Chapter 3

An approximation approach for fixed -charge transportation-location problem[1]

This chapter describes *fixed-charge transportation-location problem* (FCT-LP) that integrates FCTP and FLP. In fact, FCT-LP, speculation of an FCTP looks for where and how to impose the facilities with the goal that the overall transportation cost with a fixed-charge cost from the existing facility sites to potential facility sites is reduced. A novel approximation approach is incorporated to solve the proposed model for extracting optimal solution. An experimental design is consolidated to demonstrate the proficiency and viability of the proposed consideration. The chapter ends with conclusions.

3.1 Introduction

FLP and FCTP are correlated with the distribution framework. Locating the facilities in the best places and transportation with the fixed costs from the existing facility sites to the best facility locations are the main objectives in FCT-LP. It is a cost minimization problem obtained by integrating FLP and FCTP that can be solved in a continuous planner surface with Euclidean distance. In fact, FCT-LP is an NP-hard problem with nonlinear objective function which is neither convex nor concave. So, it is generally difficult to solve since local optimal does not always imply global optimality by an exact method. Due to this fact, a new approximation approach is proposed to solve large-scale FCT-LP. The main aim is to introduce a way to connect FLP and FCTP, and a new approximation approach is included to tackle it. This approach can be applied to plant location problems where minimizing total transportation costs (overall fixed and variable cost) is taken into consideration as the main priority. We believe that this model is more reasonable than classical FCTP and FLP. It may be useful to the model of emergency services and online shopping systems.

[1]The content of this chapter has been communicated (revised version) in ***Journal of Industrial & Management Optimization, SCIE, I.F: 1.366***.

3.2 Mathematical description

Here, the proposed problem is delineated. Based on the problem, the mathematical model is depicted on the premise of the following assumptions and notations. The model formulation, a connection between this formulation and FCTP and the characteristic properties are displayed.

3.2.1 Problem definition

Here, a new geographical practical problem is inspected from an economical aspect. Figure 3.1 illustrates the FCT-LP network including various existing facility sites, add up to transportation cost, fixed costs (represent setup costs underway frameworks, toll charges on an expressway, costs for building streets, or landing expenses at air terminals) and potential facility sites. Products are sent from existing facility sites to potential facility sites with the point of minimizing the overall fixed cost and variable cost. Assume that, there are three existing facility sites, for example, O1, O2, and O3, four toll charges like F1, F2, F3, and F4 and four potential facility sites such as P1, P2, P3, and P4. The related supply and demand of the existing facility sites and the potential facility sites are known. Furthermore, the locations of O1, O2, and O3 are given. But, the locations of F1, F2, F3, and F4 are not known on the planner surface. The line denotes the transportation cost function per unit commodity from O1, O2 and O3 to P1, P2, P3, and P4 along with fixed costs for toll charges F1, F2, F3, and F4. In this circumstance, the DM has to choose the optimal locations of the potential facility sites in such a way that the total fixed-charge transportation cost from existing facility sites to potential facility sites is minimized. Due to this reason, a connection between FLP and FCTP is made.

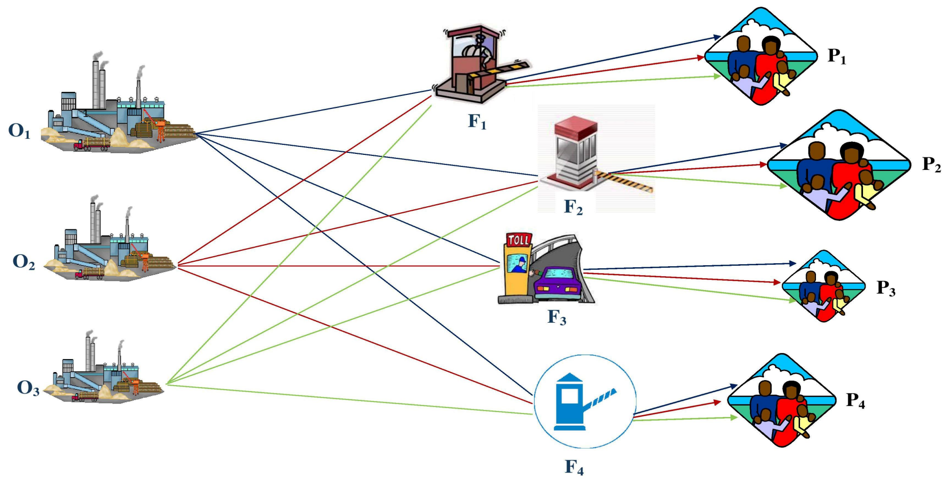

Fig. 3.1: Network for FCT-LP.

3.2.2 Assumptions and Notations

The following assumptions and notations are used to design the model:

- The solution space is continuous;
- Parameters are deterministic;
- The space in which potential facility sites are located is planner;
- Potential facility sites are assumed as points;
- Type of distance is Euclidean distance $\left(\phi(a_i, b_i; x_j, y_j) = \sqrt{(a_i - x_j)^2 + (b_i - y_j)^2}\right)$;
- Facilities are capacitated;
- No relationship between potential facility sites;
- Ignoring the opening cost of new potential facility sites;
- Fixed cost for opening each route;

m : number of existing facility sites (origins);

p : number of potential facility sites (demand points);

α_i : non-negative weights of existing facility sites $(i = 1, 2, \ldots, m)$;

r_i : availability at the i-th existing facility site $(i = 1, 2, \ldots, m)$;

s_j : demand at the j-th potential facility site $(j = 1, 2, \ldots, p)$;

(a_i, b_i): co-ordinate of i-th existing facility site $(i = 1, 2, \ldots, m)$;

(x_j, y_j): co-ordinate of j-th potential facility site $(j = 1, 2, \ldots, p)$;

w_{ij} : decision variables (amounts of flow to be shipped from the i-th existing facility site to the j-th potential facility site);

F : feasible set;

f_{ij} : fixed cost (for opening the i-th existing facility site to the j-th potential facility site route);

u_{ij} : (0-1) variable that equals 1 iff $x_{ij} > 0$ (0 otherwise);

ϕ : transportation cost function per unit commodity from the i-th existing facility site to the j-th potential facility site;

3.2.3 Model formulation

Here, a mathematical model is designed based on FLP and FCTP. In addition to minimize the transportation cost, we are mainly interested to find the optimal locations by choosing the potential facility sites. The mathematical model of FCT-LP is considered as follows:

Model 3.1

$$\text{minimize} \quad Z = \sum_{i=1}^{m}\sum_{j=1}^{p} \alpha_i w_{ij} \phi(a_i, b_i; x_j, y_j) + \sum_{i=1}^{m}\sum_{j=1}^{p} f_{ij} u_{ij}, \tag{3.1}$$

$$\text{subject to} \quad \sum_{j=1}^{p} w_{ij} \leq r_i \quad (i = 1, 2, \dots, m), \tag{3.2}$$

$$\sum_{i=1}^{m} w_{ij} \geq s_j \quad (j = 1, 2, \dots, p), \tag{3.3}$$

$$w_{ij} \geq 0 \ \forall \ i \text{ and } j, \tag{3.4}$$

$$u_{ij} = 0 \ \text{ if } \ w_{ij} = 0, \tag{3.5}$$

$$u_{ij} = 1 \ \text{ if } \ w_{ij} > 0. \tag{3.6}$$

The objective function (3.1) intends to minimize the total transportation cost along with fixed cost from existing facility sites to potential facility sites. Constraints (3.2) enforce that the total items of each existing facility site which cannot surpass its sum accessible. Constraints (3.3) force that the total flow to each potential facility site ought to fulfill its desired demand. Constraints (3.4) are the non-negativity conditions. Constraints (3.5) to (3.6) suggest that cost of distributing no items along route (i, j) is zero; yet any positive goods acquires a fixed cost.

3.2.4 Connection between FCT-LP and FCTP

The objective function (3.1) of Model 3.1 relies upon the location of potential facility sites. From Figure 3.1, it can be seen that if the locations of potential facility sites are riveted, then the set of cost functions become a constant cost function i.e., $\phi(a_i, b_i; x_j, y_j) = t_{ij}$ and if $\alpha_i t_{ij}$ is picked as c_{ij} (unit transportation cost from source to demand point). Consequently, the objective function of Model 3.1 is lessened; and it progresses toward becoming as Model 3.2.

Model 3.2

$$\text{minimize} \quad Z = \sum_{i=1}^{m}\sum_{j=1}^{p} (c_{ij} w_{ij} + f_{ij} u_{ij}), \tag{3.7}$$

$$\text{subject to} \quad \text{the constraints (3.2) to (3.6).}$$

which is the traditional form of FCTP. Hence, for the constant cost function of FCT-LP becomes a FCTP.

3.2.5 Characteristics of FCT-LP

Herein, some fundamental propositions and theorem are described to perceive the nature of FCT-LP.

Proposition 3.1 *A necessary and sufficient condition for the problem FCT-LP is that* $\sum_{i=1}^{m} r_i \geq \sum_{j=1}^{p} s_j$.

Proof : Straightforward. □

Proposition 3.2 *The feasible solution of FCT-LP is never unbounded.*

Proof : The constraints of the FCT-LP are as follows:

$$\begin{aligned}
&\sum_{j=1}^{p} w_{ij} \leq r_i \quad (i = 1,2,\ldots,m),\\
&\sum_{i=1}^{m} w_{ij} \geq s_j \quad (j = 1,2,\ldots,p),\\
&w_{ij} \geq 0 \ \ \forall \ \ i \text{ and } j,\\
&u_{ij} = 0 \ \text{ if } \ w_{ij} = 0,\\
&u_{ij} = 1 \ \text{ if } \ w_{ij} > 0.
\end{aligned}$$

So, $s_j \leq w_{ij} \leq r_i \, \forall \; i$ and j, and furthermore $w_{ij} \geq 0 \ \forall \ i$ and j. It can be easily inferred that $\inf(0,s_j) \leq w_{ij} \leq r_i \ \forall \ i$ and j, presently since $s_j > 0 \ \forall \ j$ then $0 \leq w_{ij} \leq r_i \ \forall \ i$ and j. This demonstrates the proof of the proposition. □

Proposition 3.3 *The number of basic variables in FCT-LP is at most* $(m + p - 1)$.

Proof : This property depends on the constraints. Here, the constraints of two problems are accepted to be same. Thus, this proposition is moreover same as FCTP. □

Proposition 3.4 *For the problem minimize* $Z = \sum_{i=1}^{m} \sum_{j=1}^{p} \left[\alpha_i w_{ij} \phi(a_i,b_i;x_j,y_j) + f_{ij} u_{ij}\right]$, $w_{ij} \in F$ *an optimal solution exists at an extreme point of the convex set F of feasible solutions to FCT-LP.*

Proof : Let $(x,y) \in \{(x_j,y_j),\ j = 1,2,...,p\}$, $w \in \{w_{ij},\ i = 1,2,\ldots,m,\ j = 1,2,\ldots,p\}$ and $w_E \in \{w^E_{ij},$ set of all extreme points$\}$. If the destination such that $(x,y) = (x^*_j,y^*_j)$ is chosen by finding the optimal location then the objective function becomes minimize Z $=$ $\sum_{i=1}^{m} \sum_{j=1}^{p} [\alpha_i w_{ij} \phi(a_i,b_i;x^*_j,y^*_j) + f_{ij} u_{ij}]$, $w_{ij} \in$ F, which is a traditional FCTP. Then it always has a solution at an extreme point $w_E \in$ F. Hence, we conclude that (x^*,y^*,w_E) is an optimal solution at an extreme point of FCT-LP. This completes the proof of the proposition. □

Proposition 3.5 *The number of basic feasible solutions of FCT-LP is at most* $\binom{mp}{m+p-1}$.

Proof : FCT-LP has mp variables and at most $m + p - 1$ basic variables. So, the number of basic feasible solutions of FCT-LP is at most $\binom{mp}{m+p-1}$. □

Theorem 3.1 *The function minimize* $Z = \sum_{i=1}^{m} \sum_{j=1}^{p} \left[\alpha_i w_{ij} \phi(a_i,b_i;x_j,y_j) + f_{ij} u_{ij}\right]$ *is neither a convex function nor a concave function for all values of* (x_j,y_j,w_{ij}).

Proof : Here, a particular case is considered where all $w_{ij} = 0$ except w_{22}. Further let, $b_2 = y_2$ and $a_2 > x_2$. Here, Z= $\alpha_2 w_{22}(a_2 - x_2) + K$, where K is the sum of all fixed charge costs. Now, it is known to all that a function will be neither convex nor concave if the Hessian matrix associated with Z is neither positive nor negative semi definite [130]. The Hessian matrix for Z is

$$H = \begin{pmatrix} \frac{\partial^2 Z}{\partial x_2^2} & \frac{\partial^2 Z}{\partial w_{22} \partial x_2} \\ \frac{\partial^2 Z}{\partial w_{22} \partial x_2} & \frac{\partial^2 Z}{\partial w_{22}^2} \end{pmatrix} = \begin{pmatrix} 0 & -\alpha_2 \\ -\alpha_2 & 0 \end{pmatrix}$$

which indicates indefinite. Hence, Z is neither convex nor concave for particular value of (x_j, y_j, w_{ij}). Therefore, Z is neither convex nor concave for all values of (x_j, y_j, w_{ij}). This shows the proof of the theorem. □

3.3 Methodology

In this section, an approximation approach with its algorithm is discussed for the proposed model.

3.3.1 Approximation approach

Here, a new approximation approach is presented based on the Balinski's approximation [11] and a Loc-Alloc heuristic [29], which always provides an optimal solution within a relatively short computational time for large scale entries. It is observed that the feasible region of FCT-LP is a bounded convex set with neither convex nor concave objective function. The optimal solution occurs at an extreme point of the constraint set, every extreme point of the feasible region is a local minimum (from Theorem 3.1, Propositions 3.2 and 3.4 and Appendix A.2). The proposed approach comprises two parts. In the first part, the approach finds an initial location and in the second part, it develops steps for the optimum location. The process of finding an initial location for the proposed approach depends on Loc-Alloc heuristic. Here, at first, the locations are selected for p-facilities from m-existing locations. Then, the distances between p-facilities and m-existing locations are computed. When $p \leq m$, we can easily calculate such distances but, when $p \geq m$, then there is a problem to calculate such distances. Because of that, a positive number is relegated for such distances which cannot be calculated. Now, it is already assumed that the distances are cost functions per unit commodity from i-th existing facility site to the j-th potential facility site. We take these distances as the cost coefficients then the problem converts into classical FCTP (Model 3.2.). To find an initial basic feasible solution, an approximation is utilized which is known as Balinski's approximation. Balinski [11] observed that there exists an optimal solution to the relaxed version of FCTP (formed by relaxing the integer restriction on u_{ij}), with the property that

$$u_{ij} = \frac{w_{ij}}{m_{ij}}, \text{where } m_{ij} = \min(r_i, s_j). \tag{3.8}$$

So, the relaxed TP of a FCTP would be simply a standard TP with unit transportation cost as $C_{ij} = c_{ij} + \frac{f_{ij}}{m_{ij}}$. The optimal solutions $\left\{w_{ij}^B\right\}$ could be transformed into an optimal solution of $\left\{w_{ij}^B, u_{ij}^B\right\}$ of FCTP as bellow:

$$u_{ij}^B = \begin{cases} 0, & w_{ij}^B = 0, \\ 1, & w_{ij}^B > 0. \end{cases}$$

The initial feasible solutions are designated as $\left\{w_{ij}^B, u_{ij}^B\right\}$, then using each such solution we solve the problem.

$$\text{minimize} \qquad Z^B = \sum_{i=1}^{m}\sum_{j=1}^{p}\left[\alpha_i w_{ij}^B \sqrt{(a_i - x_j)^2 + (b_i - y_j)^2} + f_{ij} u_{ij}^B\right]. \tag{3.9}$$

Now we can write the problem as

$$\text{minimize} \qquad Z^B = \sum_{j=1}^{p} \text{minimize } Z_j^B, \tag{3.10}$$

where,

$$\text{minimize} \qquad Z_j^B = \sum_{i=1}^{m}\left[\alpha_i w_{ij}^B \sqrt{(a_i - x_j)^2 + (b_i - y_j)^2} + f_{ij} u_{ij}^B\right] (j = 1, 2, \ldots, p) \tag{3.11}$$

Now we minimize $Z_j^B \quad (j = 1, 2, \ldots, p)$ for minimize Z^B. Here, the iterative formula are derived in similar way of Appendix A.1 to solve the problem (3.11). The iterations for (x_j, y_j) are as follows:

$$x_j^0 = \frac{\sum_{i=1}^m \alpha_i w_{ij}^B a_i}{\sum_{i=1}^m \alpha_i w_{ij}^B} \quad (j = 1, 2, \ldots, p), \tag{3.12}$$

$$y_j^0 = \frac{\sum_{i=1}^m \alpha_i w_{ij}^B b_i}{\sum_{i=1}^m \alpha_i w_{ij}^B} \quad (j = 1, 2, \ldots, p), \tag{3.13}$$

$$x_j^{k+1} = \frac{\sum_{i=1}^m \frac{\alpha_i w_{ij}^B a_i}{\phi(a_i, b_i; x_j^k, y_j^k)}}{\sum_{i=1}^m \frac{\alpha_i w_{ij}^B}{\phi(a_i, b_i; x_j^k, y_j^k)}} \quad (j = 1, 2, \ldots, p; k \in \mathbb{N}), \tag{3.14}$$

$$y_j^{k+1} = \frac{\sum_{i=1}^m \frac{\alpha_i w_{ij}^B b_i}{\phi(a_i, b_i; x_j^k, y_j^k)}}{\sum_{i=1}^m \frac{\alpha_i w_{ij}^B}{\phi(a_i, b_i; x_j^k, y_j^k)}} \quad (j = 1, 2, \ldots, p; k \in \mathbb{N}), \tag{3.15}$$

$$\text{where} \quad \phi(a_i, b_i; x_j^k, y_j^k) = [(a_i - x_j^k)^2 + (b_i - y_j^k)^2]^{1/2}. \tag{3.16}$$

If we denote the optimum value of Z^B for n-th basic feasible solution as Z_n^*, then the optimal value of the objective function, Z^* for FCT-LP will be

$$Z^* = \text{minimize } Z_n^* \quad \forall\, n \in \mathbb{N}. \tag{3.17}$$

If the optimum value is occurred at $n = l$, then the optimal solutions are (x_{jl}^*, y_{jl}^*) and $\left\{w_{ijl}^*, u_{ijl}^*\right\}, (i = 1, 2, \ldots, m, j = 1, 2, \ldots, p)$, where (x_{js}, y_{js}) and $\left\{w_{ijs}^B, u_{ijs}^B\right\}$ designate the s-th solution of (x_j, y_j) and $\left\{w_{ij}^B, u_{ij}^B\right\}$.

3.3.2 Algorithm

Here, an approximation approach is depicted for solving FCT-LP briefly. The schematic diagram of the proposed approach is displayed in Figure 3.2. The following steps are appraised for selection of optimal potential facility sites to the objective function in FCT-LP as:

Step 1: First, an initial location is chosen for each of p-facilities from m-existing locations.

Step 2: Therefore, two cases arise when $p \leq m$ then it can easily find the distances between the existing and the potential facility sites. But, when $p > m$ then, it cannot find all the distances. So, in that case, a positive number is assigned for each distance, and avoid to calculate such distance.

Step 3: Without loss of generality, it is assumed that the distances are proportional to the cost functions. So, these distances are taken as the cost coefficients. Then, it is converted to a classical FCTP.

Step 4: Using Balinski's approximation formula (3.8), FCTP converts into a relaxed transportation problem.

Step 5: It can easily find the set of initial basic feasible solutions to relaxed transportation

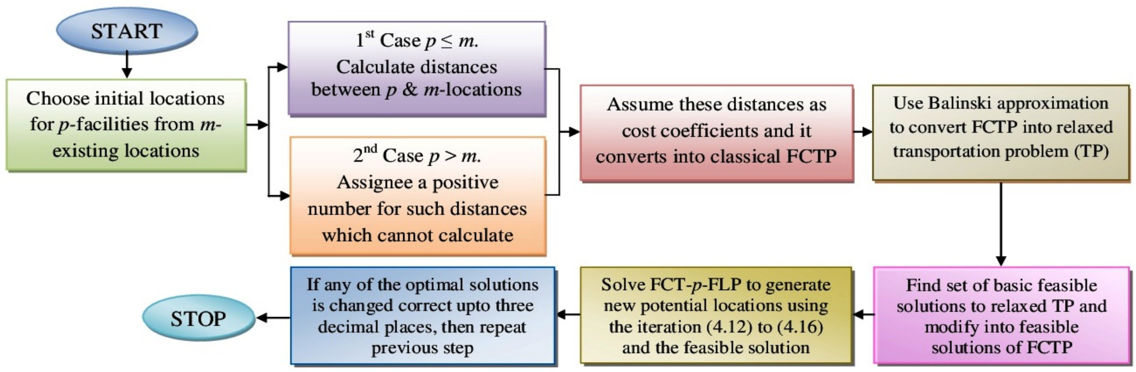

Fig. 3.2: Schematic diagram of proposed approach for solving FCT-LP.

by LINGO iterative scheme. And the initial basic feasible solutions can be transformed into feasible solutions of FCTP.

Step 6: Using initial feasible solution from Step 4 and the iteration formula from (3.12) to (3.16), FCT-LP is solved to generate a new set of potential locations.

Step 7: If any of the optimal solution has changed correct upto three decimal places, then repeat Step 6; otherwise Stop.

3.4 Experimental design

Herein, a real-life experiment is taken to exhibit the proposed model; and the delineated procedure is more capable to locate the potential facility sites in the Euclidean plane with an objective to minimize the total transportation cost along with a fixed cost. A reckoned company wishes to establish some new wings in such a way that the total transportation cost along with fixed cost from the existing plants is minimized. The company has four plants S1, S2, S3 and S4; and the company wants to set-up three new wings D1, D2, and D3. The capacities of supply at S1, S2, S3 and S4; and the requirement to the wings D1, D2, and D3, the position and the weights of the plants S1, S2, S3, and S4 are also given. Data of the problem are provided in Tables 3.1 and 3.2.

Table 3.1: Cost matrix (f_{ij}, c_{ij}).

	D1	D2	D3	Supply (r_i)
S1	$(10, c_{11})$	$(30, c_{12})$	$(20, c_{13})$	10
S2	$(40, c_{21})$	$(90, c_{22})$	$(50, c_{23})$	70
S3	$(70, c_{31})$	$(150, c_{32})$	$(80, c_{33})$	60
S4	$(60, c_{41})$	$(160, c_{42})$	$(100, c_{43})$	30
Demand (s_j)	20	100	50	

Table 3.2: The positions and weights of the existing plants.

	Position (a_i, b_i)	Weight (α_i)
S1	(0,1)	0.1
S2	(0,0)	0.2
S3	(1,0)	0.3
S4	(1,1)	0.4

The approach is coded in C++ and executed using code-block compiler on a Lenovo z580 computer with 2.50 GHz Intel (R) core (TM) i5-3210M CPU and 4 GB RAM. To contrast, the obtained results are compared from Linux terminal on a computer with Intel(R) Core (TM) i3-4130 CPU @3.40 GHz and 4 GB RAM.

3.4.1 Performance of approximation approach

Here, we mainly concentrate on the following topics for solving FCT-LP by approximation approach:

- First, three initial locations are chosen for each of 3-wings from Table 3.2. In that case four possible cases are arisen and they are displayed in Tables 3.3 to 3.6.

Table 3.3: Case 3.1.

	Position	Weight
D1	(0,1)	0.1
D2	(0,0)	0.2
D3	(1,0)	0.3

Table 3.4: Case 3.2.

	Position	Weight
D1	(0,0)	0.2
D2	(1,0)	0.3
D3	(1,1)	0.4

Table 3.5: Case 3.3.

	Position	Weight
D1	(1,0)	0.3
D2	(1,1)	0.4
D3	(0,1)	0.1

Table 3.6: Case 3.4.

	Position	Weight
D1	(1,1)	0.4
D2	(0,1)	0.1
D3	(0,0)	0.2

- Now, the distances are calculated between them for cost coefficient by using Tables 3.3 to 3.6 and put them in Tables 3.7 to 3.10, respectively.

Table 3.7: Cost Coefficient (c_{ij}) for Table 3.3.

	D1	D2	D3	r_i
S1	0	1	1.41	10
S2	1	0	1	70
S3	1.41	1	0	60
S4	1	1.41	1	30
s_j	20	100	50	

Table 3.8: Cost Coefficient (c_{ij}) for Table 3.4.

	D1	D2	D3	r_i
S1	1	1.41	1	10
S2	0	1	1.41	70
S3	1	0	1	60
S4	1.41	1	0	30
s_j	20	100	50	

Table 3.9: Cost Coefficient (c_{ij}) for Table 3.5.

	D1	D2	D3	r_i
S1	1.41	1	0	10
S2	1	1.41	1	70
S3	0	1	1.41	60
S4	1	0	1	30
s_j	20	100	50	

Table 3.10: Cost Coefficient (c_{ij}) for Table 3.6.

	D1	D2	D3	r_i
S1	1	0	1	10
S2	1.41	1	0	70
S3	1	1.41	1	60
S4	0	1	1.41	30
s_j	20	100	50	

- Balinski's approximation is used in 3.7 to 3.10 and the obtained results are shown in Tables 3.11 to 3.14, respectively.

Table 3.11: Balinski Cost matrix ($C_{ij} = c_{ij} + f_{ij}/m_{ij}$) for Table 3.7.

	D1	D2	D3	r_i
S1	1	4	3.41	10
S2	3	1.29	2	70
S3	4.91	3.5	1.6	60
S4	4	6.74	4.33	30
s_j	20	100	50	

Table 3.12: Balinski Cost matrix ($C_{ij} = c_{ij} + f_{ij}/m_{ij}$) for Table 3.8.

	D1	D2	D3	r_i
S1	2	4.41	3	10
S2	2	2.29	2.41	70
S3	4.5	2.5	2.6	60
S4	4.41	6.33	3.33	30
s_j	20	100	50	

Table 3.13: Balinski Cost matrix ($C_{ij} = c_{ij} + f_{ij}/m_{ij}$) for Table 3.9.

	D1	D2	D3	r_i
S1	2.41	4	2	10
S2	3	2.70	2	70
S3	3.5	3.5	3.01	60
S4	4	5.33	4.33	30
s_j	20	100	50	

Table 3.14: Balinski Cost matrix ($C_{ij} = c_{ij} + f_{ij}/m_{ij}$) for Table 3.10.

	D1	D2	D3	r_i
S1	2	3	3	10
S2	3.41	2.29	1	70
S3	4.5	3.91	2.6	60
S4	3	6.33	4.74	30
s_j	20	100	50	

- LINGO 16.0 iterative scheme is used for initial BFSs by utilizing Tables 3.11 to 3.14, and the obtained results are shown in Tables 3.15 to 3.18, respectively.

Table 3.15: Initial BFS (w_{ij}^B, u_{ij}^B) for Table 3.11.

	D1	D2	D3	r_i
S1	(10,1)	(0,0)	(0,0)	10
S2	(0,0)	(70,1)	(0,0)	70
S3	(0,0)	(30,1)	(30,1)	60
S4	(10,1)	(0,0)	(20,1)	30
s_j	20	100	50	

Table 3.16: Initial BFS (w_{ij}^B, u_{ij}^B) for Table 3.12.

	D1	D2	D3	r_i
S1	(10,1)	(0,0)	(0,0)	10
S2	(10,1)	(60,1)	(0,0)	70
S3	(0,0)	(40,1)	(20,1)	60
S4	(0,0)	(0,0)	(30,1)	30
s_j	20	100	50	

Table 3.17: Initial BFS (w_{ij}^B, u_{ij}^B) for Table 3.13.

	D1	D2	D3	r_i
S1	(0,0)	(0,0)	(10,1)	10
S2	(0,0)	(40,1)	(30,1)	70
S3	(0,0)	(60,1)	(0,0)	60
S4	(20,1)	(0,0)	(10,1)	30
s_j	20	100	50	

Table 3.18: Initial BFS (w_{ij}^B, u_{ij}^B) for Table 3.14.

	D1	D2	D3	r_i
S1	(0,0)	(10,1)	(0,0)	10
S2	(0,0)	(70,1)	(0,0)	70
S3	(0,0)	(20,1)	(40,1)	60
S4	(20,1)	(0,0)	(10,1)	30
s_j	20	100	50	

- Using C++ programming language, the obtained computational results for Tables 3.15 to 3.18 are finally placed in Table 3.19.

Table 3.19: Computational results for Tables 3.15 to 3.18.

Initial BFS	Location of D1	Location of D2	Location of D3	Value of Z
Table 3.15	(1.000,1.000)	(0.015,0.000)	(1.000,0.267)	508.341
Table 3.16	(0.000,0.001)	(0.500,0.000)	(1.000,0.999)	489.005
Table 3.17	(1.000,1.000)	(1.000,0.000)	(0.066,0.089)	506.049
Table 3.18	(1.000,1.000)	(0.000,0.000)	(1.000,0.000)	522.314

3.4.2 Computational results and discussion

Here, the optimal solutions of the experimental design are exhibited. The following optimal solution is obtained by approximation approach, utilising Table 3.19, which is displayed in Table 3.20. The convergence performance of the approximation approach and the solutions are depicted in Figures 3.3 and 3.4.

Table 3.20: The optimal solution of proposed FCT-LP.

Initial BFS	Location of D1	Location of D2	Location of D3	Value of Z
Table 3.16	(0.000,0.001)	(0.500,0.000)	(1.000,0.999)	489.005

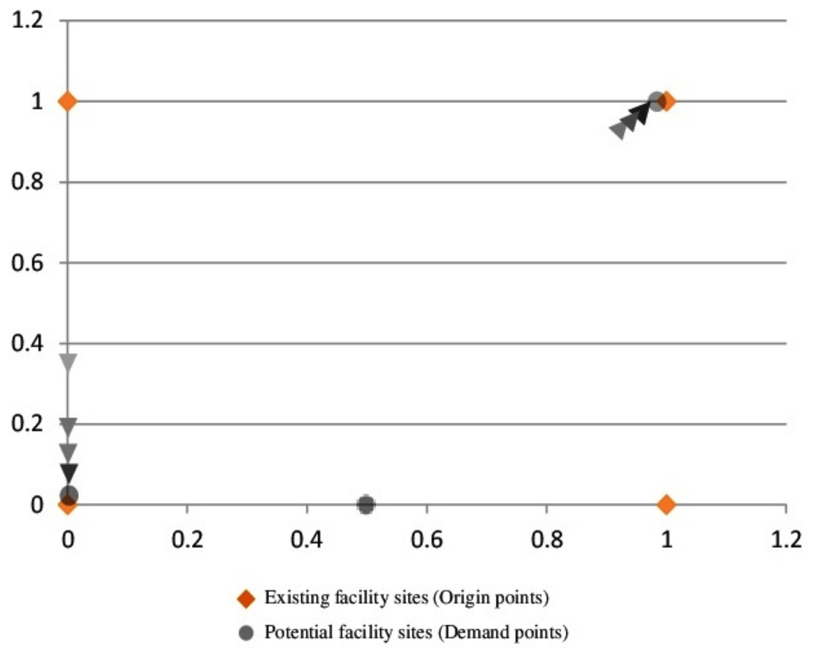

Fig. 3.3: Performance of the approximation approach.

3.5 Sensitivity analysis

In this section, the resiliency of the optimal solution for FCT-LP is inspected by varying the changes of the coefficients in the objective function and the right hand side constraints. For this purpose, the sensitivity of the optimal potential facility sites for new facility are

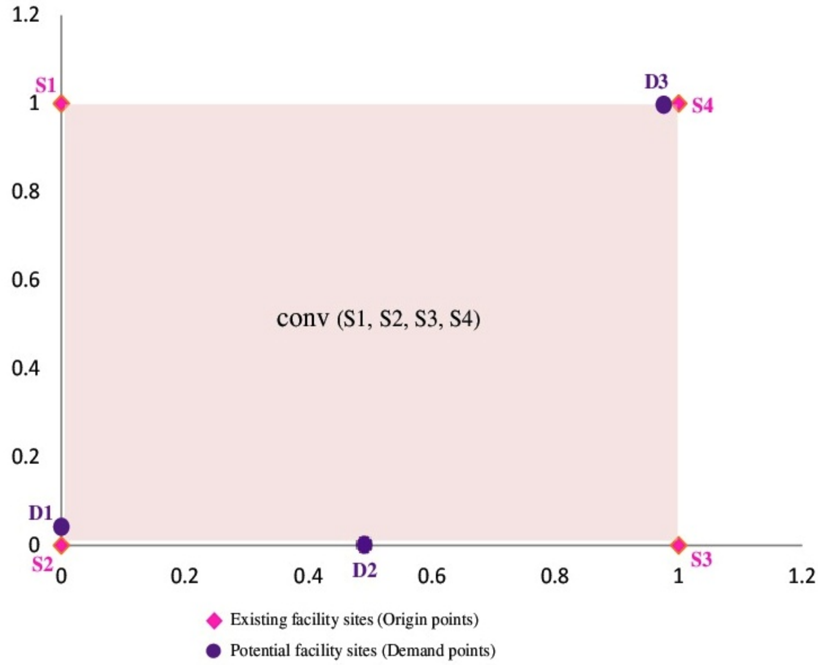

Fig. 3.4: The existing and potential facility sites in Experimental design.

found while changing the estimation of the existing locations, weights of the existing facility sites, supply and demand parameters. For FCT-LP, it is difficult to choose the ranges of parameters after small changes in which a given optimal solution stays optimal. But difficulty arises when the number of variables and constraints are in large size. To overcome the circumstance, a simple procedure [47, 88] is adopted to sensitivity analysis of FCT-LP on the fact that the basic variables remain same yet their values undergo changes. The range of the parameters in FCT-LP is determined by the following steps:

Step 1. First, keeping the basic variables, proceed as before, for a given optimal solution of FCT-LP, extracted from approximation approach.

Step 2. Then changing the values of each parameter taken each one at a time and unaltered others in FCT-LP utilizing progressive trial. After that solve the corresponding FCT-LP.

Step 3. Step 2 is proceeded until the solution appears for FCT-LP either “no feasible solution" or “change the basic variable in optimal solution".

Step 4. Finally, the range of each parameter is calculated from Step 3.

Sensitivity analysis for supply and demand parameters:

Let r_i be changed to $r_i^* = r_i + \beta_i$ $(i = 1,2,3,4)$ and s_j changed to $s_j^* = s_j + \beta_j$ $(j = 1,2,3)$. Using the proposed approach, it can easily find the values of r_i^* and s_j^* which is displayed in Table 3.21. Note that the ranges of the other parameters in FCT-LP are resolved in similar way.

Table 3.21: The ranges of supply and demand parameters.

Actual values of r_i and s_j	Changing values of r_i and s_j
$r_1 = 10$	$10 \leq r_1^* \leq 19.9$
$r_2 = 70$	$70 \leq r_2^* \leq 99.9$
$r_3 = 60$	$60 \leq r_3^* \leq 89.9$
$r_4 = 30$	$30 \leq r_4^* < \infty$
$s_1 = 20$	$10.1 \leq s_1^* \leq 20$
$s_2 = 100$	$70.1 \leq s_2^* \leq 100$
$s_3 = 50$	$20.1 \leq s_3^* \leq 50$

3.6 Conclusion

This study has been portrayed a new practical problem for a transportation network that aims to minimize total transportation cost along with the fixed cost on the entire supply chain and to select potential facility sites for different plants. To the best of the knowledge, for the first time in research, the proposed design has been provided a way of analyzing the connection between FLP and FCTP. Thereafter, some fundamental propositions and theorem on FCT-LP have been included to investigate the nature of FCT-LP. Despite the over, the development of a new version of approach has been incorporated to tackle the proposed problem in a proficient way. The studied model and developed procedure have been tested by a real-life example. Finally, the obtained computational outcomes from the approximation approach have been discussed with the suggestions for selecting the potential facility sites. The approximation approach has been used to find optimal solutions for FCT-LP with the larger size in a less computational timeframe. However, the formulation has been presented here can be applied in industrial applications such as the manufacturing of plants and applications.