introduced by Huang et al. (42). Liu and Han

(62) studied the influence of prey refuge on a diffusive prey-predator system in 2011. Global stability and bifurcation analysis of prey-predator system in the presence of prey refuge was explored by Jana et al. (48) in 2012. The effect of prey refuge on a competitive prey-predator system has been investigated by Sarwardi et al. (93) in 2012. In 2013, Devi (23) reported effects of prey refuge on a ratio-dependent predator-prey model. Prey-predator model in drainage system with migration and harvesting has been discussed by Roy and Roy (89). The impact of additional food on pest management and biological controls of predators have been reported by Srinivasu et al. [(98), (94)]. Krivan [(55), (54)] studied effects of optimal anti predator behavior of prey and the history of the gause prey-predator model in a reserve region. The role of supplying additional food on a exploited predator-prey system has been investigated by Kar et al. (52) in 2012. In 2012, by supplying additional food, control of chaos in a prey-predator system has been studied by Gakkhar and Singh (31). The effects of additional food to the dynamics of predator-prey system has been reported with mutually interfering predators by Prasad et al. (81) in 2013. In 2014, the role of additional food to control of chaos and its effects on time delay in a epidemiological model have been investigated by Sahoo and Poria (92). Sen et al. (96) studied the impact of supplying additional food in a predator prey model with harvesting in 2015. In 2016, Roy and Roy (88) analyzed a prey-predator fishery model with harvesting in a reserve zone.

Gestation is an important factor in any population, so many researchers have shown their interest to study time delay. Gopalsamy (36) wrote a book about stability and oscillations in delay differential equations of population dynamics in 1992. In 2007, Sakera and Alzabutb (95) discussed about existence of periodic solutions, global attractivity and oscillation of impulsive delay population. After that Pal and Mandal (73) also studied about time delay in the formulated model.

Depending on time in a life cycle, feeding capacities of species are changed. Sometimes, resources of food are also different on different stages of species' life cycle. For this reason, prey of predator is also changed based on life cycle of predator. In different stages of life cycle of an organism has different food habits. Mathematical representation for single species stage structure model was developed by Aiello and Freedman (2) in 1990. Also Wang *et al.* (106) discussed performance and stability of a stage-structured predator-prey model. After that, many researchers showed their interest ((20), (61), (87)) on it. Kent and Kumar (49) discussed dynamical behavior of a stage structured prey-predator model. Cai and Song (11) described stage structure for predator.

Harvesting is a process to catch or specially for food. Enlargement of harvesting is harmful for the species's existence in an ecological system. In 1962, Pontryagin et al. (80) studied the mathematical theory of optimal process. In 1983, effects of harvesting on an ecologically interdependent fish species have been reported by Hannesson (38). The harvesting policies and non-market valuation of a prey-predator system have been defined by Ragozin and Brown (83) in 1985. Then the bio-economic modelling, fishery management and also the optimal management of renewable resources have been studied by Clark [(17),(18)]. Role of bio-economics of sustainable harvest between competing species has been discussed by Flaaten (24) in 1991. In 1998, the effects of harvesting on the global dynamics of prey-predator system have been investigated by Dai and Tang (19). In 2004, Purohit and Chaudhuri (82) designed a bioeconomic model of non-selective harvesting of two competing fish species. Xiao and Jenning (107) discussed bifurcation of a system with constant rate of harvesting. After that, many researchers [(51),(108),(16)] analyzed optimality of different fishery models. Peng et al. (77) studied the impact of harvesting on prey-predator dynamics in 2009. Das et al. (20) studied harvesting of a fishery in presence of toxicity. Chakrborty et al. (12) discussed about global dynamics and bifurcation in a stage structured fishery model with harvesting. In 2014, Ghosh and Kar (34) discussed about sustainable yield. In 2015, Pal et al. (72) reported the effects of predator harvesting in a fuzzy parameter based predator model.

Plankton are multiple group of organisms that live in the water column of large bodies of water on sea, lake, river etc. Phytoplankton are the primary food source for the living animal of aquatic ecosystem. Phytoplankton are directly consumed by zooplankton. So, the study about the dynamics of phytoplankton and zooplankton is important since these are the primary food source for all the living organisms in the earth. In 1961, the paradox of the plankton has been studied by Hutchison (43). Steel and Henderson (99) studied a simple plankton model in 1981. After that in 1992, the impact of predation in plankton system has been reported by Steel and Henderson (100). The food web stability and the influence of trophic flow on the prey-predator system has been investigated by Huxel and MaCann (44). In 2002, Chattopadhyay and Pal [(14),(15)] studied the impact of viral infection on the zooplankton-phytoplankton system through mathematical model and dynamics of nutrient -phytoplankton interaction in presence of viral infection. Pal et al. [(74),(75)] investigated the impact of toxin for the occurrence and termination of planktonic bloom and the effects of competition for the occurrence and control of planktonic bloom. Some deterministic mathematical models on population ecology.

1.4 Objective and Scope of the Thesis

The main objective of this thesis is to develop some prey-predator relationships in view of real world situations. From the literature survey it is observed that many researchers have developed different prey-predator relationships from 1925, still there are some gaps in their formulated models. The main aim of this thesis is to fill up the gaps by introducing some realistic models and analyzing the solutions based on hypothetical and real-life data.

The objectives of the present work are as follows:

1: To study on prey-predator three species model depending on different groups of organism, different types of functional response are exemplified.

2: The effect of two predators is investigated which is also related with prey-predator relationship.

3: To study a prey-predator relationship where prey lives in refuse region and predatory region. Migration as well as emigration between two regions are studied.

4: To exemplify a reserve region of predator where generalist predator cannot enter. Harvesting of generalist predator is studied.

5: Gestation time delay of predator and the dynamic stability of time delay preventing system are studied.

6: Stage structure of predator is studied. Different preys are exemplified in different stages of predator.

1.5 Organization of the Thesis

In the proposed thesis, some real-life ecological problem has been considered and briefly discussed about it. The whole thesis contains eight chapters. A brief introduction related to the proposed research work is presented to Chapter-1. In Chapter-2, prey-predator three species model with vertebral and invertebral predators is analyzed. The Chapter-3 is discussed about effects on prey-predator with different functional responses. In Chapter-4, prey-predator three species fishery model with harvesting including prey refuge and migration is described. The Chapter-5 is depicted about a prey-predator model with migration and harvesting in a reserve region of predator. Chapter-6 is introduced the concept of time delay in Holling-Tanner model with Beddington-DeAngelis functional response and time delay introducing harvesting. In Chapter-7, the concept of stage structure population model with different prey is provided. In the last Chapter, the conclusions and scope of future works are presented regarding our research work.

The chapter wise summary of the proposed research works is given below:

Chapter 1 introduces the study of mathematical biology. A brief survey on prey-predator relationship, prey-predator model, continuous population model, discrete population model is furnished. We discuss in short exponential population growth model, logistic population model, harvesting. Finally, the organization of the thesis is presented.

Chapter 2

Analysis of prey-predator three species models with vertebral and invertebral predators

In this chapter, a mathematical model has been considered involving three species namely prey, predator and generalist predator. Different types of functional responses have been considered to formulate the mathematical model for predator and generalist predator. Main intention of this study is to establish the local and global stabilities for the proposed model around its interior equilibrium point. A numerical example is considered to illustrate the proposed system of our paper. The stability of the system has been analyzed using some graphical representations.

A part of this chapter has been appeared in ***International Journal of Dynamics and Control***, Springer, SCOPUS, 3(3), 306-312, (2015).

Chapter 3

Effects on prey-predator with different functional responses

In this chapter, the effects on prey of two predators which are also related in terms of prey-predator relationship has been investigated. Different types of functional responses are considered to formulate the mathematical model for predator and generalist predator of our proposed model. Harvesting effort for the generalist predator is considered and the density dependent mortality rate for predator and generalist predator are incorporated in our proposed model. Local stability as well as global stability for the system are discussed. The different bifurcation parameters to evaluate Hopf bifurcation in the neighborhood of interior equilibrium point are analyzed. Finally, some numerical simulations and graphical figures are provided to verify our analytical results with the help of different sets of parameters.

A part of this chapter has been appeared in ***International Journal of Biomathematics***, World Scientific, **IF: 1.05**, 10(8), 1750113 (22 pages), (2017).

Chapter 4

Analysis of prey-predator three species fishery model with harvesting including prey refuge and migration

In this chapter, a prey-predator system with Holling-type II functional response for the predator population including prey refuge region has been analyzed. Also a harvesting effort has been considered for the predator population. The density-dependent mortality rate for the prey, predator and generalist predator has been considered. The equilibria of the proposed system have been determined. Local and global stabilities for the system have been discussed. The analytic approach to derive the global asymptotic stabilities of the system is used. The maximal predator per capita consumption rate has been considered as a bifurcation parameter to evaluate Hopf bifurcation in the neighborhood of interior equilibrium point. Also, the fishing effort to harvest predator population of the system as a control to develop a dynamic framework to investigate the optimal utilization of the resource, sustainability properties of the stock and the resource rent is earned from the resource are described. Finally, some numerical simulations to verify the analytic results are presented and the system has been analyzed through graphical illustrations.

A part of this chapter has been appeared in ***International journal of bifurcation and chaos***, World Scientific, SCI, **IF: 1.329**, 26(2), 1650022 (19 pages), (2016)

Chapter 5

Prey-predator model with migration and harvesting

In this chapter, a prey-predator model with a reserve region of predator where generalist predator cannot enter is considered. The predator population which consumes the prey population with Holling type-II functional response and generalist predator population consume the predator population with Beddington-DeAngelis functional response are introduced. The density-dependent mortality rate for prey and generalist predator are considered. The equilibria of proposed system are determined. Local stability for the system is discussed. The environmental carrying capacity is considered as a bifurcation parameter to evaluate Hopf bifurcation in the neighborhood of interior equilibrium point. Here the fishing effort is used as a control parameter to harvest the predator population of the system. With the help of this control parameter, a dynamic

framework is developed to investigate the optimal utilization of resources, sustainability properties of the stock and the resource rent. Finally, a numerical simulation to verify the analytical results is described and the system is analyzed through graphical illustrations.
A part of this chapter has been communicated to the International Journal.

Chapter 6
Holling-Tanner model with Beddington-DeAngelis functional response and time delay introducing harvesting

The chapter is designed with the Holling-Tanner prey-predator model with Beddington-DeAngelis functional response including prey harvesting. Gestational time delay of predator and the dynamic stability of time delay preventing system are incorporated into the system of this chapter. The equilibria of the proposed system are determined and the existence of interior equilibrium point for the proposed system is described. Local stability of the system with the magnitude of time delay at the interior equilibrium point is discussed. Thereafter, the direction and the stability of Hopf bifurcation are established with the help of normal theory and center manifold theorem. Furthermore, profit function is calculated with the help of bionomic equilibrium and it is optimized using optimal control. Finally, some numerical simulations are introduced to verify the validity of analytic results of our proposed model.
A part of this chapter has been appeared in ***Mathematics and Computers in Simulation***, Elsevier, **IF: 1.218**, 142, 1-14, (2017).

Chapter 7
Stage structure population model in different prey

A prey-predator system with stage structure of predator has been considered. In the proposed model, prey of immature predator and that of mature predator are different. Consumption rate of prey by the immature predator has been described by Holling type II functional response and consumption rate of prey by the mature predator has been described by Holling type III functional response. Both prey obey logistic growth rate. Immature predator transfers to mature predator in a constant rate. Mortality rate of immature predator and mature predator are different. Local stability of the system has been discussed. Transform rate of immature predator to mature predator is considered as bifurcation parameter. Finally, some numerical simulations to verify the analytic results are depicted and the system has been analyzed through graphical illustrations.
A part of this chapter has been communicated to the International Journal.

Finally, the concluding remarks on the work carried out in Chapters 2 to 7 are described in **Chapter 8**. Future scope of further research works on the presented topic is also discussed.