Chapter 7

Multi-objective green solid transportation-location problem with dwell time under two-fold uncertainty[1]

This chapter introduces an unprecedented integrated mathematical model for a green solid transportation system with dwell time to execute the carbon tax, cap and offset regulation. Due to market fluctuations, the supply and demand parameters are not always of a crisp nature. Hence, a two-fold (type-2 intuitionistic) uncertainty is incorporated in this study to provide a realistic transportation system. A new ranking defuzzification technique is presented for conversion into a deterministic form. After that, a fuzzy technique and a non-fuzzy technique are used to get a Pareto-optimal solution of the proposed problem. The performances of our findings are discussed with industrial-based application examples. Moreover, a comparative study with particular cases is explored among the other existing techniques. Managerial insights and conclusions are offered at the end of this study.

7.1 Introduction

We are increasingly witnessing visit outrageous climate incidents because of global warming. There is a dire requirement for governments, enterprises, the overall population, and academics to take facilitated activities so as to handle the difficulties forced by environmental change. The most important strategic issue is to design an effective and environmentally concerned logistics system as transportation is one of the fundamental reason for carbon emanations. To reduce carbon discharge, governments and other policymakers endorse a couple of strategies, wherein the carbon emission tax, cap and offset policy is commonly accepted. The motivation of this study is to design a strategic green transportation network to reduce CO_2 emission in the atmosphere. FLP and STP are the key factors of logistics network system. Deciding about the optimal locations for the facilities such as retailer-outlets,

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plants, terminals, workplaces, fire stations, railroad stations, etc., and optimizing the overall logistics cost, customer service level and environmental concerns by different transportation modes can significantly affect the management system. Therefore, the significance of the integrated model helps an organization to increase efficiency and decrease the number of unnatural incidents. Here, an unprecedented mathematical model is introduced by incorporating FLP and MOSTP under two-fold (type-2 intuitionistic) uncertainty. Consequently, the stated model is referred as *multi-objective green solid transportation-location problem* (MOGST-LP). The aims of the stated formulation are multi-fold: *(i)* seek the optimum locations and budgets for potential facilities in the Euclidean plane, *(ii)* find the amount of distributed commodities, and *(iii)* optimize three conflicting objectives such as financial costs, customers' satisfaction and carbon emission simultaneously. In the proposed study, variable carbon emission is considered due to the variable locations of facilities and also the flows of conveyed goods. It is believed that the designed transportation model will be more relevant for recent environmental concerns. Nowadays, the parameters of MOGST-LP are vague because of insufficient information from the DM. For that reason, the proposed problem is difficult to handle by conventional solution procedures. Therefore, here, a novel 4-dimensional type-2 trapezoidal fuzzy number with the degree of hesitation is introduced to overcome the uncertainties in a green solid transportation system. For more explanations of the type-2 fuzzy environment, one may see to the Chapter 1.

The major contributions of this chapter may be listed as:

A1. An unprecedented nonlinear mathematical formulation based on FLP and multi-objective solid green logistics modeling under a carbon tax, cap and offset policy is presented.

A2. The formulation provides a decision regarding the assignment from numerous existing sites to several potential sites in the Euclidean plane with a distance function.

A3. The overall logistics cost including maintaining and fixed charge cost, transportation time with dwell, loading and unloading time, and carbon emission cost by different modes of transportation are also considered.

A4. Variable budget constraints are incorporated to find the optimal budgets of the potential facilities, a novel contribution in this direction.

A5. A new form of trapezoidal type-2 fuzzy number is introduced to handle the uncertainties, which is defuzzified by a proposed ranking function.

A6. A fuzzy technique and a non-fuzzy technique are described to achieve the best Pareto-optimal solution of MOGST-LP.

7.2 Preliminaries

Herein, some preliminaries about the type-2 fuzzy set, IFS and *intuitionistic fuzzy number* (IFN) are presented subsequently. Type-1 fuzzy set or the classical fuzzy set was the beginning of fuzzy logic. Thereafter, Zadeh [170] gave the idea of a *type-2 fuzzy set* (T2FS).

Definition 7.1 ***Type-2 fuzzy set**: Let us consider that $\tilde{F}[0,1]$ be the universe of all fuzzy sets in $[0,1]$. A T2FS $\tilde{\tilde{E}}$ in the universe X is characterized by a function $\mu_{\tilde{\tilde{E}}} : X \to \tilde{F}[0,1]$. Therefore, $\tilde{\tilde{E}}$ can be represented as $\tilde{\tilde{E}} = \left\{ \left(x, \mu_{\tilde{\tilde{E}}}(x)\right) : \forall\, x \in X, \forall \mu_{\tilde{\tilde{E}}}(x) \in \tilde{F}[0,1] \right\}$. According to Mendel and John [106], $\tilde{\tilde{E}}$ can be expressed as $\tilde{\tilde{E}} = \left\{ \left((x,u), \mu_{\tilde{\tilde{E}}}(x,u)\right) : \forall\, x \in X, \forall\, u \in J_x \subseteq [0,1], 0 < \mu_{\tilde{\tilde{E}}}(x,u) < 1 \right\}$, where x,u are the primary and secondary variables, respectively, and $J_x, \mu_{\tilde{\tilde{E}}}(x,u)$ are known as primary and secondary membership grades.*

Definition 7.2 ***Intuitionistic fuzzy set** (Atanassov [8]): Let X be a universal set and $x \in X$. An IFS $\hat{C}$ in X is described by a set of ordered triplet as the following form $\hat{C} = \{\langle x, \mu_{\hat{C}}(x), \gamma_{\hat{C}}(x)\rangle : x \in X\}$, where the functions $\mu_{\hat{C}}(x) : X \to [0,1]$ and $\gamma_{\hat{C}}(x) : X \to [0,1]$ denote the degree of membership and non-membership, respectively, such that $0 \le \mu_{\hat{C}}(x) + \gamma_{\hat{C}}(x) \le 1,\ \forall\, x \in X$. Furthermore, $1 - \mu_{\hat{C}}(x) - \gamma_{\hat{C}}(x)$ represents the* degree of hesitation of x in $\hat{C}$.

Definition 7.3 ***Trapezoidal intuitionistic fuzzy number (TIFN)** (Li [89]): A trapezoidal type-1 IFN, or simply the TIFN, $\hat{D}$ is denoted as $\hat{D} = \langle(\zeta_1,\zeta_2,\zeta_3,\zeta_4); \mu_{\hat{D}}, \gamma_{\hat{D}}\rangle$, with the membership and non-membership functions as given below:*

$$\mu_{\hat{D}}(x) = \begin{cases} \alpha_{\hat{D}}\left(\dfrac{x-\zeta_1}{\zeta_2-\zeta_1}\right), & \text{if } \zeta_1 \le x < \zeta_2, \\ \alpha_{\hat{D}}, & \text{if } \zeta_2 \le x < \zeta_3, \\ \alpha_{\hat{D}}\left(\dfrac{\zeta_4-x}{\zeta_4-\zeta_3}\right), & \text{if } \zeta_3 \le x < \zeta_4, \\ 0, & \text{if } x < \zeta_1 \text{ or } x > \zeta_4, \end{cases}, \quad \gamma_{\hat{D}}(x) = \begin{cases} \dfrac{\zeta_2 - x + \beta_{\hat{D}}(x-\zeta_1)}{\zeta_2-\zeta_1}, & \text{if } \zeta_1 \le x < \zeta_2, \\ \beta_{\hat{D}}, & \text{if } \zeta_2 \le x < \zeta_3, \\ \dfrac{x-\zeta_3+\beta_{\hat{D}}(\zeta_4-x)}{\zeta_4-\zeta_3}, & \text{if } \zeta_3 \le x < \zeta_4, \\ 1, & \text{if } x < \zeta_1 \text{ or } x > \zeta_4. \end{cases}$$

Here, $\alpha_{\hat{D}}$ and $\beta_{\hat{D}}$ are the degree of membership and non-membership such that $\alpha_{\hat{D}}, \beta_{\hat{D}} \in [0,1]$ and $0 \le \alpha_{\hat{D}} + \beta_{\hat{D}} \le 1$.

7.2.1 Trapezoidal type-2 intuitionistic fuzzy number (TT2IFN)

Here, we present a two-fold uncertainty environment based on the concepts of TIFN and type-2 fuzzy set. Thereafter, its definition and arithmetic operations are also addressed. The choice of two-fold uncertainty plays a vital role in the scenario of the proposed problem due to the attributes of trapezoidal fuzzy numbers. The stated fuzzy number can handle both symmetric and asymmetric uncertainties, is more acceptable to formulate a real-life problem.

Definition 7.4 ***Trapezoidal type-2 intuitionistic fuzzy number (TT2IFN)**:*
A TT2IFN $\tilde{\hat{A}}$ in X is the following form:

$$\tilde{\hat{A}} = \langle(\hat{A}_1, \hat{A}_2, \hat{A}_3, \hat{A}_4); \omega_{1\tilde{\hat{A}}}, \omega_{2\tilde{\hat{A}}}\rangle, \tag{7.1}$$

where $\hat{A}_1$, $\hat{A}_2$, $\hat{A}_3$ *and* $\hat{A}_4$ *are also TIFNs, and* $\omega_{1\tilde{\tilde{A}}}$ *and* $\omega_{2\tilde{\tilde{A}}}$ *denote the membership and non-membership degree of* $\tilde{\tilde{A}}$*, respectively. Thus, (7.1) can be expressed as*

$$\begin{aligned}\tilde{\tilde{A}} &= \langle(\hat{A}_1,\hat{A}_2,\hat{A}_3,\hat{A}_4);\omega_{1\tilde{\tilde{A}}},\omega_{2\tilde{\tilde{A}}}\rangle,\\ &= \langle(\langle(a_{11},a_{12},a_{13},a_{14});\mu_{\hat{A}_1},\gamma_{\hat{A}_1}\rangle,\langle(a_{21},a_{22},a_{23},a_{24});\mu_{\hat{A}_2},\gamma_{\hat{A}_2}\rangle,\\ &\quad \langle(a_{31},a_{32},a_{33},a_{34});\mu_{\hat{A}_3},\gamma_{\hat{A}_3}\rangle,\langle(a_{41},a_{42},a_{43},a_{44});\mu_{\hat{A}_4},\gamma_{\hat{A}_4}\rangle);\omega_{1\tilde{\tilde{A}}},\omega_{2\tilde{\tilde{A}}}\rangle,\end{aligned}$$

where $\omega_{1\tilde{\tilde{A}}} = \min\{\mu_{\hat{A}_1},\mu_{\hat{A}_2},\mu_{\hat{A}_3},\mu_{\hat{A}_4}\}$, $\omega_{2\tilde{\tilde{A}}} = \max\{\gamma_{\hat{A}_1},\gamma_{\hat{A}_2},\gamma_{\hat{A}_3},\gamma_{\hat{A}_4}\}$.

Arithmetic Operations on TT2IFNs:

Let us consider that

$$\begin{aligned}\tilde{\tilde{A}} &= \langle(\langle(a_{11},a_{12},a_{13},a_{14});\mu_{\hat{A}_1},\gamma_{\hat{A}_1}\rangle,\langle(a_{21},a_{22},a_{23},a_{24});\mu_{\hat{A}_2},\gamma_{\hat{A}_2}\rangle,\\ &\quad \langle(a_{31},a_{32},a_{33},a_{34});\mu_{\hat{A}_3},\gamma_{\hat{A}_3}\rangle,\langle(a_{41},a_{42},a_{43},a_{44});\mu_{\hat{A}_4},\gamma_{\hat{A}_4}\rangle);\omega_{1\tilde{\tilde{A}}},\omega_{2\tilde{\tilde{A}}}\rangle\\ \text{and } \tilde{\tilde{B}} &= \langle(\langle(b_{11},b_{12},b_{13},b_{14});\mu_{\hat{B}_1},\gamma_{\hat{B}_1}\rangle,\langle(b_{21},b_{22},b_{23},b_{24});\mu_{\hat{B}_2},\gamma_{\hat{B}_2}\rangle,\\ &\quad \langle(b_{31},b_{32},b_{33},b_{34});\mu_{\hat{B}_3},\gamma_{\hat{B}_3}\rangle,\langle(b_{41},b_{42},b_{43},b_{44});\mu_{\hat{B}_4},\gamma_{\hat{B}_4}\rangle);\omega_{1\tilde{\tilde{B}}},\omega_{2\tilde{\tilde{B}}}\rangle\end{aligned}$$

be two TT2IFNs, and ρ be any real number. Then, the arithmetic operations (i.e., addition, subtraction and scalar multiplication) of these two numbers are as follows:

Addition:

$$\begin{aligned}\tilde{\tilde{A}}\oplus\tilde{\tilde{B}} = \langle(&\langle(a_{11}+b_{11},a_{12}+b_{12},a_{13}+b_{13},a_{14}+b_{14});\mu_{\hat{A}_1}\wedge\mu_{\hat{B}_1},\gamma_{\hat{A}_1}\vee\gamma_{\hat{B}_1}\rangle,\\ &\langle(a_{21}+b_{21},a_{22}+b_{22},a_{23}+b_{23},a_{24}+b_{24});\mu_{\hat{A}_2}\wedge\mu_{\hat{B}_2},\gamma_{\hat{A}_2}\vee\gamma_{\hat{B}_2}\rangle,\\ &\langle(a_{31}+b_{31},a_{32}+b_{32},a_{33}+b_{33},a_{34}+b_{34});\mu_{\hat{A}_3}\wedge\mu_{\hat{B}_3},\gamma_{\hat{A}_3}\vee\gamma_{\hat{B}_3}\rangle,\\ &\langle(a_{41}+b_{41},a_{42}+b_{42},a_{43}+b_{43},a_{44}+b_{44});\mu_{\hat{A}_4}\wedge\mu_{\hat{B}_4},\gamma_{\hat{A}_4}\vee\gamma_{\hat{B}_4}\rangle);\omega_{1(\tilde{\tilde{A}}\oplus\tilde{\tilde{B}})},\omega_{2(\tilde{\tilde{A}}\oplus\tilde{\tilde{B}})}\rangle,\end{aligned}$$

where, $\omega_{1(\tilde{\tilde{A}}\oplus\tilde{\tilde{B}})} = \min\{\mu_{\hat{A}_1}\wedge\mu_{\hat{B}_1},\mu_{\hat{A}_2}\wedge\mu_{\hat{B}_2},\mu_{\hat{A}_3}\wedge\mu_{\hat{B}_3},\mu_{\hat{A}_4}\wedge\mu_{\hat{B}_4}\}$, $\omega_{2(\tilde{\tilde{A}}\oplus\tilde{\tilde{B}})} = \max\{\gamma_{\hat{A}_1}\vee\gamma_{\hat{B}_1},\gamma_{\hat{A}_2}\vee\gamma_{\hat{B}_2},\gamma_{\hat{A}_3}\vee\gamma_{\hat{B}_3},\gamma_{\hat{A}_4}\vee\gamma_{\hat{B}_4}\}$.

Subtraction:

$$\begin{aligned}\tilde{\tilde{A}}\ominus\tilde{\tilde{B}} = \langle(&\langle(a_{11}-b_{44},a_{12}-b_{43},a_{13}-b_{42},a_{14}-b_{41});\mu_{\hat{A}_1}\wedge\mu_{\hat{B}_4},\gamma_{\hat{A}_1}\vee\gamma_{\hat{B}_4}\rangle,\\ &\langle(a_{21}-b_{34},a_{22}-b_{33},a_{23}-b_{32},a_{24}-b_{31});\mu_{\hat{A}_2}\wedge\mu_{\hat{B}_3},\gamma_{\hat{A}_2}\vee\gamma_{\hat{B}_3}\rangle,\\ &\langle(a_{31}-b_{24},a_{32}-b_{23},a_{33}-b_{22},a_{34}-b_{21});\mu_{\hat{A}_3}\wedge\mu_{\hat{B}_2},\gamma_{\hat{A}_3}\vee\gamma_{\hat{B}_2}\rangle,\\ &\langle(a_{41}-b_{14},a_{42}-b_{13},a_{43}-b_{12},a_{44}-b_{11});\mu_{\hat{A}_4}\wedge\mu_{\hat{B}_1},\gamma_{\hat{A}_4}\vee\gamma_{\hat{B}_1}\rangle);\omega_{1(\tilde{\tilde{A}}\ominus\tilde{\tilde{B}})},\omega_{2(\tilde{\tilde{A}}\ominus\tilde{\tilde{B}})}\rangle,\end{aligned}$$

where, $\omega_{1(\tilde{\tilde{A}}\ominus\tilde{\tilde{B}})} = \min\{\mu_{\hat{A}_1}\wedge\mu_{\hat{B}_4},\mu_{\hat{A}_2}\wedge\mu_{\hat{B}_3},\mu_{\hat{A}_3}\wedge\mu_{\hat{B}_2},\mu_{\hat{A}_4}\wedge\mu_{\hat{B}_1}\}$, $\omega_{2(\tilde{\tilde{A}}\ominus\tilde{\tilde{B}})} = \max\{\gamma_{\hat{A}_1}\vee\gamma_{\hat{B}_4},\gamma_{\hat{A}_2}\vee\gamma_{\hat{B}_3},\gamma_{\hat{A}_3}\vee\gamma_{\hat{B}_2},\gamma_{\hat{A}_4}\vee\gamma_{\hat{B}_1}\}$.

Multiplication with Scalar:

$$\rho\tilde{\tilde{A}} = \begin{cases}\langle(\langle(\rho a_{11},\rho a_{12},\rho a_{13},\rho a_{14});\mu_{\hat{A}_1},\gamma_{\hat{A}_1}\rangle,\langle(\rho a_{21},\rho a_{22},\rho a_{23},\rho a_{24});\mu_{\hat{A}_2},\gamma_{\hat{A}_2}\rangle,\\ \langle(\rho a_{31},\rho a_{32},\rho a_{33},\rho a_{34});\mu_{\hat{A}_3},\gamma_{\hat{A}_3}\rangle,\langle(\rho a_{41},\rho a_{42},\rho a_{43},\rho a_{44});\mu_{\hat{A}_4},\gamma_{\hat{A}_4}\rangle);\omega_{1\tilde{\tilde{A}}},\omega_{2\tilde{\tilde{A}}}\rangle,\\ \text{if } \rho\geq 0,\\ \langle(\langle(\rho a_{41},\rho a_{42},\rho a_{43},\rho a_{44});\mu_{\hat{A}_4},\gamma_{\hat{A}_4}\rangle,\langle(\rho a_{31},\rho a_{32},\rho a_{33},\rho a_{34});\mu_{\hat{A}_3},\gamma_{\hat{A}_3}\rangle,\\ \langle(\rho a_{21},\rho a_{22},\rho a_{23},\rho a_{24});\mu_{\hat{A}_2},\gamma_{\hat{A}_2}\rangle,\langle(\rho a_{11},\rho a_{12},\rho a_{13},\rho a_{14});\mu_{\hat{A}_1},\gamma_{\hat{A}_1}\rangle);\omega_{1\tilde{\tilde{A}}},\omega_{2\tilde{\tilde{A}}}\rangle,\\ \text{if } \rho<0.\end{cases}$$

7.2.2 Proposed defuzzification technique

Defuzzification of fuzzy number plays a significant role to overcome the uncertainties in real-life applications. In fact, several techniques such as α-cut, linguistic approach, critical value (CV) based reduction, integration method, etc., exist for defuzzification in the literature. In the study (Roy and Bhaumik [133]), an efficient defuzzification technique is presented to extract a triangular fuzzy number by addressing a ranking function, and it is applied on a water management problem. Motivated through this technique, we introduce a new ranking function for conversion of TT2IFNs into crisp number. The ranking function maps each TT2IFN into real line, i.e., $\mathfrak{R} : \mathbb{F}(\tilde{\tilde{A}}) \to \mathbb{R}$, where $\mathbb{F}(\tilde{\tilde{A}})$ is a set of TT2IFNs. Therefore, the ranking function for TT2IFN is mathematically defined as follows:

$$\begin{aligned} \mathfrak{R}(\tilde{\tilde{A}}) &= \left(\frac{\omega_{1\tilde{\tilde{A}}}+\omega_{2\tilde{\tilde{A}}}}{2}\right)\left(\frac{1}{4}\right)\left(\frac{a_{11}+a_{21}+a_{31}+a_{41}}{4}+\frac{a_{12}+a_{22}+a_{32}+a_{42}}{4}\right. \\ &\quad \left. +\frac{a_{13}+a_{23}+a_{33}+a_{43}}{4}+\frac{a_{14}+a_{24}+a_{34}+a_{44}}{4}\right), \end{aligned} \tag{7.2}$$

where $\tilde{\tilde{A}}$ is given in (7.1). Let $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ are two TT2IFNs. Then,

1. $\mathfrak{R}(\tilde{\tilde{A}}) > \mathfrak{R}(\tilde{\tilde{B}}) \Rightarrow \tilde{\tilde{A}} >_{\mathfrak{R}} \tilde{\tilde{B}}$, i.e., $\min\{\tilde{\tilde{A}}, \tilde{\tilde{B}}\} = \tilde{\tilde{B}}$,
2. $\mathfrak{R}(\tilde{\tilde{A}}) < \mathfrak{R}(\tilde{\tilde{B}}) \Rightarrow \tilde{\tilde{A}} <_{\mathfrak{R}} \tilde{\tilde{B}}$, i.e., $\min\{\tilde{\tilde{A}}, \tilde{\tilde{B}}\} = \tilde{\tilde{A}}$,
3. $\mathfrak{R}(\tilde{\tilde{A}}) = \mathfrak{R}(\tilde{\tilde{B}}) \Rightarrow \tilde{\tilde{A}} =_{\mathfrak{R}} \tilde{\tilde{B}}$, i.e., $\min\{\tilde{\tilde{A}}, \tilde{\tilde{B}}\} = \tilde{\tilde{A}}$ or $\tilde{\tilde{B}}$.

7.3 Mathematical identification

In this section, we initially delineate the stated problem, that is, the green multi-objective solid transportation-location problem with dwell time under two-fold uncertainty. In this regard, we incorporate the notations and state the assumptions to formulate the mathematical model.

7.3.1 Background

Herein, an unprecedented strategic formulation is investigated from an environmental, economical and customers' satisfaction frame of reference. This study deals with a solid green logistics framework, which comprises of multiple suppliers treated as existing facilities, destinations or demand points addressed as potential facilities, and commodities are distributed from some suppliers to certain demand points via different transportation modes. The important goals are: *(G1)* reduce the overall conveyance cost, time, and carbon emission cost under a carbon reduction policy, and *(G2)* find the optimal locations and budgets for potential sites simultaneously. Apart from the aforementioned objectives, the following postures are also taken into consideration: *(A)* the weights of the conveyances are based on fuel consumption, considered in the logistics cost and carbon emission cost, *(B)* processing charge, toll charges, packaging charges, safety expenses and so on, designated as fixed-charge cost *(C)* maintenance costs of the vehicles which depend on the distance of the path, *(D)* there might be a few barriers (e.g., bridge crossing, broken-down in the way and so on) in the path which affect the transportation time, considered as dwell time, *(E)* loading and unloading time of goods, which increase the accuracy level of the delivery time, *(F)* total budget of a potential site depends on the location as well as transported goods, regarded as a decision variable, *(G)* follow the *Kyoto Protocol (2007)* for controlling CO_2 emission due to transportation, reduce the carbon footprint. The main aim of this research is to formulate and resolve a green logistics modeling by taking extreme weather events into consideration for controlling CO_2 emission.

7.3.2 Notations and Assumptions

The following notations and assumptions are required to state the proposed model:

Sets

I Set of sources considered as existing facilities indexed by i,

J Set of destinations assumed as potential facilities indexed by j,

K Set of transportation modes indexed by k,

W $\{(w_{ijk}) : \forall\, i, j, k\}$: the feasible space,

W^B $\{(w^B_{ijk}) : \forall\, i, j, k\}$: the optimal feasible set,

F $\mathbb{R}^{2p} \times W$, where $(x, y) \in \mathbb{R}^{2p}$ and $w \in W$, the feasible set.

Decision variables

w_{ijk} Unknown amount to be distributed from i^{th} source to j^{th} destination by k^{th} different transportation modes,

(x_j, y_j) Coordinate of j^{th} destination,

B'_j Total budget at j^{th} destination,

$$\eta\,(w_{ijk}) = \begin{cases} 1, & \text{if } w_{ijk} > 0, \\ 0, & \text{otherwise.} \end{cases}$$

Parameters

m Number of sources,

n Number of destinations,

p Number of transportation modes,

q Number of objective functions,

(u_i, v_i) Coordinate of i^{th} source,

$\tilde{\tilde{a}}_i$ Availability of i^{th} source in TT2IFN,

$\tilde{\tilde{b}}_j$ Demand at j^{th} destination in TT2IFN,

$\tilde{\tilde{c}}_k$ Capacity of k^{th} transportation mode in TT2IFN,

α_i In a location problem, the DM may put more importance of the source with respect to transportation cost, expressed as weight. Therefore, with each i^{th} source, we associate a weight α_i,

β_i Nonnegative weight of i^{th} source with respect to transportation time,

ε_k Per unit fuel cost of k^{th} transportation mode,

δ_k There may be used k^{th} different transportation modes to distribute the goods. Depending on their fuel consumption (machine performance), the weight δ_k is assigned,

t'_k k^{th} conveyance time for per unit distance to distribute the item,

t_{ijk} Dwell time for k^{th} vehicle from i^{th} source to j^{th} destination,

l_i Loading time of the products at i^{th} source,

l'_j Unloading time of the goods at j^{th} destination,

M_k Maintenance cost of k^{th} vehicle for per unit distance,

f_{ijk} Fixed-charge cost (for example, processing charge, toll charges, loading and unloading charges, packaging charges, safety expenses, so on) to transport goods from i^{th} source to j^{th} destination by k^{th} vehicle,

e_k Per unit carbon emission by k^{th} conveyance,

γ Tax for each unit of carbon emission,

C Carbon emission cap (i.e., limited capacity of carbon emission permit),

P_c Penalty cost per unit emitted in excess of the cap,

U_q Upper bound of the q objective function,

L_q Lower bound of the q objective function.

There are the following functions and assumptions:

- $d_{ij} = \sqrt{(u_i - x_j)^2 + (v_i - y_j)^2}$: Euclidean distance function between i^{th} source and j^{th} destination.

- $\phi_k(u_i,v_i;x_j,y_j) = \varepsilon_k\sqrt{(u_i-x_i)^2+(v_i-y_j)^2+\delta_k}$: transportation cost function per unit item from i^{th} source to j^{th} destination by k^{th} conveyance.

- $\psi_k(u_i,v_i;x_j,y_j) = t'_k\sqrt{(u_i-x_i)^2+(v_i-y_j)^2}$: transportation time function for the product from i^{th} source to j^{th} destination by k^{th} conveyance.

- $\varphi_k(u_i,v_i;x_j,y_j) = e_k\sqrt{(u_i-x_i)^2+(v_i-y_j)^2+\delta_k}$: carbon emission function per unit transported item from i^{th} source to j^{th} destination by k^{th} conveyance.

- The solution space where the facilities are situated in the continuous planner surface. Furthermore, the facility plants are considered as Euclidean points. There does not exist any connection between the potential facilities.

- The facility sites have some capacity. The supply, demand and conveyance parameters are TT2IF nature. The distances are assumed as the Euclidean metric in the plane surface.

- The distributed commodity is the homogeneous type. The nature of transportation modes is heterogeneous. Transportation cost is directly proportional to the unit of shipped commodities.

- The time to transport goods between two points on the network is proportional to the Euclidean distance. The carbon emission is dependent on the distance traveled by the conveyances and their fuel consumption.

7.3.3 Model formulation

Here, a mathematical model is introduced in the light of green activities, FLP, MOSTP and dwell time. The supply, demand and conveyance parameters are considered as TT2IFNs. This formulation finds the distributed commodities, optimum locations and budgets for the potential facilities at the same time. The mathematical model of MOGST-LP along with carbon tax and offset policy can be stated as follows:

Model 7.1

$$\begin{aligned}\text{minimize} \quad Z_{1(x,y,w)} = &\sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\left[\alpha_i w_{ijk}\phi_k\left(u_i,v_i;x_j,y_j\right)+M_k d_{ij}\eta\left(w_{ijk}\right)\right] \\ &+\sum_{i\in I}\sum_{j\in J}\sum_{k\in K} f_{ijk}\eta\left(w_{ijk}\right) \end{aligned} \tag{7.3}$$

$$\begin{aligned}\text{minimize} \quad Z_{2(x,y,w)} = &\sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\left[\beta_i\psi_k\left(u_i,v_i;x_j,y_j\right)+t_{ijk}\right]\eta\left(w_{ijk}\right) \\ &+\sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\left(l_i+l'_j\right)w_{ijk} \end{aligned} \tag{7.4}$$

$$\text{minimize} \quad Z_{3(x,y,w)} = \gamma \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} w_{ijk}\varphi_k\left(u_i,v_i;x_j,y_j\right) + P_c\left(\sum_{i\in I}\sum_{j\in J}\sum_{k\in K} w_{ijk}\varphi_k\left(u_i,v_i;x_j,y_j\right)-C\right)^{+} \tag{7.5}$$

$$\text{subject to} \quad \sum_{j\in J}\sum_{k\in K} w_{ijk} \leq \tilde{\tilde{a}}_i \quad \forall\, i, \tag{7.6}$$

$$\sum_{i\in I}\sum_{k\in K} w_{ijk} \geq \tilde{\tilde{b}}_j \quad \forall\, j, \tag{7.7}$$

$$\sum_{i\in I}\sum_{j\in J} w_{ijk} \leq \tilde{\tilde{c}}_k \quad \forall\, k, \tag{7.8}$$

$$\sum_{i\in I}\tilde{\tilde{a}}_i \geq \sum_{j\in J}\tilde{\tilde{b}}_j \text{ and } \sum_{k\in K}\tilde{\tilde{c}}_k \geq \sum_{j\in J}\tilde{\tilde{b}}_j, \tag{7.9}$$

$$w_{ijk} \geq 0 \text{ and } \eta\left(w_{ijk}\right) \in \{0,1\} \quad \forall\, i,j,k, \tag{7.10}$$

$$\sum_{i\in I}\sum_{k\in K}\Big[\alpha_i\phi_k\left(u_i,v_i;x_j,y_j\right)w_{ijk} + M_k d_{ij}\eta\left(w_{ijk}\right) + \gamma\varphi_k\left(u_i,v_i;x_j,y_j\right)w_{ijk} + f_{ijk}\eta\left(w_{ijk}\right)\Big] + P_c\Big(\sum_{i\in I}\sum_{k\in K} w_{ijk}\varphi_k(u_i,v_i;x_j,y_j)-C\Big)^{+} \leq B'_j \quad \forall\, j. \tag{7.11}$$

Here, $\left(\sum_{i\in I}\sum_{j\in J}\sum_{k\in K} w_{ijk}\varphi_k\left(u_i,v_i;x_j,y_j\right)-C\right)^{+} = \max\left(\sum_{i\in I}\sum_{j\in J}\sum_{k\in K} w_{ijk}\varphi_k\left(u_i,v_i;x_j,y_j\right)-C,0\right)$

$$= \begin{cases} \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} w_{ijk}\varphi_k\left(u_i,v_i;x_j,y_j\right)-C, & \text{if } \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} w_{ijk}\varphi(u_i,v_i;x_j,y_j) \geq C, \\ 0, & \text{otherwise.} \end{cases}$$

The economic objective function (7.3) aims to determine the optimum positions for the potential facilities which minimize the overall logistics cost. Terms 1-3 of (7.3) represent the total transportation cost, maintenance cost and fixed-charge cost from i^{th} source to j^{th} destination using k^{th} conveyance, respectively. The objective function (7.4) is related to customers' satisfaction, which intents to reduce the overall conveyance time, dwell time, and loading and unloading time from i^{th} source to j^{th} destination via k^{th} vehicle. The objective function (7.5) is connected with environmental aspects, which indicates to optimize the total carbon emission cost under tax and offset policy by determining the optimum locations for the facilities. Constraints (7.6) enforce that the overall distributed quantity of each source must be less or equal to its capacity. Constraints (7.7) impose that the overall shipped units of each destination fulfill the demand. Constraints (7.8) demonstrate that the overall transported flows of each transportation mode cannot surpass its ability. Constraints (7.9) refer to the feasibility criterion of the problem. Constraints (7.10) are the non-negativity conditions and binary restrictions. Ultimately, Constraints (7.11) ensure that the total expenditure of j^{th} destination under a carbon tax and offset regulation is not higher than the optimal budget.

7.3.4 Deterministic formulation

Type-2 intuitionistic fuzzy MOGST-LP model cannot be directly solved due to the existence of TT2IFNs as supply, demand and conveyance parameters. Therefore, a ranking defuzzification technique is introduced (see Subsection 7.2.2) for conversation of Type-2 intuitionistic fuzzy MOGST-LP (i.e., Model 7.1) into a deterministic MOGST-LP (i.e., Model 7.2).

Model 7.2

$$\begin{aligned}
\text{minimize} \quad & Z_{1(x,y,w)} = \sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\left[\alpha_i w_{ijk}\phi_k\left(u_i,v_i;x_j,y_j\right)+M_k d_{ij}\eta\left(w_{ijk}\right)\right] \\
& \qquad +\sum_{i\in I}\sum_{j\in J}\sum_{k\in K} f_{ijk}\eta\left(w_{ijk}\right) \\
\text{minimize} \quad & Z_{2(x,y,w)} = \sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\left[\beta_i\psi_k\left(u_i,v_i;x_j,y_j\right)+t_{ijk}\right]\eta\left(w_{ijk}\right) \\
& \qquad +\sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\left(l_i+l'_j\right)w_{ijk} \\
\text{minimize} \quad & Z_{3(x,y,w)} = \gamma\sum_{i\in I}\sum_{j\in J}\sum_{k\in K} w_{ijk}\varphi_k\left(u_i,v_i;x_j,y_j\right) \\
& \qquad +P_c\left(\sum_{i\in I}\sum_{j\in J}\sum_{k\in K} w_{ijk}\varphi_k\left(u_i,v_i;x_j,y_j\right)-C\right)^{+} \\
\text{subject to} \quad & \sum_{j\in J}\sum_{k\in K} w_{ijk} \le \mathfrak{R}\left(\tilde{\tilde{a}}_i\right) \quad \forall\, i, && (7.12) \\
& \sum_{i\in I}\sum_{k\in K} w_{ijk} \ge \mathfrak{R}\left(\tilde{\tilde{b}}_j\right) \quad \forall\, j, && (7.13) \\
& \sum_{i\in I}\sum_{j\in J} w_{ijk} \le \mathfrak{R}\left(\tilde{\tilde{c}}_k\right) \quad \forall\, k, && (7.14) \\
& \sum_{i\in I}\mathfrak{R}\left(\tilde{\tilde{a}}_i\right) \ge \sum_{j\in J}\mathfrak{R}\left(\tilde{\tilde{b}}_j\right) \text{ and } \sum_{k\in K}\mathfrak{R}\left(\tilde{\tilde{c}}_k\right) \ge \sum_{j\in J}\mathfrak{R}\left(\tilde{\tilde{b}}_j\right), && (7.15) \\
& \text{the constraints (7.10) to (7.11).}
\end{aligned}$$

Now, the objective function (7.5) illustrates that, based on the carbon cap, there are two feasible regions. The first one arises when $\sum_{i\in I}\sum_{j\in J}\sum_{k\in K} w_{ijk}\varphi_k\left(u_i,v_i;x_j,y_j\right)\le C$. And the second occurs when $\sum_{i\in I}\sum_{j\in J}\sum_{k\in K} w_{ijk}\varphi_k\left(u_i,v_i;x_j,y_j\right)\ge C$.

The following model is formulated for Case 1:

Model 7.2.1

$$\begin{aligned}
\text{minimize} \quad & Z_{1(x,y,w)} = \sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\left[\alpha_i w_{ijk}\phi_k\left(u_i,v_i;x_j,y_j\right)+M_k d_{ij}\eta\left(w_{ijk}\right)\right] \\
& \qquad +\sum_{i\in I}\sum_{j\in J}\sum_{k\in K} f_{ijk}\eta\left(w_{ijk}\right) \\
\text{minimize} \quad & Z_{2(x,y,w)} = \sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\left[\beta_i\psi_k\left(u_i,v_i;x_j,y_j\right)+t_{ijk}\right]\eta\left(w_{ijk}\right) \\
& \qquad +\sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\left(l_i+l'_j\right)w_{ijk} \\
\text{minimize} \quad & Z_{3(x,y,w)} = \gamma\sum_{i\in I}\sum_{j\in J}\sum_{k\in K} w_{ijk}\varphi_k\left(u_i,v_i;x_j,y_j\right) && (7.16) \\
\text{subject to} \quad & \text{the constraints (7.10) and (7.12) to (7.15),} \\
& \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} w_{ijk}\varphi_k\left(u_i,v_i;x_j,y_j\right) \le C, && (7.17)
\end{aligned}$$

$$\sum_{i\in I}\sum_{k\in K}\left[\alpha_i w_{ijk}\phi_k\left(u_i,v_i;x_j,y_j\right)+\left(M_k d_{ij}+f_{ijk}\right)\eta\left(w_{ijk}\right)\right]$$
$$+\gamma\sum_{i\in I}\sum_{k\in K} w_{ijk}\varphi_k(u_i,v_i;x_j,y_j)\le B'_j \quad \forall\ j. \tag{7.18}$$

The Case 2 can be expressed by the model as stated below:

Model 7.2.2

$$\begin{aligned}
\text{minimize}\quad & Z_{1(x,y,w)}=\sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\left[\alpha_i w_{ijk}\phi_k\left(u_i,v_i;x_j,y_j\right)+M_k d_{ij}\eta\left(w_{ijk}\right)\right]\\
&\qquad +\sum_{i\in I}\sum_{j\in J}\sum_{k\in K} f_{ijk}\eta\left(w_{ijk}\right)\\
\text{minimize}\quad & Z_{2(x,y,w)}=\sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\left[\beta_i\psi_k\left(u_i,v_i;x_j,y_j\right)+t_{ijk}\right]\eta\left(w_{ijk}\right)\\
&\qquad +\sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\left(l_i+l'_j\right)w_{ijk}\\
\text{minimize}\quad & Z_{3(x,y,w)}=\left(\gamma+P_c\right)\sum_{i\in I}\sum_{j\in J}\sum_{k\in K} w_{ijk}\varphi_k\left(u_i,v_i;x_j,y_j\right)-P_c C \qquad (7.19)\\
\text{subject to}\quad & \text{the constraints (7.10) and (7.12) to (7.15),}\\
& \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} w_{ijk}\varphi_k\left(u_i,v_i;x_j,y_j\right)\ge C, \qquad (7.20)\\
& \sum_{i\in I}\sum_{k\in K}\left[\alpha_i w_{ijk}\phi_k\left(u_i,v_i;x_j,y_j\right)+\left(M_k d_{ij}+f_{ijk}\right)\eta\left(w_{ijk}\right)\right]\\
& +(P_c+\gamma)\sum_{i\in I}\sum_{k\in K} w_{ijk}\varphi_k(u_i,v_i;x_j,y_j)-P_c C\le B'_j \quad \forall\ j. \qquad (7.21)
\end{aligned}$$

Definition 7.5 ***Ideal solution****: An* ideal solution *of Model 7.2.1 (or, Model 7.2.2) is the one which reduces each of the goal independently, i.e.,* $Z_q(x^*,y^*,w^*)=\min_{(x,y,w)\in F} Z_q(x,y,w)$, $q=1,2,3$.

Definition 7.6 ***Anti-ideal solution****: A solution* $(x^A,y^A,w^A)\in F$ *of Model 7.2.1 (or, Model 7.2.2) is called an* anti-ideal solution *if it satisfies the condition* $Z_q(x^A,y^A,w^A)=\max_{(x,y,w)\in F} Z_q(x,y,w)$, $q=1,2,3$.

Definition 7.7 ***Pareto-optimal solution****: A solution* $(x^P,y^P,w^P)\in F$ *is said to be a* Pareto-optimal solution *(otherwise called* non-dominated solution, non-inferior or efficient solution*) of Model 7.2.1 (or, Model 7.2.2) if and only if there is no other solution* $(x,y,w)\in F$ *such that*

$$Z_q(x,y,w)\le Z_q(x^P,y^P,w^P)\ \text{for}\ q=1,2,3,\ \text{and}$$
$$Z_q(x,y,w)< Z_q(x^P,y^P,w^P)\ \text{for at least one}\ q.$$

7.4 Solution techniques

The proposed formulation is the MOGST-LP with type-2 intuitionistic fuzzy parameter. Introducing ranking index to TT2IFN parameters, the model is transformed into a deterministic MOGST-LP (i.e., Model 7.2). In a multi-objective problem, the DM needs to optimize the conflicting objective functions simultaneously. For that reason, it is difficult to choose an optimal point where all the objective functions obtain their optimum values. Thus, we have to discover a Pareto optimal solution. In the literature, there exist several fuzzy and non-fuzzy methodologies like fuzzy programming [91], global criterion method [139], goal programming approach [137], weighted goal programming [140], ε-constraint method [139], interactive algorithm [128], intuitionistic fuzzy programming [138], neutrosophic compromise programming [129] and fuzzy goal programming [51] for solving a multi-objective problem. All the previously mentioned techniques aside from fuzzy programming, global criterion method, intuitionistic fuzzy programming, and neutrosophic compromise programming need not require prior information on objectives (goals and weights) from the DM for solving the problem. Among four techniques, fuzzy programming and global criterion method provide a simple mathematical structure that makes it easier for understanding and employing. Moreover, the two techniques always give a Pareto optimal solution within a relatively short computational time and memory with respect to the other techniques. Fuzzy programming and global criterion method utilize the idea of the shortest distance from the ideal point to find a Pareto optimal solution; the techniques need not require any prior information on objective functions from the DM. In order to solve Model 7.2, we adopt a fuzzy approach, namely, a fuzzy programming and a non-fuzzy approach, specifically, a global criterion technique. Again, Model 7.2 is divided into two parts depending on the carbon cap as Model 7.2.1 and Model 7.2.2, respectively. Afterwards, two models are solved independently to extract Pareto-optimal solutions. Thereafter, the solutions are compared to find the optimal solution of Model 7.2. Nevertheless, if one of two models (i.e., Models 7.2.1 and 7.2.2) has a Pareto-optimal solution and the other has no feasible solution, then the Pareto-optimal solution of the corresponding model is the optimal solution of Model 7.2. The schematic diagram of the proposed problem and its solution techniques is displayed in Figure 7.1.

7.4.1 Fuzzy programming

After applying fuzzy programming (Li and Lai [91]), the simplified fuzzy optimization model of MOGST-LP (i.e., Model 7.2) can be described to derive a Pareto-optimal solution as stated below:

Model 7.3 (For Model 7.2.1)

$$\begin{aligned}
&\text{maximize} && \lambda\\
&\text{subject to} && Z_q(x,y,w)+\lambda(U_q-L_q)\le U_q,\quad q=1,2,3,\\
& && \text{the constraints (7.10), and (7.12) to (7.15),}\\
& && \text{the constraints (7.17) and (7.18),}\\
& && \lambda\ge 0;
\end{aligned}$$

Model 7.4 (For Model 7.2.2)

$$\begin{aligned} \text{maximize} \quad & \lambda \\ \text{subject to} \quad & Z_q(x,y,w) + \lambda(U_q - L_q) \leq U_q, \quad q = 1,2,3, \\ & \text{the constraints (7.10) and (7.12) to (7.15)}, \\ & \text{the constraints (7.20) and (7.21)}, \\ & \lambda \geq 0. \end{aligned}$$

Here, λ is the level of satisfaction of a solution, $\lambda = \min\{\mu\left(Z_q(x,y,w)\right) : q = 1,2,3\}$. The $\mu\left(Z_q(x,y,w)\right)$ is a membership function corresponding to each q^{th} objective function which is defined as follows:

$$\mu\left(Z_q(x,y,w)\right) = \begin{cases} 1, & Z_q(x,y,w) \leq L_q, \\ \frac{U_q - Z_q(x,y,w)}{U_q - L_q}, & L_q \leq Z_q(x,y,w) \leq U_q, \\ 0, & Z_q(x,y,w) \geq U_q. \end{cases}$$

Moreover, $U_q = \max\{Z_{1q}, Z_{2q}, Z_{3q}\}$, $L_q = Z_{qq}$ and $Z_{lq} := Z_q\left((x,y,w)^{(l)}\right)$, $q = 1,2,3.$ For more explanation of this technique, we refer to the study (Li and Lai [91]).

Proposition 7.1 *Let us assume that* (x^P, y^P, w^P, λ) *be an optimal solution of Model 7.3 (Model 7.4), then it is also a Pareto-optimal solution* (x^P, y^P, w^P) *of Model 7.2.1 (Model 7.2.2).*

Proof. The proof of the proposition is evident, keeping the references of the evidences of Lemma 5.3 (cf. Chapter 5) and Proposition 6.2 (cf. Chapter 6). □

7.4.2 Global criterion method

After employing global criterion method (Roy et al. [139]), the mathematical model of MOGST-LP (i.e., Model 7.2) can be formulated as:

Model 7.5 (For Model 7.2.1)

$$\begin{aligned} \text{minimize} \quad & \left[\sum_{q=1}^{3}\left(\frac{Z_q(x,y,w) - Z_q^{\min}}{Z_q^{\max} - Z_q^{\min}}\right)^2\right]^{\frac{1}{2}} \\ \text{subject to} \quad & \text{the constraints (7.10), and (7.12) to (7.15)}, \\ & \text{the constraints (7.17) and (7.18)}; \end{aligned}$$

Model 7.6 (For Model 7.2.2)

$$\begin{aligned} \text{minimize} \quad & \left[\sum_{q=1}^{3}\left(\frac{Z_q(x,y,w) - Z_q^{\min}}{Z_q^{\max} - Z_q^{\min}}\right)^2\right]^{\frac{1}{2}} \\ \text{subject to} \quad & \text{the constraints (7.10), and (7.12) to (7.15)}, \\ & \text{the constraints (7.20) and (7.21)}. \end{aligned}$$

Here, $Z_q^{\min}$ and $Z_q^{\max}$ are the ideal and anti-ideal solutions of q^{th} objective function, respectively. The readers may follow the article (Roy et al. [139]) for detailed explanations of the above method.

Proposition 7.2 *Let us assume that (x^P, y^P, w^P) be an optimal solution of Model 7.5 (Model 7.6), then it should be also a Pareto-optimal solution (x^P, y^P, w^P) of Model 7.2.1 (Model 7.2.2).*

Proof. This proposition can be proved by contradiction. Let us assume that (x^P, y^P, w^P) be an optimal solution of Model 7.5 (Model 7.6) which is not a Pareto-optimal solution of Model 7.2.1 (Model 7.2.2). Subsequently, there exists a solution (x', y', w') such that (x', y', w') dominates (x^P, y^P, w^P). It implies

$$\left[\sum_{q=1}^{3}\left(\frac{Z_q(x', y', w') - Z_q^{\min}}{Z_q^{\max} - Z_q^{\min}}\right)^2\right]^{\frac{1}{2}} < \left[\sum_{q=1}^{3}\left(\frac{Z_q(x^P, y^P, w^P) - Z_q^{\min}}{Z_q^{\max} - Z_q^{\min}}\right)^2\right]^{\frac{1}{2}}$$

which directly contradicts to the fact that (x^P, y^P, w^P) is an optimal solution of Model 7.5 (Model 7.6). This completes the proposition. □

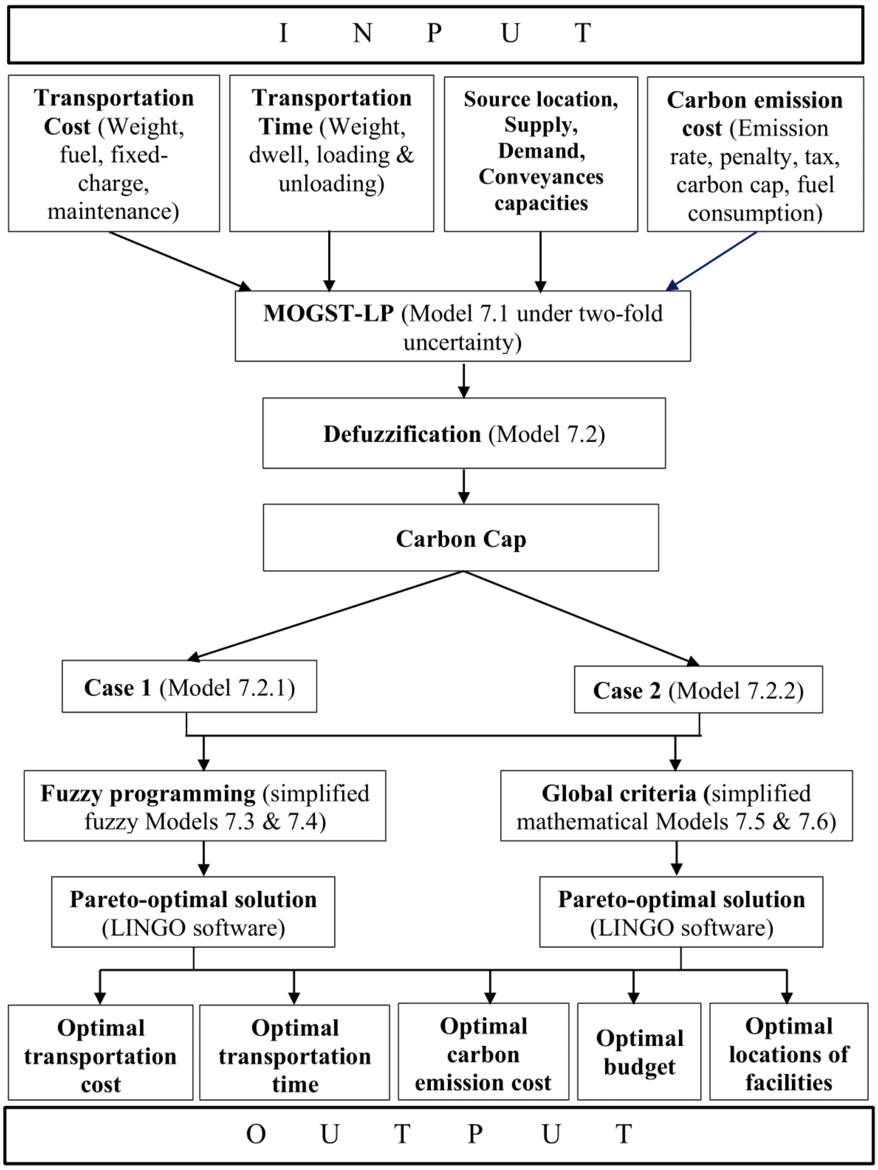

Fig. 7.1: Schematic diagram of the overall scenario.

7.5 Application examples

In order to validate the proposed model and solution techniques, here, two industrial-based application examples are presented.

Example 1 (*In plant location*): Here, we consider that an industrial organization wishes to begin a couple of firms with the goal of reducing the overall logistics cost, the delivery time with dwell time and carbon emission cost under the tax, cap and offset regulation. For simplicity, it is considered that the organization has four (04) supplier firms and he/she wants to establish two (02) new firms. He/She transports the goods from existing firms to potential firms by mode of conveyances. Products are transported by two (02) different conveyances. For that reason, the non-negative weights of fuel consumption by different conveyances are also taken into cost and emission functions. Under tax, cap and offset regulation, a carbon emission cap (C) is provided for the organization. When the organization discharges less than the cap C, then he/she can only pay the usual tax per unit emission. However, when the organization emits more than the cap, he/she has to pay an offset as a penalty along with the usual carbon tax. Supportive hypothetical data of this phenomenon are designed. The fixed-charge cost and dwell time parameters are displayed in Table 7.1. Supply, demand and conveyance type-2 fuzzy parameters and their crisp values are as follows:

$$\begin{aligned}
\tilde{\tilde{a}}_1 &= \big\langle\big(\langle(170,175,185,190);0.65,0.1\rangle,\langle(180,185,195,200);0.7,0.2\rangle,\\
&\quad\langle(190,195,205,210);0.7,0.2\rangle,\langle(195,200,205,215);0.8,0.1\rangle\big);0.6,0.2\big\rangle,\\
\Re\left(\tilde{\tilde{a}}_1\right) &= 77.625;\\
\tilde{\tilde{a}}_2 &= \big\langle\big(\langle(150,156,158,160);0.7,0.1\rangle,\langle(155,161,163,168);0.6,0.4\rangle,\\
&\quad\langle(160,166,168,173);0.6,0.3\rangle,\langle(166,170,175,180);0.7,0.3\rangle\big);0.6,0.4\big\rangle,\\
\Re\left(\tilde{\tilde{a}}_2\right) &= 82.156;\\
\tilde{\tilde{a}}_3 &= \big\langle\big(\langle(115,145,150,170);0.8,0.1\rangle,\langle(125,155,175,180);0.75,0.1\rangle,\\
&\quad\langle(135,145,165,195);0.8,0.2\rangle,\langle(150,160,170,180);0.6,0.1\rangle\big);0.6,0.2\big\rangle,\\
\Re\left(\tilde{\tilde{a}}_3\right) &= 62.875;\\
\tilde{\tilde{a}}_4 &= \big\langle\big(\langle(80,91,95,100);0.8,0.2\rangle,\langle(85,95,105,110);0.85,0.1\rangle,\\
&\quad\langle(90,99,115,120);0.9,0.1\rangle,\langle(110,120,130,135);0.6,0.3\rangle\big);0.6,0.3\big\rangle,\\
\Re\left(\tilde{\tilde{a}}_4\right) &= 47.25;\\
\tilde{\tilde{b}}_1 &= \big\langle\big(\langle(165,195,200,220);0.65,0.25\rangle,\langle(175,205,225,230);0.7,0.3\rangle,\\
&\quad\langle(185,195,215,245);0.5,0.5\rangle,\langle(200,210,220,230);0.8,0.1\rangle\big);0.5,0.5\big\rangle,\\
\Re\left(\tilde{\tilde{b}}_1\right) &= 103.593;\\
\tilde{\tilde{b}}_2 &= \big\langle\big(\langle(190,200,210,220);0.4,0.5\rangle,\langle(200,210,220,230);0.5,0.5\rangle,\\
&\quad\langle(210,220,230,240);0.8,0.2\rangle,\langle(270,275,285,290);0.9,0.1\rangle\big);0.4,0.5\big\rangle,\\
\Re\left(\tilde{\tilde{b}}_2\right) &= 104.062;
\end{aligned}$$

$$\begin{aligned}
\tilde{\tilde{c}}_1 &= \langle(\langle(190,195,205,210);0.7,0.3\rangle,\langle(200,205,215,220);0.8,0.1\rangle,\\
&\quad \langle(210,215,225,230);0.85,0.1\rangle,\langle(205,220,225,235);0.7,0.2\rangle);0.7,0.3\rangle,\\
\mathfrak{R}\left(\tilde{\tilde{c}}_1\right) &= 106.406;\\
\tilde{\tilde{c}}_2 &= \langle(\langle(220,225,235,240);0.75,0.25\rangle,\langle(230,235,245,250);0.6,0.3\rangle,\\
&\quad \langle(240,245,255,260);0.8,0.1\rangle,\langle(245,250,255,265);0.6,0.4\rangle);0.6,0.4\rangle,\\
\mathfrak{R}\left(\tilde{\tilde{c}}_2\right) &= 121.719.
\end{aligned}$$

Table 7.1: Fixed-charge cost and dwell time (f_{ijk}, t_{ijk}).

Source-Destination	Conveyance $(k = 1)$	Conveyance $(k = 2)$
$1-1$	$(120,15)$	$(250,10)$
$1-2$	$(45,12)$	$(90,10)$
$2-1$	$(200,10)$	$(300,5)$
$2-2$	$(180,20)$	$(190,40)$
$3-1$	$(145,30)$	$(150,0)$
$3-2$	$(260,0)$	$(290,60)$
$4-1$	$(300,5)$	$(400,20)$
$4-2$	$(350,25)$	$(400,15)$

Here, the other input parameters are taken as stated subsequently:
Carbon emission tax $\gamma = 5$; Penalty cost $P_c = 7$; Maintenance cost $M_1 = 0.5$, $M_2 = 0.1$; Fuel consumption rate $\delta_1 = 0.5$, $\delta_2 = 0.7$; Fuel cost $\varepsilon_1 = 30$, $\varepsilon_1 = 40$; Conveyance time $t'_1 = 1.5$, $t'_2 = 2.5$; Carbon emission rate $e_1 = 0.2$, $e_2 = 0.4$; Loading time $l_1 = 60$, $l_2 = 40$, $l_3 = 30$, $l_4 = 25$; Unloading time $l'_1 = 30$, $l'_2 = 15$; Locations of the sources $(u_1, v_1) = (1,1)$, $(u_2, v_2) = (70,7)$, $(u_3, v_3) = (90,70)$, $(u_4, v_4) = (10,50)$; Weights of the source points $\alpha_1 = 0.2$, $\alpha_2 = 0.4$, $\alpha_3 = 0.1$, $\alpha_4 = 0.3$, $\beta_1 = 0.3$, $\beta_2 = 0.5$, $\beta_3 = 0.1$, $\beta_4 = 0.1$; Carbon emission cap $C = 1500$.

Example 2 (*In plant location*): Here, we consider the carbon emission cap $C = 800$, and the others parameters remain the same as in Example 1.

7.6 Result and discussion

In this section, the Pareto-optimal solutions of Model 7.2 are derived by employing a fuzzy technique and a non-fuzzy technique. The solution techniques are coded in LINGO 17.0.79 optimization tool on a MacBook Air computer with the configuration 1.8 GHz Intel Core i5, 8 GB RAM. The optimal solutions of both the examples are given in Tables 7.2-7.3. The steps of solution techniques are shown in Figures 7.2-7.3.
From Table 7.2, we conclude that the solution obtained from the proposed global criterion method is better in comparison with the solution which is received by the fuzzy programming. However, from Table 7.3, we notice that it happens exactly the opposite one. Therefore, the comparison of the proposed two techniques is not likely. In fact, the DM has a choice to pick any one of the solution techniques according to the circumstance of the problem. Moreover,

Table 7.2: The Pareto-optimal solution of Example 1.

Solution technique	Optimal solution of Model 2: Solution of Model 2.1	Average CPU time (s)	Memory (K)	Solution of Model 2.2
Fuzzy programming	$\lambda = 0.714$, $Z_1 = 25740.85$, $Z_2 = 13450.696$, $Z_3 = 4810.478$, $B'_1 = 24318.52$, $B'_2 = 8648.487$, $(x_1,y_1) = (1.189,1,650)$, $(x_2,y_2) = (70.015,7.047)$, $w_{112} = 58.198$, $w_{221} = 39.105$, $w_{222} = 43.05$, $w_{311} = 14.418$, $w_{321} = 21.906$, $w_{411} = 30.976$, and remaining all $w_{ijk} = 0$.	0.32	84	No feasible solution
Global criterion	$\mathbf{Z_1 = 23909.661}$, $\mathbf{Z_2 = 13225.287}$, $\mathbf{Z_3 = 4537.228}$, $B'_1 = 21893.02$, $B'_2 = 8878.958$, $(x_1,y_1) = (9.892,49.195)$, $(x_2,y_2) = (70.025,7.077)$, $w_{111} = 52.894$, $w_{222} = 82.156$, $w_{311} = 3.449$, $w_{321} = 21.906$, $w_{411} = 28.157$, $w_{412} = 19.093$, and remaining all $w_{ijk} = 0$.	0.33	77	No feasible solution

Boldface indicates the optimal solution.

Table 7.3: The Pareto-optimal solution of Example 2.

Solution technique	Optimal solution of Model 2: Solution of Model 2.1	Solution of Model 2.2	Average CPU time (s)	Memory (K)
Fuzzy programming	No feasible solution	$\lambda = 0.847$, $\mathbf{Z_1 = 39373.227}$, $\mathbf{Z_2 = 12307.214}$, $\mathbf{Z_3 = 12452.79}$, $B'_1 = 38763.93$, $B'_2 = 24439.49$, $(x_1,y_1) = (78.083,67.009)$, $(x_2,y_2) = (69.907,6.991)$, $w_{121} = 21.906$, $w_{222} = 82.156$, $w_{311} = 43.782$, $w_{312} = 19.093$, $w_{411} = 40.718$, and remaining all $w_{ijk} = 0$.	0.27	84
Global criterion	No feasible solution	$Z_1 = 43361.27$, $Z_2 = 12940.23$, $Z_3 = 18293.65$, $B'_1 = 25302.96$, $B'_2 = 38170.42$, $(x_1,y_1) = (9.887,49.389)$, $(x_2,y_2) = (80.561,40.268)$, $w_{111} = 56.343$, $w_{221} = 41.187$, $w_{321} = 8.876$, $w_{322} = 53.999$, $w_{412} = 47.25$, and remaining all $w_{ijk} = 0$.	2.07	71

Boldface indicates the optimal solution.

the analytical results reveal that the financial, customers' satisfaction and environmental objectives are optimized, and the optimum locations and budgets of the new facilities are also found. We explore that if the cap is larger than the threshold, then an organization will select the cheaper conveyances which emit more carbon. For this fact, the total transportation cost, as well as carbon emission cost decrease as he/she has to pay only the usual carbon emission tax no excess offset. Because of that, he/she will make more benefit. Again if the margin is

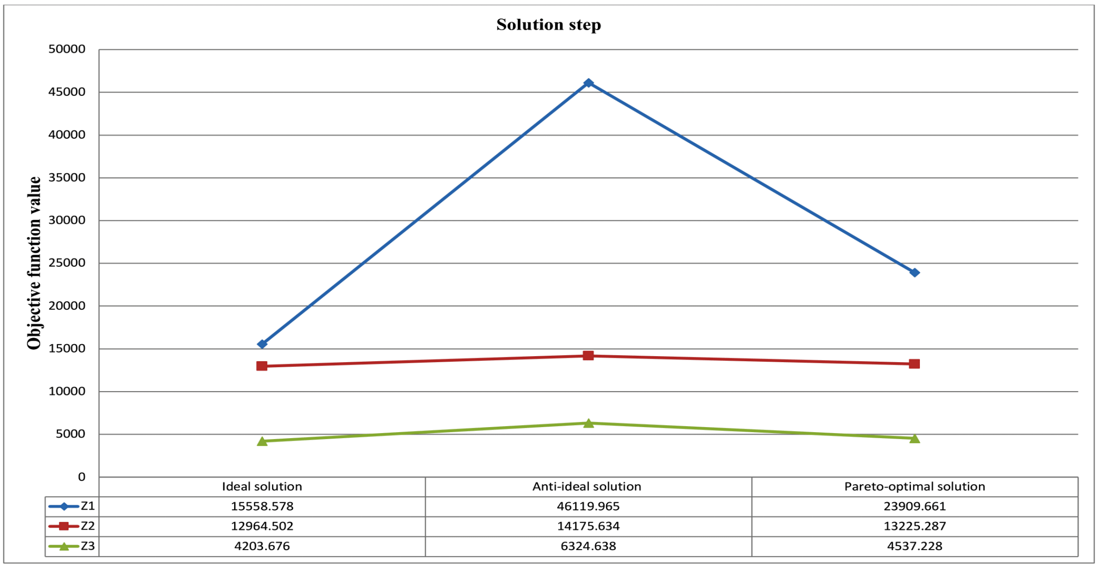

Fig. 7.2: Graphical representation of the global criterion for Example 1.

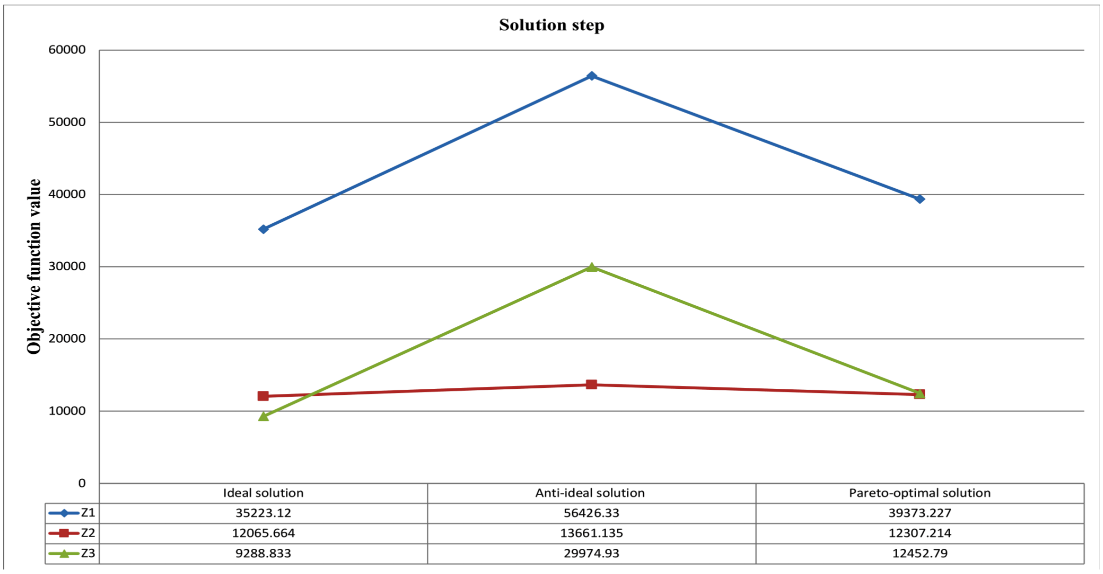

Fig. 7.3: Pictorial diagram of the fuzzy programming for Example 2.

less than the total emission, the organization chooses the costly conveyances which emit less. Therefore, the total conveyances cost as well as carbon emission cost increase as he/she has to give an offset as a penalty, which will reduce his/her profit. For this reason, the organization will always be concerned about carbon emission due to the transportation of goods. Additionally, he/she can invest in carbon offset projects to increase its carbon cap. Afterwards, the carbon policy not only helps the organization to choose optimal decisions for improving their economic performance but also supports the policymaker for reducing carbon emission.

7.7 A comparative study with particular cases

Here, we consider a few particular cases of our problem so that a comparison can be drawn with the existing studies. They are as follows; *(I)* the locations of the facilities are known, then the Euclidean functions are converted into constants, *(II)* consider all parameters in crisp number, *(III)* goods are transported by one kind of conveyance, and *(IV)* fixed-charge cost, maintenance cost, carbon emission, dwell time, loading and unloading time are not taken into consideration. With these particular cases, the stated problem translates into an MOTP. For more mathematical details of MOTP, we refer to Rizk-Allah et al. [129]. Thereafter, a numerical example is adopted from Rizk-Allah et al. [129].

Example 3 (*In transportation*): The input data are as follows:
Supply: $a_1 = 5$, $a_2 = 4$, $a_3 = 2$, $a_4 = 9$; Demand: $b_1 = 4$, $b_2 = 4$, $b_3 = 6$, $b_4 = 2$, $b_5 = 4$.
The coefficients of three objectives are as listed below:

$$C_1 = \begin{bmatrix} 9 & 12 & 9 & 6 & 9 \\ 7 & 3 & 7 & 7 & 5 \\ 6 & 5 & 9 & 11 & 3 \\ 6 & 8 & 11 & 2 & 2 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 2 & 9 & 8 & 1 & 4 \\ 1 & 9 & 9 & 5 & 2 \\ 8 & 1 & 8 & 4 & 5 \\ 2 & 8 & 6 & 9 & 8 \end{bmatrix}, \quad C_3 = \begin{bmatrix} 2 & 4 & 6 & 3 & 6 \\ 4 & 8 & 4 & 9 & 2 \\ 5 & 3 & 5 & 3 & 6 \\ 6 & 9 & 6 & 3 & 1 \end{bmatrix}.$$

Therefore, the optimal solution obtained by the proposed and existing techniques is displayed in Table 7.4. From this Table 7.4, we conclude that the proposed approaches provide a good Pareto-optimal solution with respect to the others. Besides, the proposed methods give convenient mathematical structures, which always yield Pareto-optimal solutions with less computational burden and memory.

Table 7.4: Comparison between proposed and others existing approaches.

Solution techniques	Z_1	Z_2	Z_3	Average CPU time (s)	Memory (K)
Global criterion	**126.958**	**104.33**	**75.835**	0.08	30
Fuzzy programming	**124.98**	**103.76**	**79.04**	0.10	30
Roy and Midya [138]	125	104	79.047	0.35	40
Rizk-Allah et al. [129]	132	100	76	0.40	50
El-Wahed and Lee [51]	126.79	103.10	77.52	0.65	70
Ringuest and Rinks [128]	127	104	76	0.80	75

Boldface indicates the optimal solution of the proposed techniques.

7.8 Sensitivity analysis

Here, we check the resiliency of Pareto-optimal solutions in MOGST-LP by changing the parameters values. For MOGST-LP, the complexity occurs when the ranges are calculated after parametric changes to the object that the obtained Pareto-optimal solutions still remains the same. Indeed, the difficulty enlarges when the decision variables and restrictions are large in number. Because of that, a simple procedure is already carried out in Chapter 3 (see Section 3.5) to analyze the sensitivity of parameters. Here, the same steps (Steps 1- 4) are repeated to obtain the validity ranges of the parameters in MOGST-LP.

Sensitivity analysis for supply, demand and capacity parameters:

Let us consider that a_i be converted to a_i^* $(i=1,2,3,4)$, b_j be changed to b_j^* $(j=1,2)$ and c_k be changed to c_k^* $(l=1,2)$. Using the steps, the values of a_i^*, b_j^* and c_k^* are easily computed, which are displayed in Tables 7.5-7.6. In fact, the ranges of the alternate parameters in MOGST-LP are likewise achieved comparably.

Table 7.5: The range of supply, demand and capacity parameters of Example 1.

Real values of a_i, b_j and c_k	Ranges of a_i, b_j and c_k
$a_1 = 77.625$	$77.625 \leq a_1^* \leq 100$
$a_2 = 82.156$	$77.9 \leq a_2^* \leq 84.3$
$a_3 = 62.875$	$22.5 \leq a_3^* \leq 70.6$
$a_4 = 47.25$	$47.25 \leq a_4^* \leq 65.5$
$b_1 = 103.593$	$103.593 \leq b_1^* \leq 120.5$
$b_2 = 104.062$	$103 \leq b_2^* \leq 110$
$c_1 = 106.406$	$106.406 \leq c_1^* \leq 130.2$
$c_2 = 121.719$	$101.3 \leq c_2^* < \infty$

Table 7.6: The range of supply, demand and capacity parameters of Example 2.

Real values of a_i, b_j and c_k	Ranges of a_i, b_j and c_k
$a_1 = 77.625$	$36.7 \leq a_1^* < \infty$
$a_2 = 82.156$	$70.1 \leq a_2^* \leq 89.9$
$a_3 = 62.875$	$62.875 \leq a_3^* \leq 80.1$
$a_4 = 47.25$	$47.25 \leq a_4^* \leq 60.9$
$b_1 = 103.593$	$103.593 \leq b_1^* \leq 125.5$
$b_2 = 104.062$	$104.062 \leq b_2^* \leq 115.9$
$c_1 = 106.406$	$106 \leq c_1^* \leq 135.5$
$c_2 = 121.719$	$101.3 \leq c_2^* < \infty$

7.9 Managerial insights

Profitable and vital managerial insights are received through this research, which would be valuable to the different kinds of governmental and private organizations associated with the logistics system. From the outcome, organizations can select the best Pareto-optimal solution when Model 7.2 is decomposed into two sub-models. In fact, organizations may determine the best potential sites so that they can distribute the commodities with the stated objectives. A brief discussion of the impact of CO_2 emission under a carbon tax, cap and offset policy is analyzed. From that analysis, organizations are able to understand when their profit will be less (more). Accordingly, they can adjust their benefits and environmental awareness, which may lead to a gain of reputation in the worldwide market. On the other hand, the fuel consumption of the vehicles is displayed in conveyance cost and emission functions. In cases where the fuel consumption is less, then the overall logistics cost along with carbon discharge will be reduced. Hence, organizations can easily select which kinds of vehicles are most suitable for distributing products. Once more, the dwell time for the barriers of the paths is also incorporated into the conveyance time, so that organizations are ready to calculate a more accurate delivery time which improves their customers' service.

7.10 Conclusion

In this research work, a strategic problem of integrated green logistics systems and location decisions has been introduced by considering economical, customers' satisfaction level and environmental objectives under two-fold uncertainty. In order to support the decision, an unprecedented multi-objective model has been formulated with the above three conflicting objectives under a carbon policy. At the same time, it also asks the optimal locations for the potential facilities in the Euclidean plane as well as the amounts of distributed goods by different transportation modes simultaneously. In addition, this study makes various major contributions such as fixed-charge cost, maintenance cost, dwell time, variable budget constraints, loading and unloading time. Thereafter, a new form of trapezoidal type-2 fuzzy number has been presented to handle the uncertainties. A simple linear ranking function has been introduced, significantly defuzzified the aforementioned uncertainty under a smaller computational effort. A fuzzy technique and a non-fuzzy technique are used to solve the stated formulation in a successful way. Thereafter, the aforementioned model and solution procedures have been validated by two examples. Finally, decisions regarding reducing CO_2 due to transportation systems have been discussed, too. We have also drawn the conclusion that our problem formulation and solution can control the carbon emission due to the logistics system.