Chapter 4

Heuristic approaches for solid transportation-location problem[1]

This chapter delineates *solid transportation-location problem* (ST-LP), a generalization of the classical STP in which location of potential facility sites are sought so that the total transportation cost by means of conveyances from existing facility sites to potential facility sites will be minimized. This is one of the most important problems in the transportation system and the location research areas. Two heuristic approaches are developed to solve such type of problem: a locate-allocate heuristic and an approximate heuristic. Thereafter, the performance of the proposed model and the heuristics are evaluated by an application example, and the obtained results are compared. Moreover, a sensitivity analysis is introduced to investigate the resiliency of the proposed model. Finally, conclusions are provided.

4.1 Introduction

Determining optimum places for the facilities and optimum transportation from existing sites to the facilities belongs to the main problems in supply chain management. FLP and STP are well connected with the transportation network. In ST-LP, one has to determine where and how to locate the new facilities among several existing facilities so that the total transportation cost by different types of conveyances from the existing facilities to the potential facilities will be minimized. Hereafter, ST-LP, a nonlinear cost minimization problem, is solved over a continuous surface with a hyperbolic approximation of Euclidean distance. In the present era, there have been many solution approaches to tackle several traditional facility location problems. Among them, the most commonly used approaches are locate-allocate (Loc-Alloc) heuristic, approximate heuristic, genetic algorithm, Lagrangean relaxation, etc. From the aforementioned solution approaches, the Loc-Alloc heuristic and the approximate heuristic are widely used for solving the different facility location problems [19, 29, 71, 158]. In fact, they always provide a good solution within a relatively short

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computational time [29, 71]. Due to this fact, we develop heuristic approaches for solving ST-LP with multiple existing and potential facility sites. The main aim is to incorporate a way to connect FLP and STP, and we develop heuristic approaches for tackling our stated problem. We extend the concept of STP by taking the origins as existing sites and destinations as facilities that are to be found. We determine the best location of a facility and the effective transportation cost by means of conveyances from sources to this facility locations simultaneously by solving ST-LP. The formulation can be applied to plant location problems where minimizing the total transportation cost by different types of conveyances is taken into consideration as the main priority. We believe that this formulation is more applicable than classical STP and FLP. It will be very useful to the model of emergency services and online shopping systems.

4.2 Mathematical description

Here, we first introduce the proposed problem, i.e., ST-LP. Thenceforth, the mathematical identification is stated on the premise of the following notations and assumptions. The connection between this formulation and STP, and its characteristic properties are illustrated.

4.2.1 Problem background

In this subsection, a new strategical problem is investigated from an economic point of view. Figure 4.1 exhibits the ST-LP network with various existing facility sites, the total transportation cost by different types of conveyances (e.g., trucks, air freight, goods trains, and ships) and potential facility sites. Goods are transported from existing facility sites to potential facility sites with the objective to minimize the total transportation cost by different types of transport modes. Let us assume that there are three existing facility sites, S_1, S_2 and S_3, two different types of conveyances, E_1 and E_2, and three potential facility sites, D_1, D_2 and D_3. In fact, the associated supply and demand of the existing facility sites and potential facility sites are known. Furthermore, the locations of S_1, S_2 and S_3 are given. But, the locations of D_1, D_2 and D_3 are not known in the Euclidean plane. Consequently, the lines (paths) denote the transportation cost function per unit commodity from S_1, S_2 and S_3 to D_1, D_2 and D_3, by the conveyances E_1 and E_2, respectively. Under this circumstance, one has to determine the optimal locations of the potential facility sites in such a way that the total transportation cost from existing facility sites to potential facility sites is minimized. Due to this reason, an integration between the FLP and the STP is made.

4.2.2 Notations and Assumptions

The following notations and assumptions are used to design the model:

m : number of existing facility sites,

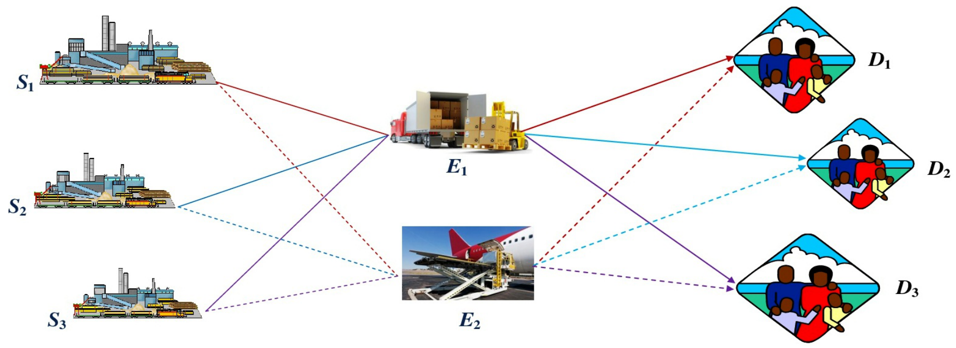

Fig. 4.1: Pictorial diagram of ST-LP.

p : number of potential facility sites,

l : number of conveyances,

γ_i : nonnegative weight of existing facility sites $(i = 1,2,\ldots,m)$,

a_i : availability at i-th existing facility site $(i = 1,2,\ldots,m)$,

b_j : demand at j-th potential facility site $(j = 1,2,\ldots,p)$,

c_k : limitation on capacity to transport the product by k-th conveyance $(k = 1,2,\ldots,l)$,

ε_k : nonnegative k-th conveyance cost per unit commodity $(k = 1,2,\ldots,l)$,

δ_k : nonnegative weights of k-th conveyance $(k = 1,2,\ldots,l)$,

(u_i, v_i): coordinates of i-th existing facility site $(i = 1,2,\ldots,m)$,

(x_j, y_j): coordinates of j-th potential facility site $(j = 1,2,\ldots,p)$,

w_{ijk} : amount of flow to be transported from i-th existing facility site to j-th potential facility site by means of each conveyance k,

W : $\{(w_{ijk}) : (i = 1,2,\ldots,m;\ j = 1,2,\ldots,p;\ k = 1,2,\ldots,l)\}$: the feasible set with respect to the matrix variable w,

W_B : $(w^B_{ijk} : i = 1,2,\ldots m;\ j = 1,2,\ldots,p;\ k = 1,2,\ldots,l)$, the initial basic feasible solution,

F : $\mathbb{R}^{2p} \times W$, where $(x,y) \in \mathbb{R}^{2p}$ and $w \in W$, the feasible set,

ϕ_k $= \varepsilon_k \varphi_k$: transportation cost function per unit flow from an existing facility site to a potential facility site by means of conveyance k.

- The solution space is continuous. The parameters are deterministic.
- The space in which potential facility sites are located is the planner. The potential facility sites are assumed as points.

- The transported products are homogeneous in nature. The modes of transport are heterogeneous in nature.
- The distance function is a hyperbolic approximation of Euclidean distance $\left(\varphi_k(u_i,v_i;x_j,y_j) = \sqrt{(u_i-x_j)^2+(v_i-y_j)^2+\delta_k}\right)$.
- The facilities are capacitated. No relationship exists between potential facility sites. The opening cost of new potential facility sites are ignored.

4.2.3 Model identification

In this subsection, a mathematical model is introduced based on FLP and STP. In fact, instead of determining the potential facility sites, this model asks for the amounts of flow to be transported from all existing facility sites to all potential facility sites by means of any conveyances such that the total transportation cost is minimized. The mathematical model of ST-LP can be stated as follows:

Model 4.1

$$\text{minimize} \quad Z_{(x,y,w)} = \sum_{i=1}^{m}\sum_{j=1}^{p}\sum_{k=1}^{l} \gamma_i w_{ijk} \phi_k(u_i,v_i;x_j,y_j) \tag{4.1}$$

$$\text{subject to} \quad \sum_{j=1}^{p}\sum_{k=1}^{l} w_{ijk} \le a_i \quad (i=1,2,\ldots,m), \tag{4.2}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} w_{ijk} \ge b_j \quad (j=1,2,\ldots,p), \tag{4.3}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ijk} \le c_k \quad (k=1,2,\ldots,l), \tag{4.4}$$

$$w_{ijk} \ge 0 \;\; \forall \;\; i,j,k. \tag{4.5}$$

The objective function (4.1) indicates to minimize the total transportation cost from existing facility sites to potential facility sites by means of any conveyance. Constraints (4.2) impose that the total amounts of each existing facility site cannot go surplus its availability. Constraints (4.3) enforce that the total items of each potential facility site satisfy its desired demand. Constraints (4.4) suggest that the total flow of each conveyance cannot exceed its capacity. Constraints (4.5) are non-negativity conditions.

4.2.4 Connection between ST-LP and STP

The transportation cost function of (4.1) is only dependent on the locations of the potential facility sites. If we determine the optimal location of potential facility sites, the cost functions convert into constant cost functions. Consequently, we use short notations (x_j^*, y_j^*) for optimal location and $\gamma_i \phi_k(u_i,v_i;x_j^*,y_j^*) = c_{ijk}$ (the unit transportation cost from i-th source to the j-th demand point by means of k-th conveyance). Hence, Model 4.1 is represented as follows:

Model 4.2

$$\text{minimize} \quad Z_{(w)} = \sum_{i=1}^{m}\sum_{j=1}^{p}\sum_{k=1}^{l} c_{ijk} w_{ijk} \tag{4.6}$$
$$\text{subject to} \quad \text{the constraints (4.2) to (4.5)},$$

which is the traditional form of an STP.

4.2.5 Characteristics properties

Herein, few fundamental propositions and a theorem are introduced to understand the nature of ST-LP.

Proposition 4.1 *A necessary and sufficient condition for a feasible solution of ST-LP is* $\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{p} b_j$ *and* $\sum_{k=1}^{l} c_k \geq \sum_{j=1}^{p} b_j$.

Proof: This property is known as feasibility condition, and it depends on the constraints of ST-LP. In fact, two problems, i.e., Models 4.1 and 4.2 have the same constraints. Moreover, the proof is given in [150] for the case of an STP. □

Proposition 4.2 *The feasible solution of ST-LP is never unbounded.*

Proof: From ST-LP constraints, we obtain:

$$\sum_{j=1}^{p}\sum_{k=1}^{l} w_{ijk} \leq a_i \quad (i = 1,2,\ldots,m),$$
$$\sum_{i=1}^{m}\sum_{k=1}^{l} w_{ijk} \geq b_j \quad (j = 1,2,\ldots,p),$$
$$\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ijk} \leq c_k \quad (k = 1,2,\ldots,l),$$
$$w_{ijk} \geq 0 \ \forall \ i,j,k.$$

From the above constraints, we can write $b_j \leq w_{ijk} \leq a_i$, $b_j \leq w_{ijk} \leq c_k \ \forall \ i,j$ and k, and furthermore $w_{ijk} \geq 0 \ \forall \ i,j$ and k. Then, $\inf\{0,b_j\} \leq w_{ijk} \leq \inf\{a_i,c_k\} \ \forall \ i,j$ and k. As $b_j > 0 \ \forall \ j$, we conclude that $0 \leq w_{ijk} \leq \inf\{a_i,c_k\} \ \forall \ i,j$ and k. This ends the proof of the proposition. □

Proposition 4.3 *The number of non-degenerated basic variables in ST-LP is at most* $(m+p+l-2)$.

Proof: This property also depends on the constraints. Here, both problems, i.e., Models 4.1 and 4.2 reveal the same constraints. Thus, this proposition coincides with the STP. Once again, the proof is provided in [150] for the STP. □

Proposition 4.4 *For the problem minimize*$_{(x,y,u)}$ $Z = \sum_{i=1}^{m}\sum_{j=1}^{p}\sum_{k=1}^{l} \gamma_i w_{ijk} \phi_k(u_i,v_i;x_j,y_j)$, $(w_{ijk}) \in W$, *the optimal solution exists at some extreme points of the feasible set* W *of ST-LP.*

Proof: Let $(x,y)=(x_j,y_j)$ $(j=1,2,\ldots,p)$ and $w_E \in \{(w^E_{ijk})$ $(i=1,2,\ldots,m;\ j=1,2,\ldots,p;$ $k=1,2,\ldots,l)$, extreme points of $W\}$. If we choose potential facility sites such that $(x^*,y^*)=(x^*_j,y^*_j)$ by determining optimal locations, then the problem becomes minimize$_{(u)}$ $Z=\sum_{i=1}^m\sum_{j=1}^p\sum_{k=1}^l \gamma_i w_{ijk}\phi_k(u_i,v_i;x^*_j,y^*_j)$, $(w_{ijk})\in W$, which is the traditional form of a STP. Then, it is never unbounded but has a solution at some extreme point $w_E \in W$. Hence, we conclude that (x^*,y^*,w_E) is the optimal solution at the extreme points of W to ST-LP. □

Proposition 4.5 *The number of basic feasible solutions of ST-LP is at most* $\binom{mpl}{m+p+l-2}$.

Proof: ST-LP has mpl variables and at most $m+p+l-2$ basic variables. Consequently, the number of basic feasible solutions of ST-LP is at most $\binom{mpl}{m+p+l-2}$. □

Theorem 4.1 *The objective function* $Z=\sum_{i=1}^m\sum_{j=1}^p\sum_{k=1}^l \gamma_i w^B_{ijk}\phi_k(u_i,v_i;x_j,y_j)$ *is a convex function in the joint variable* (x,y) *on* $\mathbb{R}^{2p}$.

Proof: It is well-known that a smooth function Z is convex on the region iff the corresponding Hessian matrix with Z is positive semidefinite on the region [130]. Let $Z=\sum_{j=1}^p Z_j$, where $Z_j=\sum_{i=1}^m\sum_{k=1}^l \gamma_i w^B_{ijk}\phi_k(u_i,v_i;x_j,y_j)$ and the terms w^B_{ijk} are constants. Here, let Z_j only depends on the variables x_j and y_j. The Hessian matrix for Z_j at (x_j,y_j) is

$$H_j=\begin{pmatrix} \frac{\partial^2 Z_j}{\partial x_j^2} & \frac{\partial^2 Z_j}{\partial x_j\partial y_j} \\ \frac{\partial^2 Z_j}{\partial y_j\partial x_j} & \frac{\partial^2 Z_j}{\partial y_j^2} \end{pmatrix}.$$

The principal minors of H_j are $\frac{\partial^2 Z_j}{\partial x_j^2}$ and $\det H_j$ (determinant of H_j).

Now,

$$\frac{\partial^2 Z_j}{\partial x_j^2}=\sum_{i=1}^m\sum_{k=1}^l \frac{\gamma_i\varepsilon_k w^B_{ijk}[(v_i-y_j)^2+\delta_k]}{[(u_i-x_j)^2+(v_i-y_j)^2+\delta_k]^{3/2}},$$

and $\det H_j = \dfrac{\partial^2 Z_j}{\partial x_j^2}\dfrac{\partial^2 Z_j}{\partial y_j^2}-\Big(\dfrac{\partial^2 Z_j}{\partial x_j\partial y_j}\Big)^2 \quad \Big(\text{since } \dfrac{\partial^2 Z_j}{\partial x_j\partial y_j}=\dfrac{\partial^2 Z_j}{\partial y_j\partial x_j}\Big)$

$$=\Big(\sum_{i=1}^m\sum_{k=1}^l \frac{\gamma_i\varepsilon_k w^B_{ijk}[(v_i-y_j)^2+\delta_k]}{[(u_i-x_j)^2+(v_i-y_j)^2+\delta_k]^{3/2}}\Big)\Big(\sum_{i=1}^m\sum_{k=1}^l \frac{\gamma_i\varepsilon_k w^B_{ijk}[(u_i-x_j)^2+\delta_k]}{[(u_i-x_j)^2+(v_i-y_j)^2+\delta_k]^{3/2}}\Big)$$
$$-\Big(\sum_{i=1}^m\sum_{k=1}^l \frac{\gamma_i\varepsilon_k w^B_{ijk}(u_i-x_j)(v_i-y_j)}{[(u_i-x_j)^2+(v_i-y_j)^2+\delta_k]^{3/2}}\Big)^2$$
$$=\Big(\sum_{i=1}^m\sum_{k=1}^l \big(\frac{\sqrt{\gamma_i\varepsilon_k w^B_{ijk}}(v_i-y_j)}{[(u_i-x_j)^2+(v_i-y_j)^2+\delta_k]^{3/4}}\big)^2\Big)\Big(\sum_{i=1}^m\sum_{k=1}^l \big(\frac{\sqrt{\gamma_i\varepsilon_k w^B_{ijk}}(u_i-x_j)}{[(u_i-x_j)^2+(v_i-y_j)^2+\delta_k]^{3/4}}\big)^2\Big)$$
$$-\Big(\sum_{i=1}^m\sum_{k=1}^l \frac{\sqrt{\gamma_i\varepsilon_k w^B_{ijk}}(v_i-y_j)}{[(u_i-x_j)^2+(v_i-y_j)^2+\delta_k]^{3/4}}\frac{\sqrt{\gamma_i\varepsilon_k w^B_{ijk}}(u_i-x_j)}{[(u_i-x_j)^2+(v_i-y_j)^2+\delta_k]^{3/4}}\Big)^2$$
$$+2\Big(\sum_{i=1}^m\sum_{k=1}^l \frac{\gamma_i\varepsilon_k w^B_{ijk}\delta_k}{[(u_i-x_j)^2+(v_i-y_j)^2+\delta_k]^{3/2}}\Big).$$

Now, $\left(\sum_{i=1}^{m}\sum_{k=1}^{l}\left(\frac{\sqrt{\gamma_i\varepsilon_k w_{ijk}^{B}}(v_i-y_j)}{[(u_i-x_j)^2+(v_i-y_j)^2+\delta_k]^{3/4}}\right)^2\right)\left(\sum_{i=1}^{m}\sum_{k=1}^{l}\left(\frac{\sqrt{\gamma_i\varepsilon_k w_{ijk}^{B}}(u_i-x_j)}{[(u_i-x_j)^2+(v_i-y_j)^2+\delta_k]^{3/4}}\right)^2\right) \geq$ $\left(\sum_{i=1}^{m}\sum_{k=1}^{l}\frac{\sqrt{\gamma_i\varepsilon_k w_{ijk}^{B}}(v_i-y_j)}{[(u_i-x_j)^2+(v_i-y_j)^2+\delta_k]^{3/4}}\frac{\sqrt{\gamma_i\varepsilon_k w_{ijk}^{B}}(u_i-x_j)}{[(u_i-x_j)^2+(v_i-y_j)^2+\delta_k]^{3/4}}\right)^2$ (by Cauchy-Schwarz inequality). As $\gamma_i > 0, \varepsilon_k \geq 0, w_{ijk}^{B} \geq 0, \delta_k \geq 0, (u_i - x_j)^2 \geq 0$ and $(v_i - y_j)^2 \geq 0$, we conclude that $\frac{\partial^2 Z_j}{\partial x_j^2} \geq 0$ and $\det H_j \geq 0$ for all values of x_j and y_j. Hence, Z_j is convex with respect to x_j and y_j. Let $\big((x_1,y_1),(x_2,y_2),\ldots,(x_p,y_p)\big)$ and $\big((x_1',y_1'),(x_2',y_2'),\ldots,(x_p',y_p')\big)$ be two arbitrary points of $\mathbb{R}^{2p}$, and $\alpha \in [0,1]$.

Herewith,

$$\begin{aligned}
&Z\Big(\alpha\big((x_1,y_1),(x_2,y_2),\ldots,(x_p,y_p)\big)+(1-\alpha)\big((x_1',y_1'),(x_2',y_2'),\ldots,(x_p',y_p')\big)\Big)\\
&=\sum_{j=1}^{p} Z_j\Big(\alpha\big((x_1,y_1),(x_2,y_2),\ldots,(x_p,y_p)\big)+(1-\alpha)\big((x_1',y_1'),(x_2',y_2'),\ldots,(x_p',y_p')\big)\Big)\\
&\leq \alpha\sum_{j=1}^{p} Z_j\big((x_1,y_1),(x_2,y_2),\ldots,(x_p,y_p)\big)+(1-\alpha)\sum_{j=1}^{p} Z_j\big((x_1',y_1'),(x_2',y_2'),\ldots,(x_p',y_p')\big)\\
&=\alpha Z\big((x_1,y_1),(x_2,y_2),\ldots,(x_p,y_p)\big)+(1-\alpha)Z\big((x_1',y_1'),(x_2',y_2'),\ldots,(x_p',y_p')\big).
\end{aligned}$$

Therefore, Z is convex in the variable (x,y) on $\mathbb{R}^{2p}$. □

4.3 Methodology

In this section, we first present a locate-allocate (Loc-Alloc) heuristic with its algorithm, and introduce an approximate heuristic algorithm for the proposed model.

4.3.1 Loc-Alloc heuristic approach

The Loc-Alloc heuristic approach was first introduced to solve the large-scale classical facility location problems by Cooper [29], which gives always a good solution in less computational time. We develop it to solve ST-LP. Here, the feasible set of our ST-LP program is a bounded convex set with a convex objective function. The optimal solution arises at an extreme point of the constraint set (by Propositions 4.2 and 4.4, and Theorem 4.1). The proposed heuristic is comprised of 2 parts. In Part 1, the heuristic seeks the initial location, and in Part 2 it finds the optimum locations. Here, at first, the locations are placed for p facilities from m existing sites. Then, the distances between p facilities and m existing locations are computed. If $p \leq m$, we easily calculate such distances; however, if $p > m$, we introduce a heuristic concept to resolve this issue. Initially, we consider the first m facility allocations as m existing sites, and we allocate the remaining $(p-m)$ facilities in some Euclidean points with large coordinates such that the distances of those coordinates become a very large number from facilities. Because of that, a large positive number is assigned for such distances which cannot be calculated. When we take the minimum among the distances, these large distances will not be affected. Now, it is already assumed that the distances are cost functions per unit commodity from the i-th existing facility site to the j-th potential facility site by means of k-th conveyances. We take these distances as the cost coefficients; herewith, then the problem converts into a classical STP (Model 4.2). Therefore, we solve it to generate the initial basic feasible solutions which are $W_B = (w^B_{ijk} : i = 1,2,\dots m;\ j = 1,2,\dots,p;\ k = 1,2,\dots,l)$; then for each such solution we solve the problem:

$$\text{minimize}_{(x,y)} \qquad Z^B = \sum_{j=1}^{p} Z^B_j \tag{4.7}$$
$$\text{subject to} \qquad \text{the constraints (4.2) to (4.5),}$$

where $Z^B_j = \sum_{i=1}^m \sum_{k=1}^l \gamma_i w^B_{ijk} \phi_k(u_i, v_i; x_j, y_j)$ $(j = 1,2,\dots,p)$. Then, we minimize all the functions $Z^B_j (j = 1,2,\dots,p)$ for minimizing Z^B. Here, the iterative formula are derived in similar way of Appendix A.1 to solve the problem (4.7). The iterations for (x_j, y_j) are as follows:

$$x^0_j = \frac{\sum_{i=1}^m \sum_{k=1}^l \gamma_i w^B_{ijk} u_i}{\sum_{i=1}^m \sum_{k=1}^l \gamma_i w^B_{ijk}} \quad (j = 1,2,\dots,p), \tag{4.8}$$

$$y^0_j = \frac{\sum_{i=1}^m \sum_{k=1}^l \gamma_i w^B_{ijk} v_i}{\sum_{i=1}^m \sum_{k=1}^l \gamma_i w^B_{ijk}} \quad (j = 1,2,\dots,p), \tag{4.9}$$

$$x^{r+1}_j = \frac{\sum_{i=1}^m \sum_{k=1}^l \frac{\varepsilon_k \gamma_i w^B_{ijk} u_i}{\varphi_k(u_i, v_i; x^r_j, y^r_j)}}{\sum_{i=1}^m \sum_{k=1}^l \frac{\varepsilon_k \gamma_i w^B_{ijk}}{\varphi_k(u_i, v_i; x^r_j, y^r_j)}} \quad (j = 1,2,\dots,p;\ r \in \mathbb{N}), \tag{4.10}$$

$$y^{r+1}_j = \frac{\sum_{i=1}^m \sum_{k=1}^l \frac{\varepsilon_k \gamma_i w^B_{ijk} v_i}{\varphi_k(u_i, v_i; x^r_j, y^r_j)}}{\sum_{i=1}^m \sum_{k=1}^l \frac{\varepsilon_k \gamma_i w^B_{ijk}}{\varphi_k(u_i, v_i; x^r_j, y^r_j)}} \quad (j = 1,2,\dots,p;\ r \in \mathbb{N}), \tag{4.11}$$

where $\varphi_k(u_i, v_i; x^r_j, y^r_j) = [(u_i - x^r_j)^2 + (v_i - y^r_j)^2 + \delta_k]^{1/2}$. Let $S = \{Z^*_n :$ optimum value for Z^B for n-th basic feasible solution$\}$. Clearly, S is a finite set according to Proposition 4.5. Hence, it has a minimum; the optimal value of the objective function Z^* for ST-LP will be $Z^* = \min S$. If the optimum has been attained at $n = q$, then the optimal solutions are (x^q_j, y^q_j), $(j = 1,2,\dots,p)$, and $w^B_{ijk_q}$ $(i = 1,2,\dots,m; j = 1,2,\dots,p;\ k = 1,2,\dots,l)$, where (x^q_j, y^q_j) indicates (x_j, y_j) for the q-th basic feasible solution and $w^B_{ijk_q}$ are the values of w^B_{ijk} at this solution.

4.3.2 A Loc-Alloc algorithm

The steps of the Loc-Alloc heuristic algorithm are as follows:

Step 1: First, the initial locations are chosen for each of the p facilities from m existing sites.

Step 2: Therefore, two cases occur: if $p \leq m$, then it can be easily calculated the distances between the existing and the potential facility sites. But, if $p > m$, then the first m facility allocations are considered as m existing sites, and the remaining $(p - m)$ facilities are allocated in some large number of coordinates such that the distances of those coordinates become a very large number from facilities. Because of that, a large positive number is assigned for such distances which cannot be calculated. When the minimum among the distances is taken, these large distances will stay unaffected.

Step 3: It is already assumed that the distances are taken as the cost coefficients. Then, the formulation is converted into an STP.

Step 4: Using MATLAB computation software, we can easily find the set of initial basic feasible solutions W_B.

Step 5: Employing W_B from Step 4 and the iterations given by (4.8) to (4.11), we solve ST-LP to seek a new set of potential locations.

Step 6: If any of the locations has been rounded up to 4 decimal places, then repeat Step 5; otherwise: stop.

Remark 1: The optimal solutions can be sought where the existing facility sites are allocated to the potential facility sites $(j = 1, 2, \ldots, p)$. Subsequently, subsests are provided by $P_1 = \{(u_1, v_1), \ldots, (u_{j_1}, v_{j_1})\}, P_2 = \{(u_{j_1+1}, v_{j_1+1}), \ldots, (u_{j_2}, v_{j_2})\}, \ldots, P_p = \{(u_{j_{p-1}+1}, v_{j_{p-1}+1}), \ldots, (u_m, v_m)\}$, respectively [93].

4.3.3 An approximate heuristic algorithm

Herein, we introduce an approximate heuristic algorithm for solving ST-LP program. This heuristic approach was also proposed by Cooper [29] to solve the traditional FLP. Here, we employ a trick to reduce the possible cases obtained by our Loc-Alloc heuristic. Due to this matter, the approximate heuristic is more suitable for large-scale entries in less computational effort. The following steps are illustrated for locating the optimal facility sites:

Step 1: We generate an $m \times m$ matrix of distances between the existing sites. We address all such distances as cost coefficients.

Step 2: We consider two cases: if $p = 1$, then for each of the columns we built its sum; the column index with the minimum sum indicates at place, the single facility is to be located. If $p > 1$, then we generate all possible combinations of the p facility sites from m existing sites. For each combination, the existing sites are considered as potential facility sites, and other existing sites are designated depending on which potential facility sites have the smallest distance. Finally, all designated distances are summed up. The final possible locations are those with the minimum sum of costs.

Step 3: With these possible locations, we determine the cost coefficients. Afterwards, the set of initial basic feasible solutions W_B are easily obtained by MATLAB computation software.

Step 4: Using W_B from Step 3 and the formula (4.8) to (4.11), we locate the facilities at the best place.

Step 5: Repeat Step 4 until no further changes are possible when correcting up to 4 decimal places; otherwise: stop.

4.4 Application example

In this section, a real-life based example is incorporated to illustrate that the stated model and the approaches work well in locating the potential facility sites into the Euclidean plane with the objective of minimizing the total transportation cost by different modes of transport. Here, we consider an industrial organization wishing to set-up some new plants with the goal of minimizing the total transportation cost by means of conveyances from the existing plants to new plants. The organization has 4 plants: S_1, S_2, S_3 and S_4, and it wants to establish 3 new plants: D_1, D_2 and D_3. Products are transported by 3 different conveyances: E_1, E_2 and E_3. The availability at S_1, S_2, S_3 and S_4, the requirement of the plants D_1, D_2, and D_3, and the capacity of conveyances E_1, E_2 and E_3 are given. The position and the weights of the plants S_1, S_2, S_3 and S_4 are also known. Data of real-life scenario are provided in Tables 4.1-4.3.

Table 4.1: Cost of matrix (c_{ijk}).

Conveyance	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	Capacity (e_k) 40 80 85
	D_1			D_2			D_3			Supply (a_i)
S_1	c_{111}	c_{112}	c_{113}	c_{121}	c_{122}	c_{123}	c_{131}	c_{132}	c_{133}	20
S_2	c_{211}	c_{212}	c_{213}	c_{221}	c_{222}	c_{223}	c_{231}	c_{232}	c_{233}	85
S_3	c_{311}	c_{312}	c_{313}	c_{321}	c_{322}	c_{323}	c_{331}	c_{332}	c_{333}	40
S_4	c_{411}	c_{412}	c_{413}	c_{421}	c_{422}	c_{423}	c_{431}	c_{432}	c_{433}	60
Demand (b_j)	50			85			70			

Table 4.2: Positions & weights of the existing plants.

	Position (u_i, v_i)	Weight (γ_i)
S_1	$(4,6)$	0.2
S_2	$(6,10)$	0.1
S_3	$(8,7)$	0.4
S_4	$(10,10)$	0.3

Table 4.3: Conveyances cost and weights.

	Conveyance cost	Weight (δ_k)
E_1	10	0.3
E_2	30	0.5
E_3	20	0.2

The heuristic approaches are coded in C++ and conducted using a code-block compiler on a Lenovo z580 computer with 2.50 GHz Intel (R) core (TM) i5-3210M CPU with 4 GB RAM. In contrast, the computational results are compared with Linux terminal on a computer with Intel(R) Core (TM) i3-4130 CPU @3.40 GHz with 4 GB RAM.

4.4.1 Performance of the Loc-Alloc heuristic

Here, we mainly focus on the following topics for solving ST-LP by our Loc-Alloc heuristic:

- First, 3 initial locations are selected for each of 3 plants based on Table 4.2. Then, four *cases* occur, which are displayed in Tables 4.4-4.7.

Table 4.4: Case 4.1.

	Position	Weight
D_1	$(4,6)$	0.2
D_2	$(6,10)$	0.3
D_3	$(8,7)$	0.4

Table 4.5: Case 4.2.

	Position	Weight
D_1	$(6,10)$	0.1
D_2	$(8,7)$	0.4
D_3	$(10,10)$	0.3

Table 4.6: Case 4.3.

	Position	Weight
D_1	$(8,7)$	0.4
D_2	$(10,10)$	0.3
D_3	$(4,6)$	0.2

Table 4.7: Case 4.4.

	Position	Weight
D_1	$(10,10)$	0.3
D_2	$(4,6)$	0.2
D_3	$(6,10)$	0.1

- We determine the distances between existing and initial locations of plants by using Tables 4.4-4.7, and put the distances as cost coefficients in Tables 4.8-4.11, respectively.

Table 4.8: Cost Coefficients (c_{ijk}) for Table 4.4.

$c_{111} = 0$	$c_{221} = 0$	$c_{311} = 41.593$	$c_{411} = 72.318$
$c_{112} = 0$	$c_{212} = 135.830$	$c_{312} = 124.779$	$c_{412} = 217.370$
$c_{113} = 0$	$c_{213} = 82.945$	$c_{313} = 82.945$	$c_{413} = 144.449$
$c_{121} = 45.055$	$c_{211} = 45.055$	$c_{321} = 36.469$	$c_{421} = 40.373$
$c_{122} = 135.830$	$c_{222} = 0$	$c_{322} = 109.407$	$c_{422} = 121.860$
$c_{123} = 89.888$	$c_{223} = 0$	$c_{323} = 72.663$	$c_{423} = 80.498$
$c_{131} = 41.593$	$c_{231} = 36.469$	$c_{331} = 0$	$c_{431} = 36.469$
$c_{132} = 124.779$	$c_{232} = 109.407$	$c_{332} = 0$	$c_{432} = 110.227$
$c_{133} = 82.945$	$c_{233} = 72.663$	$c_{333} = 0$	$c_{433} = 72.663$

Table 4.9: Cost Coefficients (c_{ijk}) for Table 4.5.

$c_{111} = 45.055$	$c_{221} = 36.469$	$c_{311} = 36.469$	$c_{411} = 40.373$
$c_{112} = 135.830$	$c_{212} = 0$	$c_{312} = 110.227$	$c_{412} = 121.860$
$c_{113} = 89.888$	$c_{213} = 0$	$c_{313} = 72.663$	$c_{413} = 80.498$
$c_{121} = 41.593$	$c_{211} = 0$	$c_{321} = 0$	$c_{421} = 36.469$
$c_{122} = 125.499$	$c_{222} = 110.227$	$c_{322} = 0$	$c_{422} = 110.227$
$c_{123} = 82.945$	$c_{223} = 72.663$	$c_{323} = 0$	$c_{423} = 72.663$
$c_{131} = 72.318$	$c_{231} = 40.373$	$c_{331} = 36.469$	$c_{431} = 0$
$c_{132} = 217.370$	$c_{232} = 121.860$	$c_{332} = 110.227$	$c_{432} = 0$
$c_{133} = 144.499$	$c_{233} = 80.498$	$c_{333} = 72.663$	$c_{433} = 0$

Table 4.10: Cost Coefficients (c_{ijk}) for Table 4.6.

$c_{111} = 41.593$	$c_{221} = 72.249$	$c_{311} = 0$	$c_{411} = 36.469$
$c_{112} = 125.499$	$c_{212} = 110.227$	$c_{312} = 0$	$c_{412} = 110.227$
$c_{113} = 82.945$	$c_{213} = 72.663$	$c_{313} = 0$	$c_{413} = 75.663$
$c_{121} = 72.249$	$c_{211} = 36.469$	$c_{321} = 36.469$	$c_{421} = 0$
$c_{122} = 217.370$	$c_{222} = 217.370$	$c_{322} = 110.227$	$c_{422} = 0$
$c_{123} = 144.499$	$c_{223} = 144.499$	$c_{323} = 72.663$	$c_{423} = 0$
$c_{131} = 0$	$c_{231} = 45.055$	$c_{331} = 41.593$	$c_{431} = 72.249$
$c_{132} = 0$	$c_{232} = 135.830$	$c_{332} = 125.499$	$c_{432} = 217.370$
$c_{133} = 0$	$c_{233} = 89.988$	$c_{333} = 82.945$	$c_{433} = 144.499$

Table 4.11: Cost Coefficients (c_{ijk}) for Table 4.7.

$c_{111} = 72.318$	$c_{221} = 45.055$	$c_{311} = 36.469$	$c_{411} = 0$
$c_{112} = 217.370$	$c_{212} = 121.119$	$c_{312} = 110.227$	$c_{412} = 0$
$c_{113} = 144.499$	$c_{213} = 80.498$	$c_{313} = 72.663$	$c_{413} = 0$
$c_{121} = 0$	$c_{211} = 40.373$	$c_{321} = 41.593$	$c_{421} = 72.318$
$c_{122} = 0$	$c_{222} = 44.434$	$c_{322} = 125.499$	$c_{422} = 217.370$
$c_{123} = 0$	$c_{223} = 89.888$	$c_{323} = 83.945$	$c_{423} = 144.499$
$c_{131} = 45.055$	$c_{231} = 0$	$c_{331} = 36.469$	$c_{431} = 40.373$
$c_{132} = 44.434$	$c_{232} = 0$	$c_{332} = 110.227$	$c_{432} = 121.860$
$c_{133} = 89.888$	$c_{233} = 0$	$c_{333} = 72.663$	$c_{433} = 80.498$

- MATLAB computation software is employed for possible initial BFSs, which are shown in Tables 4.12-4.15.

Table 4.12: Initial BFS (w^B_{ijk}) for Table 4.8.

$w^B_{111} = 0$	$w^B_{211} = 0$	$w^B_{311} = 0$	$w^B_{411} = 30$
$w^B_{112} = 0$	$w^B_{212} = 0$	$w^B_{312} = 0$	$w^B_{412} = 0$
$w^B_{113} = 20$	$w^B_{213} = 0$	$w^B_{313} = 0$	$w^B_{413} = 0$
$w^B_{121} = 0$	$w^B_{221} = 0$	$w^B_{321} = 0$	$w^B_{421} = 0$
$w^B_{122} = 0$	$w^B_{222} = 40$	$w^B_{322} = 0$	$w^B_{422} = 0$
$w^B_{123} = 0$	$w^B_{223} = 45$	$w^B_{323} = 0$	$w^B_{423} = 0$
$w^B_{131} = 0$	$w^B_{231} = 0$	$w^B_{331} = 0$	$w^B_{431} = 10$
$w^B_{132} = 0$	$w^B_{232} = 0$	$w^B_{332} = 40$	$w^B_{432} = 0$
$w^B_{133} = 0$	$w^B_{233} = 0$	$w^B_{333} = 0$	$w^B_{433} = 20$

Table 4.13: Initial BFS (w^B_{ijk}) for Table 4.9.

$w^B_{111} = 0$	$w^B_{211} = 0$	$w^B_{311} = 0$	$w^B_{411} = 0$
$w^B_{112} = 0$	$w^B_{212} = 0$	$w^B_{312} = 0$	$w^B_{412} = 0$
$w^B_{113} = 0$	$w^B_{213} = 50$	$w^B_{313} = 0$	$w^B_{413} = 0$
$w^B_{121} = 20$	$w^B_{221} = 10$	$w^B_{321} = 0$	$w^B_{421} = 0$
$w^B_{122} = 0$	$w^B_{222} = 0$	$w^B_{322} = 20$	$w^B_{422} = 0$
$w^B_{123} = 0$	$w^B_{223} = 15$	$w^B_{323} = 20$	$w^B_{423} = 0$
$w^B_{131} = 0$	$w^B_{231} = 10$	$w^B_{331} = 0$	$w^B_{421} = 0$
$w^B_{132} = 0$	$w^B_{232} = 0$	$w^B_{332} = 60$	$w^B_{432} = 0$
$w^B_{133} = 0$	$w^B_{233} = 0$	$w^B_{333} = 0$	$w^B_{433} = 0$

Table 4.14: Initial BFS (w^B_{ijk}) for Table 4.10.

$w^B_{111} = 0$	$w^B_{211} = 0$	$w^B_{311} = 0$	$w^B_{411} = 0$
$w^B_{112} = 0$	$w^B_{212} = 0$	$w^B_{312} = 20$	$w^B_{412} = 0$
$w^B_{113} = 0$	$w^B_{213} = 10$	$w^B_{313} = 20$	$w^B_{413} = 0$
$w^B_{121} = 0$	$w^B_{221} = 25$	$w^B_{321} = 0$	$w^B_{421} = 0$
$w^B_{122} = 0$	$w^B_{222} = 0$	$w^B_{322} = 0$	$w^B_{422} = 60$
$w^B_{123} = 0$	$w^B_{223} = 0$	$w^B_{323} = 0$	$w^B_{423} = 0$
$w^B_{131} = 0$	$w^B_{231} = 15$	$w^B_{331} = 0$	$w^B_{421} = 0$
$w^B_{132} = 0$	$w^B_{232} = 0$	$w^B_{332} = 0$	$w^B_{432} = 0$
$w^B_{133} = 20$	$w^B_{233} = 35$	$w^B_{333} = 0$	$w^B_{433} = 0$

Table 4.15: Initial BFS (w^B_{ijk}) for Table 4.11.

$w^B_{111} = 0$	$w^B_{211} = 0$	$w^B_{311} = 0$	$w^B_{411} = 0$
$w^B_{112} = 0$	$w^B_{212} = 0$	$w^B_{312} = 0$	$w^B_{412} = 50$
$w^B_{113} = 0$	$w^B_{213} = 0$	$w^B_{313} = 0$	$w^B_{413} = 0$
$w^B_{121} = 0$	$w^B_{221} = 15$	$w^B_{321} = 15$	$w^B_{421} = 15$
$w^B_{122} = 0$	$w^B_{222} = 0$	$w^B_{322} = 0$	$w^B_{422} = 0$
$w^B_{123} = 20$	$w^B_{223} = 0$	$w^B_{323} = 25$	$w^B_{423} = 0$
$w^B_{131} = 0$	$w^B_{231} = 0$	$w^B_{331} = 0$	$w^B_{421} = 0$
$w^B_{132} = 0$	$w^B_{232} = 30$	$w^B_{332} = 0$	$w^B_{432} = 0$
$w^B_{133} = 0$	$w^B_{233} = 40$	$w^B_{333} = 0$	$w^B_{433} = 0$

- Finally, we explore the C++ programming language to execute the results for Tables 4.12-4.15, and place the obtained results in Table 4.16.

Table 4.16: Computational results for Tables 4.12-4.15.

Initial BFS	Location of D_1	Location of D_2	Location of D_3	Value of Z
Table 4.12	$(8.791, 9.194)$	$(6.000, 10.000)$	$(8.128, 7.192)$	1595.000
Table 4.13	$(6.000, 10.000)$	$(7.913, 7.034)$	$(9.987, 10.000)$	1061.960
Table 4.14	$(7.984, 7.024)$	$(9.968, 10.000)$	$(5.222, 8.443)$	1170.334
Table 4.15	$(10.000, 10.000)$	$(7.874, 7.030)$	$(6.000, 10.000)$	1031.294

4.4.2 Performance of the approximate heuristic

This heuristic is same as the Loc-Alloc heuristic without a trick of reducing possible cases for locating the initial potential sites. Employing this trick (i.e., Step 2), we reduce the cases which are derived from the Loc-Alloc heuristic. For that reason, we need less computational burden.

- Step 1 is the same as our Loc-Alloc heuristic, 3 initial locations are chosen from 4 existing plants. The *cases* are already provided in Tables 4.4-4.7.
- Step 2 is the most important for this heuristic. Here, we first calculate the distances between assigned locations and the rest site for each case. Then, the distances are summed up for each case. They are given by 896.279 as Case 1, 955.001 as Case 2, 732.866 as Case 3, and 688.756 as Case 4. Eventually, the smallest distance is taken from the above cases, which arises in Case 4. Therefore, the final initial locations for the plants are stated in Case 4.
- Now, we repeat our Loc-Alloc heuristic only for Case 4. Finally, the optimal solution is obtained; it is shown in Table 4.17.

Table 4.17: Computational results for Table 4.15 (for Case 4).

Initial BFS	Location of D_1	Location of D_2	Location of D_3	Value of Z
Table 4.15	$(10.000, 10.000)$	$(7.874, 7.030)$	$(6.000, 10.000)$	1031.294

4.4.3 Computational results and discussion

Here, optimal solutions of the application example are exhibited from 2 heuristics. The following optimal solution is obtained by both the heuristic approaches using Tables 4.16 and 4.17; and we display it in Table 4.18. The convergence performance of both the heuristics and the solution are depicted in Figures 4.2-4.3.

Table 4.18: The optimal solution of the proposed ST-LP.

Initial BFS	Location of D_1	Location of D_2	Location of D_3	Value of Z
Table 4.15	$(10.000, 10.000)$	$(7.874, 7.030)$	$(6.000, 10.000)$	1031.294

4.5 Sensitivity analysis

In this section, the range of the optimal solution in ST-LP is investigated by varying the parameters in the objective function and the constraints. For this purpose, the resiliency of optimal potential facility sites for new facilities is analyzed while changing the estimation

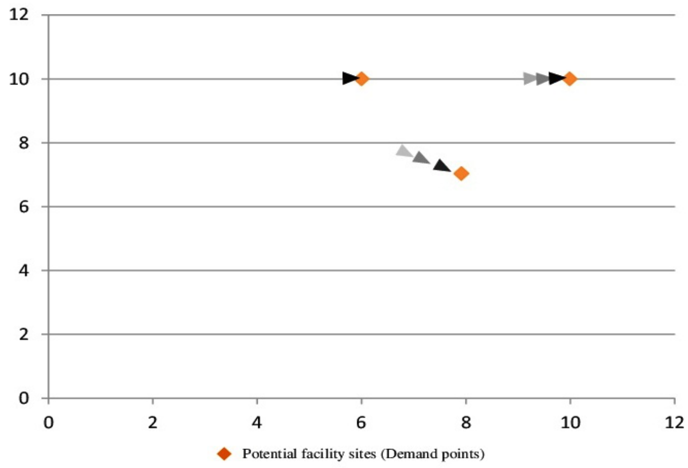

Fig. 4.2: Performance of both the heuristic approaches.

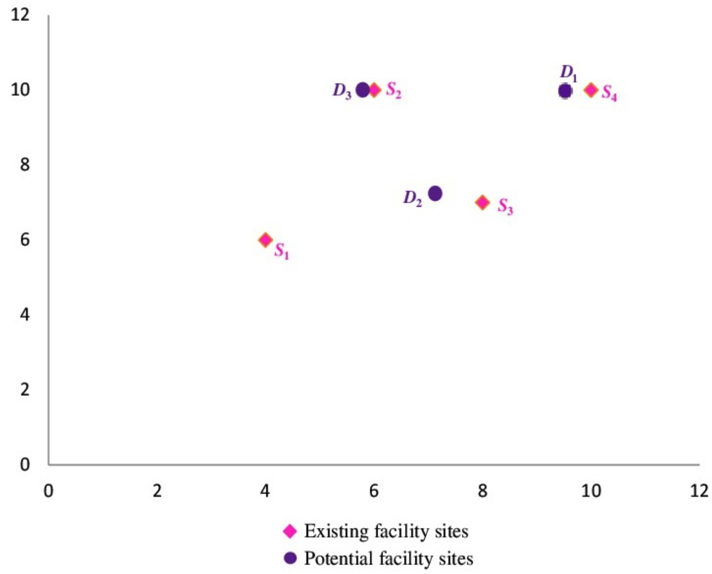

Fig. 4.3: The existing and potential facilities in the example.

of weights of the existing facility sites, supply, demand, and conveyance parameters. For the ST-LP program, it is difficult to choose the ranges where parametric changes can be made and the given solution still remains optimal. But the main problem arises when the number of variables and constraints is of large size. For that reason, a simple procedure is already introduced in Chapter 3 (see Section 3.5) to analyze the sensitivity of parameters. Here, we repeat the same steps (Steps 1- 4) to obtain the validity ranges of the parameters in ST-LP.

Sensitivity analysis for supply, demand and conveyance parameters:
Let a_i be changed to $a_i^* = a_i + \beta_i$ $(i = 1,2,3,4)$, b_j be changed to $b_j^* = b_j + \beta_j$ $(j = 1,2,3)$ and c_k be changed to $c_k^* = c_k + \beta_k$ $(k = 1,2,3)$. Using the proposed procedure, we easily find the values of a_i^*, b_j^* and c_k^*; they are displayed in Table 4.19. Note that the ranges of the other parameters in ST-LP are resolved in a similar way.

Table 4.19: The ranges of supply, demand and conveyance parameters.

Actual values of a_i, b_j and c_k	Changing values of a_i, b_j and c_k
$a_1 = 20$	$20 \leq a_1^* < \infty$
$a_2 = 85$	$85 \leq a_2^* < \infty$
$a_3 = 40$	$40 \leq a_3^* < \infty$
$a_4 = 60$	$60 \leq a_4^* < \infty$
$b_1 = 50$	$-\infty < b_1^* \leq 50$
$b_2 = 85$	$-\infty < b_2^* \leq 85$
$b_3 = 70$	$-\infty < b_3^* \leq 70$
$c_1 = 40$	$40 \leq c_1^* < \infty$
$c_2 = 80$	$80 \leq c_2^* < \infty$
$c_3 = 85$	$85 \leq c_3^* < \infty$

4.6 Conclusion

This chapter has been presented a new practical problem for a transportation network system with the goal of minimizing the total transportation cost by different types of transportation modes on the entire supply chain and to seek facilities sites for plants. According to our knowledge, for the first time in research, we have introduced a connection between STP and FLP. Afterwards, a theorem and few fundamental propositions on ST-LP have been stated to inspect the nature of our formulation. In addition to the preceding achievements, we have improved two heuristic approaches to solve the stated problem in an efficient way. The proposed formulation and developed heuristics have been evaluated by a real-life based example. Therefore, the derived computational outcomes from our two heuristics have been compared with suggestions for locating the facilities. In comparison, the Loc-Alloc heuristic approach is appropriate to solve the ST-LP program with small sizes. The approximate heuristic is more suitable for ST-LP of larger size since it can generate optimal solutions in less computational burden. Finally, a sensitivity analysis has been provided to validate the ranges of the parameters in our formulation. Moreover, the formulation presented here can be employed in large-scale industrial applications, such as the manufacturing of plants, genetic-metabolic, financial and further applications.