Chapter 2

An exact and a heuristic approach for transportation-location problem[1]

This chapter addresses *transportation-location problem* (T-LP) which makes a connection between FLP and TP. In fact, T-LP is a generalization of the classical TP in which we have to seek where and how we impose the new facilities such that the total transportation cost from existing facility sites to the potential facility sites will be minimized. The exact approach, based on the iterative procedure, and a heuristic approach as applied to T-LP are discussed and corresponding results are compared. An experimental analysis is incorporated to expose the efficiency and effectiveness of our proposed study in reality. Finally, a summary is given at the end of this chapter.

2.1 Introduction

In the present decade, FLP and TP are a “hot topic" in supply chain management as well as the transportation planning system. Determining the best locations for the facilities (i.e., plants, depots, warehouses, offices, fire stations, railway stations, etc.) and minimizing the total transportation cost from existing sites to facilities can significantly affect transportation planning system. The main aim is to introduce a way to connect FLP and TP. We generalize the concept of TP by taking the sources as existing facility sites and demand points as potential facility sites that are to be determined. In fact, T-LP is a cost minimization problem obtained by integrating FLP and TP. Thereafter, T-LP can be solved in a continuous planner surface with Euclidean distance. We determine the best location of a facility and the effective transportation cost from sources to this facility locations simultaneously by solving T-LP. Our formulation can be applied to plant location problems where minimizing transportation cost is the main priority. We believe that this model is more reasonable than the classical TP and FLP approach. The proposed formulation will be useful to the models of transportation systems, emergency services, and online-shopping systems.

[1]The content of this chapter has been published in ***Computational Management Science***, Springer, Scopus, 17:389-407, 2020, https://doi.org/10.1007/s10287-020-00363-8.

2.2 Mathematical identification

In this section, we first incorporate the proposed problem. Thereafter, the mathematical formulation is stated based on the following notations and assumptions. The model formulation, a connection between this formulation and TP, and the structural properties are presented.

2.2.1 Problem description

Herein, a new logistical problem is inspected from an economical point of view. Our proposed problem deals with a transportation network that consists of multiple existing sites or sources, potential facility sites or demand points, and products are transported from existing sites to potential facility sites. The main aim is to minimize the total transportation cost by locating potential facility sites simultaneously. A network is depicted in Figure 2.1 to illustrate T-LP. For example, there are three existing facility sites such as O_1, O_2 and O_3, and four potential facility sites like D_1, D_2, D_3 and D_4. The corresponding supply and demand of the existing facility sites and the potential facility sites are given. Furthermore, the locations of O_1, O_2 and O_3 are known. But, the locations of D_1, D_2, D_3 and D_4 are not known in the planner surface (Euclidean plane). The line denotes the transportation cost function per unit commodity from O_1, O_2 and O_3 to D_1, D_2, D_3 and D_4, respectively. In this situation, the DM has to seek the optimal locations for the potential facility sites in such a way that the total transportation cost from existing facility sites to potential facility sites will be minimized.

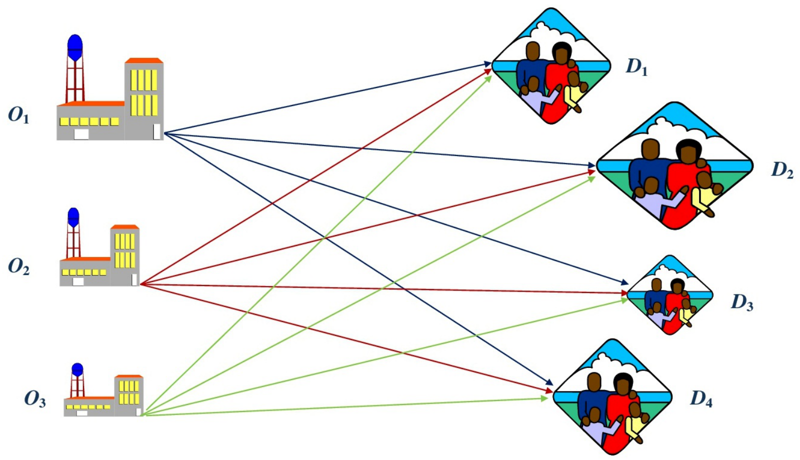

Fig. 2.1: Pictorial representation of T-LP.

2.2.2 Notations and Assumptions

Notations and assumptions are as follows:

m : number of existing facility sites (origins).

p : number of potential facility sites (demand points).

α_i : non-negative weights of existing facility sites $(i = 1, 2, \ldots, m)$.

a_i : availability at the i-th existing facility site $(i = 1, 2, \ldots, m)$.

b_j : demand at the j-th potential facility site $(j = 1, 2, \ldots, p)$.

(c_i, d_i): coordinates of i-th existing facility sites $(i = 1, 2, \ldots, m)$.

(x_j, y_j): coordinates of j-th potential facility sites $(j = 1, 2, \ldots, p)$.

w_{ij} : amounts of flow to be transported from the i-th existing facility site to the j-th potential facility site $(i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, p)$.

F : $\{(w_{ij}) : \forall\ i, j\}$: the feasible set with respect to the matrix variable w.

W_B : $\{(w_{ij}^B) : i = 1, 2, \ldots m;\ j = 1, 2, \ldots, p\}$: the initial basic feasible solution.

ϕ : transportation cost function per unit commodity from an existing facility site to a potential facility site.

- The solution space is continuous. The space in which potential facility sites are located is the planner. Potential facility sites are assumed as points. Parameters are deterministic.
- Facilities are capacitated. No relationship between potential facility sites. Ignoring the opening cost of new potential facility sites.
- The objective function is to be minimized. Type of distance is the usual Euclidean distance in 2-dimensional space $(\phi(c_i, d_i; x_j, y_j) = [(c_i - x_j)^2 + (d_i - y_j)^2]^{1/2})$.

2.2.3 Model formulation

Here, we introduce a formulation based on the classical FLP and TP. However, instead of minimizing the total transportation cost, this model finds optimal locations by determining the potential facility sites. We consider the mathematical model of T-LP as follows:
Model 2.1

$$\text{minimize}_{(x,y,w)} \quad Z = \sum_{i=1}^{m} \sum_{j=1}^{p} \alpha_i w_{ij} \phi(c_i, d_i; x_j, y_j) \tag{2.1}$$

$$\text{subject to} \quad \sum_{j=1}^{p} w_{ij} \leq a_i \quad (i = 1, 2, \ldots, m), \tag{2.2}$$

$$\sum_{i=1}^{m} w_{ij} \geq b_j \quad (j = 1, 2, \ldots, p), \tag{2.3}$$

$$w_{ij} \geq 0\ \ \forall\ \ i \text{ and } j. \tag{2.4}$$

The objective function (2.1) aims to minimize the total transportation cost from existing facility sites to potential facility sites. Constraints (2.2) enforce that the total flow from each existing facility site cannot exceed its amount available. Constraints (2.3) impose that the total flow to each potential facility site should satisfy its demand. Constraints (2.4) consist of non-negativity conditions.

2.2.4 Connection between T-LP and TP

The objective function (2.1) of Model 2.1 depends on the location of potential facility sites. From Figure 2.1, it can be seen that if we fix the locations of potential facility sites, then the set of cost functions converts into the set of constant cost functions. Subsequently, in view of objective function (2.1) we use the short notation $\phi(c_i,d_i;x_j,y_j) = s_{ij}$ (constant cost functions), now $\alpha_i s_{ij}$ is chosen as t_{ij} (unit transportation cost from sources to demand points). Hence, the objective function of Model 2.1 is reduced; and the subsequent Model 2.2 as follows:

Model 2.2

$$\text{minimize}_{(w)} \quad Z = \sum_{i=1}^{m}\sum_{j=1}^{p} t_{ij}w_{ij} \tag{2.5}$$

$$\text{subject to} \quad \text{the constraints (2.2) to (2.4);}$$

this is the classical form of a TP. Hence, for constant cost function T-LP becomes a TP.

2.2.5 Structural properties

In this subsection, we discuss some fundamental propositions and a theorem to recognize the nature of T-LP.

Proposition 2.1 *A necessary and sufficient conditions for the problem T-LP is that* $\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{p} b_j$.

Proof: This property is called feasibility condition. The feasibility condition depends on the constraints. In fact, both the problems have the same constraints. This illustrates the proof of the proposition. □

Proposition 2.2 *The feasible solution of T-LP is never unbounded.*

Proof: The constraints of T-LP are as follows:

$$\sum_{j=1}^{p} w_{ij} \leq a_i \quad (i = 1,2,\ldots,m),$$

$$\sum_{i=1}^{m} w_{ij} \geq b_j \quad (j = 1,2,\ldots,p),$$

$$w_{ij} \geq 0 \;\; \forall \;\; i \text{ and } j.$$

So, $b_j \leq w_{ij} \leq a_i \,\forall\;\, i$ and j, and also $w_{ij} \geq 0 \;\;\forall\;\; i$ and j. We conclude that $\inf\{0,b_j\} \leq w_{ij} \leq a_i \;\;\forall\;\; i$ and j. As $b_j > 0 \;\;\forall\; j$, now $\inf\{0,b_j\} = 0$ and $0 \leq w_{ij} \leq a_i$. This completes the proof of the proposition. □

Proposition 2.3 *The number of basic variables in T-LP is at most* $(m+p-1)$.

Proof: This property is also dependent on the constraints. Here, we see that the constraints of two problems are the same. So, this proposition is also the same as the TP. □

Proposition 2.4 *For the problem minimize*$_{(x,y,w)}$ $Z=\sum_{i=1}^{m}\sum_{j=1}^{p}\alpha_i w_{ij}\phi(c_i,d_i;x_j,y_j)$, $(w_{ij})\in F$*, an optimal solution exists at an extreme point of the convex set* F *of feasible solutions to T-LP.*

Proof: Let $(x,y)=(x_j,y_j)$ $(j=1,2,\ldots,p)$, $w=(w_{ij}:\ i=1,2,\ldots,m;\ j=1,2,\ldots,p)$ and $w_E\in\{(w_{ij}^E)\ (i=1,2,\ldots,m;\ j=1,2,\ldots,p):\text{extreme points of } F\}$. If we choose the destination such that $(x,y)=(x_j^*,y_j^*)$ by seeking the optimal location, then the objective function to minimize is $Z=\sum_{i=1}^{m}\sum_{j=1}^{p}\alpha_i w_{ij}\phi(c_i,d_i;x_j^*,y_j^*)$, $(w_{ij})\in F$, which is a classical TP. Then it always has a solution at an extreme point $w_E\in F$. So, we conclude that (x^*,y^*,w_E) is an optimal solution at an extreme point of F to T-LP. □

Proposition 2.5 *The number of basic feasible solutions of T-LP is at most* $\binom{mp}{m+p-1}$.

Proof: T-LP has mp variables and at most $m+p-1$ basic variables. So, the number of basic feasible solutions of T-LP is at most $\binom{mp}{m+p-1}$. □

Theorem 2.1 *The objective function* $Z=\sum_{i=1}^{m}\sum_{j=1}^{p}\alpha_i w_{ij}^B\phi(c_i,d_i;x_j,y_j)$ *is a convex function in the joint variable* (x,y) *on* $\mathbb{R}^{2p}$.

Proof: We know that a function Z is convex over the region iff the Hessian matrix associated with Z is positive semidefinite over the region [130]. Let $Z=\sum_{j=1}^{p}Z_j$, where $Z_j=\sum_{i=1}^{m}\alpha_i w_{ij}^B\phi(c_i,d_i;x_j,y_j)$ and the terms w_{ij}^B are constants. Here, Z_j only depends on the variables x_j and y_j. Hence we can consider Z_j to be a function in two variables x_j and y_j. The Hessian matrix for Z_j at (x_j,y_j) is

$$H_j=\begin{pmatrix}\frac{\partial^2 Z_j}{\partial x_j^2} & \frac{\partial^2 Z_j}{\partial x_j\partial y_j}\\ \frac{\partial^2 Z_j}{\partial y_j\partial x_j} & \frac{\partial^2 Z_j}{\partial y_j^2}\end{pmatrix}.$$

The principal minors of H_j are $\frac{\partial^2 Z_j}{\partial x_j^2}$ and $\det H_j$ (determinant of H_j).
Now,

$$\frac{\partial^2 Z_j}{\partial x_j^2}=\sum_{i=1}^{m}\frac{\alpha_i w_{ij}^B(d_i-y_j)^2}{[(c_i-x_j)^2+(d_i-y_j)^2]^{3/2}},$$

$$\begin{aligned}\text{and }\det H_j &= \frac{\partial^2 Z_j}{\partial x_j^2}\frac{\partial^2 Z_j}{\partial y_j^2}-\Big(\frac{\partial^2 Z_j}{\partial x_j\partial y_j}\Big)^2\quad\Big(\text{since }\frac{\partial^2 Z_j}{\partial x_j\partial y_j}=\frac{\partial^2 Z_j}{\partial y_j\partial x_j}\Big)\\
&=\Big(\sum_{i=1}^{m}\frac{\alpha_i w_{ij}^B(d_i-y_j)^2}{[(c_i-x_j)^2+(d_i-y_j)^2]^{3/2}}\Big)\Big(\sum_{i=1}^{m}\frac{\alpha_i w_{ij}^B(c_i-x_j)^2}{[(c_i-x_j)^2+(d_i-y_j)^2]^{3/2}}\Big)\\
&\quad-\Big(\sum_{i=1}^{m}\frac{\alpha_i w_{ij}^B(c_i-x_j)(d_i-y_j)}{[(c_i-x_j)^2+(d_i-y_j)^2]^{3/2}}\Big)^2\\
&=\Big(\sum_{i=1}^{m}\big(\frac{\sqrt{\alpha_i w_{ij}^B}(d_i-y_j)}{[(c_i-x_j)^2+(d_i-y_j)^2]^{3/4}}\big)^2\Big)\Big(\sum_{i=1}^{m}\big(\frac{\sqrt{\alpha_i w_{ij}^B}(c_i-x_j)}{[(c_i-x_j)^2+(d_i-y_j)^2]^{3/4}}\big)^2\Big)\end{aligned}$$

$$-\Big(\sum_{i=1}^{m}\frac{\sqrt{\alpha_i w_{ij}^B}(d_i-y_j)}{[(c_i-x_j)^2+(d_i-y_j)^2]^{3/4}}\frac{\sqrt{\alpha_i w_{ij}^B}(c_i-x_j)}{[(c_i-x_j)^2+(d_i-y_j)^2]^{3/4}}\Big)^2.$$

$$\text{Now, }\Big(\sum_{i=1}^{m}\Big(\frac{\sqrt{\alpha_i w_{ij}^B}(d_i-y_j)}{[(c_i-x_j)^2+(d_i-y_j)^2]^{3/4}}\Big)^2\Big)\Big(\sum_{i=1}^{m}\Big(\frac{\sqrt{\alpha_i w_{ij}^B}(c_i-x_j)}{[(c_i-x_j)^2+(d_i-y_j)^2]^{3/4}}\Big)^2\Big)$$

$$\geq \Big(\sum_{i=1}^{m}\frac{\sqrt{\alpha_i w_{ij}^B}(d_i-y_j)}{[(c_i-x_j)^2+(d_i-y_j)^2]^{3/4}}\frac{\sqrt{\alpha_i w_{ij}^B}(c_i-x_j)}{[(c_i-x_j)^2+(d_i-y_j)^2]^{3/4}}\Big)^2$$

(by Cauchy-Schwarz inequality).

As $\alpha_i > 0, w_{ij}^B \geq 0, (c_i-x_j)^2 \geq 0$ and $(d_i-y_j)^2 \geq 0$, we conclude that $\frac{\partial^2 Z_j}{\partial x_j^2} \geq 0$ and $\det H_j \geq 0$ for all values of x_j and y_j. Hence, Z_j is convex with respect to x_j and y_j. Let $(x_1,y_1,x_2,y_2,\dots,x_p,y_p)$ and $(x'_1,y'_1,x'_2,y'_2,\dots,x'_p,y'_p)$ be two arbitrary points of $\mathbb{R}^{2p}$, and $t \in [0,1]$.

$$\begin{aligned}\text{Herewith,}\quad & Z\big(t(x_1,y_1,x_2,y_2,\dots,x_p,y_p)+(1-t)(x'_1,y'_1,x'_2,y'_2,\dots,x'_p,y'_p)\big)\\ &=\sum_{j=1}^{p} Z_j\big(t(x_1,y_1,x_2,y_2,\dots,x_p,y_p)+(1-t)(x'_1,y'_1,x'_2,y'_2,\dots,x'_p,y'_p)\big)\\ &\leq t\sum_{j=1}^{p} Z_j(x_1,y_1,x_2,y_2,\dots,x_p,y_p)+(1-t)\sum_{j=1}^{p} Z_j(x'_1,y'_1,x'_2,y'_2,\dots,x'_p,y'_p)\\ &= tZ(x_1,y_1,x_2,y_2,\dots,x_p,y_p)+(1-t)Z(x'_1,y'_1,x'_2,y'_2,\dots,x'_p,y'_p).\end{aligned}$$

Therefore, Z is convex in the variable (x,y) on $\mathbb{R}^{2p}$. This completes the proof. □

2.3 Solution methodology

In this section, we first briefly describe an exact method with its algorithm, and present a heuristic algorithm for our model.

2.3.1 Exact approach

The iterative procedure is an exact and simple solution procedure in which we find the best nearest optimal locations. We see that the objective function has a minimum value at an extreme point of the convex set F and the number of basic feasible solutions in F are finite (by Theorem 2.1 and Propositions 2.4 and 2.5). First, we find all basic feasible solutions in F by solving the constraints of T-LP. We observe that these constraints are the same as the constraints of the classical TP. Therefore, we apply the Northwest-Corner method [62] to generate the initial basic feasible solutions which are $W_B = (w_{ij}^B : i = 1,2,\dots,m;\ j = 1,2,\dots,p)$; then for each such solution we solve the problem.

$$\begin{aligned}\text{minimize}_{(x,y)}\quad & Z^B = \sum_{i=1}^{m}\sum_{j=1}^{p}\alpha_i w_{ij}^B\sqrt{(c_i-x_j)^2+(d_i-y_j)^2}\\ \text{subject to}\quad & \text{the constraints (2.2) to (2.4).}\end{aligned}$$

Now we can write the problem as

$$\text{minimize}_{(x,y)} \quad Z^B = \sum_{j=1}^{p} Z_j^B \tag{2.6}$$

$$\text{subject to} \quad \text{the constraints (2.2) to (2.4),}$$

where $Z_j^B = \sum_{i=1}^m \alpha_i w_{ij}^B \sqrt{(c_i - x_j)^2 + (d_i - y_j)^2}$ $(j = 1, 2, \ldots, p)$. Then, we minimize Z_j^B $(j = 1, 2, \ldots, p)$ for minimizing Z^B. We use the iterative formulas (A.1) to (A.6) (see Appendix A.1) to minimize the function Z_j^B. Let $S = \{Z_n^* :$ the optimum value for Z^B for n-th basic feasible solution$\}$. Clearly, S is a finite set from Proposition 2.5. Hence, it has a minimum, then the optimal value of the objective function Z^* for T-LP will be $Z^* = \min S$. If the optimum has been attained at $n = l$, then the best nearest optimal solutions are (x_j^l, y_j^l), $(j = 1, 2, \ldots, p)$, and w_{ijl}^B $(i = 1, 2, \ldots, m; j = 1, 2, \ldots, p)$, where (x_j^l, y_j^l) indicates (x_j, y_j) for the l-th basic feasible solution and $w_{ij_l}^B$ are the values of w_{ij}^B for this solution.

2.3.2 An Exact algorithm

Here, we describe an algorithm for solving T-LP. The following steps are appraised for selection of optimal potential facility sites to the objective function in T-LP as:

Step 1: First, we solve the constraints using the Northwest-Corner method to evaluate the initial basic feasible solutions.

Step 2: We address each such l-th basic feasible solution as $w_{ij_l}^B$. Based on each such solution, we consider a set of problems as indicated below:

$$\text{minimize}_{(x,y)} \quad Z_l^B = \sum_{j=1}^{p} Z_{j_l}^B$$

$$\text{subject to} \quad \text{the constraints (2.2) to (2.4),}$$

where $Z_{j_l}^B = \sum_{i=1}^m \alpha_i w_{ij_l}^B \sqrt{(c_i - x_j)^2 + (d_i - y_j)^2}$ $\quad (j = 1, 2, \ldots, p)$.

Step 3: We solve the set of problems in Step 2 by using the iterations (A.1) to (A.6).

Step 4: After a finite number of iterations, we observe that when the coordinates of some existing facility sites are equal to the potential facility sites, then the denominator of the iteration in Step 3 becomes 0. In that case, we cannot move to the next iteration. As our aim is to seek the best nearest location of the existing facility sites, we take this result as an optimal location and terminate the loop.

Step 5: Repeat Steps 3 and 4 until no further changes are possible in correct up to 4 decimal places.

Step 6: We choose optimal solutions are (x_j^l, y_j^l) and $Z^* = \min S$.

Step 7: Stop.

2.3.3 A Loc-Alloc heuristic algorithm

The locate-allocate (Loc-Alloc) heuristic algorithm was first introduced to solve large scale traditional location problems by Cooper [29], which provides always a good solution (sub-optimal) within a relatively less computational burden. We moderate it to solve T-LP. The steps of the Loc-Alloc heuristic algorithm are as follows:

Step 1: First, we choose the initial locations for the p-facilities from m-existing locations.

Step 2: Therefore, two cases arise: If $p \leq m$, then we can easily find the distances between the existing and the potential facility sites. But, if $p > m$, then we cannot find all the distances. So, in that case, we assign a positive number for each distance to less calculation burden.

Step 3: Without loss of generality, we assume that the distances are proportional to the cost functions. So, we take these distances as the cost coefficients w_{ij} (in Eqs. (2.1)). Then, it is converted to the classical TP.

Step 4: Using the LINGO optimization tool we easily find the set of initial basic feasible solutions W_B.

Step 5: Employing W_B from Step 4 and the iteration from Eqs. (A.1) to (A.6), we solve the T-LP to generate a new set of potential locations.

Step 6: If any of the locations has changed correct up to 4 decimal places, then repeat Step 5; otherwise stop.

2.4 Experimental analysis

In this section, we incorporate a real-life experiment to illustrate our model and to work out that our procedures are effective to locate the potential facility sites in the Euclidean plane with the objective to minimize the total transportation cost. A reckoned company wishes to establish some new wings in such a way that the total transportation cost from the existing plants is minimized. The company has four plants S_1, S_2, S_3 and S_4, and the company wants to set-up three new *wings* (plants) D_1, D_2 and D_3. The capacities of supply at S_1, S_2, S_3 and S_4, the requirement to the wings D_1, D_2 and D_3, the position and the weights of the plants S_1, S_2, S_3 and S_4 are also known. The supplied data of the problem are given in Tables 2.1 and 2.2. We code the approaches in C++ and execute it using a code-block compiler on a Lenovo z580 computer with 2.50 GHz Intel (R) Core (TM) i5-3210M CPU and 4 GB RAM. We set up the same configuration as above and compared its performance with our Loc-Alloc heuristic. In contrast, we compare the results obtained from the Linux terminal on a computer with Intel(R) Core (TM) i3-4130 CPU @3.40 GHz and 4 GB RAM.

Table 2.1: The capacities of supply & demand of the plants.

	D_1	D_2	D_3	a_i
S_1	w^B_{11}	w^B_{12}	w^B_{13}	10
S_2	w^B_{21}	w^B_{22}	w^B_{23}	60
S_3	w^B_{31}	w^B_{32}	w^B_{33}	50
S_4	w^B_{41}	w^B_{42}	w^B_{43}	30
b_j	20	90	40	

Table 2.2: The positions & weights of the existing plants.

	Position	Weight
S_1	(0,1)	0.1
S_2	(0,0)	0.2
S_3	(1,0)	0.3
S_4	(1,1)	0.4

2.4.1 Performance of Exact approach

Here, we mainly concentrate on the following topics:

- First, we find the possible initial BFSs by the Northwest-Corner method.
- To fix the optimum position of D_1, D_2 and D_3 for minimizing transportation cost and maximizing the profit.

Now, we use the Northwest-Corner method by utilizing Table 2.1 and get the possible initial BFS *sets*. They are placed in Tables 2.3 to 2.5.

Table 2.3: The possible BFS set 1.

	D_1	D_2	D_3	a_i
S_1	10			10
S_2	10	50		60
S_3		40	10	50
S_4			30	30
b_j	20	90	40	

Table 2.4: The possible BFS set 2.

	D_1	D_2	D_3	a_i
S_1	10			10
S_2	10	10	40	60
S_3		50		50
S_4		30		30
b_j	20	90	40	

Table 2.5: The possible BFS set 3.

	D_1	D_2	D_3	a_i
S_1	10			10
S_2	10	40	10	60
S_3		50		50
S_4			30	30
b_j	20	90	40	

The computational results for Tables 2.3 to 2.5 using the C++ programming language are shown in Table 2.6.

Table 2.6: Computational results for Tables 2.3 to 2.5.

Initial BFS	Location of D_1	Location of D_2	Location of D_3	Value of Z
Table 2.3	$(0.000000, 0.000066)$	$(0.883816, 0.000000)$	$(1.000000, 1.000000)$	14.232434
Table 2.4	$(0.000000, 0.000066)$	$(0.991361, 0.055396)$	$(0.000000, 0.000000)$	15.162587
Table 2.5	$(0.000000, 0.000066)$	$(0.999790, 0.000000)$	$(1.000000, 1.000000)$	9.829962

2.4.2 Performance of Loc-Alloc heuristic

For solving T-LP by Loc-Alloc heuristic we focus on the following:

- First, we choose three initial locations for each of 3 wings from Table 2.2. Then, 4 possible *cases* have arisen and they are displayed in Tables 2.7 to 2.10.

Table 2.7: Case 2.1.

	Position	Weight
D_1	$(0,1)$	0.1
D_2	$(0,0)$	0.2
D_3	$(1,0)$	0.3

Table 2.8: Case 2.2.

	Position	Weight
D_1	$(0,0)$	0.2
D_2	$(1,0)$	0.3
D_3	$(1,1)$	0.4

Table 2.9: Case 2.3.

	Position	Weight
D_1	$(1,0)$	0.3
D_2	$(1,1)$	0.4
D_3	$(0,1)$	0.1

Table 2.10: Case 2.4.

	Position	Weight
D_1	$(1,1)$	0.4
D_2	$(0,1)$	0.1
D_3	$(0,0)$	0.2

- Now, we calculate the distances between existing plants and initial locations of wings by using Tables 2.7 to 2.10. We put the distances as cost coefficients in Tables 2.11 to 2.14, respectively.

Table 2.11: Cost Coefficients for Table 2.7.

	D_1	D_2	D_3	a_i
S_1	0	1	$\sqrt{2}$	10
S_2	1	0	1	60
S_3	$\sqrt{2}$	1	0	50
S_4	1	$\sqrt{2}$	1	30
b_j	20	90	40	

Table 2.12: Cost Coefficients for Table 2.8.

	D_1	D_2	D_3	a_i
S_1	1	$\sqrt{2}$	1	10
S_2	0	1	$\sqrt{2}$	60
S_3	1	0	1	50
S_4	$\sqrt{2}$	1	0	30
b_j	20	90	40	

Table 2.13: Cost Coefficients for Table 2.9.

	D_1	D_2	D_3	a_i
S_1	$\sqrt{2}$	1	0	10
S_2	1	$\sqrt{2}$	1	60
S_3	0	1	$\sqrt{2}$	50
S_4	1	0	1	30
b_j	20	90	40	

Table 2.14: Cost Coefficients for Table 2.10.

	D_1	D_2	D_3	a_i
S_1	1	0	1	10
S_2	$\sqrt{2}$	1	0	60
S_3	1	$\sqrt{2}$	1	50
S_4	0	1	$\sqrt{2}$	30
b_j	20	90	40	

- We use LINGO optimization tool for initial BFSs by utilizing Tables 2.11 to 2.14, and the obtained results are shown in Tables 2.15 to 2.18, respectively.

Table 2.15: Initial BFS for Table 2.7.

	D_1	D_2	D_3	a_i
S_1		10		10
S_2		60		60
S_3		10	40	50
S_4	20	10		30
b_j	20	90	40	

Table 2.16: Initial BFS for Table 2.8.

	D_1	D_2	D_3	a_i
S_1			10	10
S_2	20	40		60
S_3		50		50
S_4			30	30
b_j	20	90	40	

Table 2.17: Initial BFS for Table 2.9.

	D_1	D_2	D_3	a_i
S_1			10	10
S_2		30	30	60
S_3	20	30		50
S_4		30		30
b_j	20	90	40	

Table 2.18: Initial BFS for Table 2.10.

	D_1	D_2	D_3	a_i
S_1		10		10
S_2		20	40	60
S_3		50		50
S_4	20	10		30
b_j	20	90	40	

- Finally, the computational results for Tables 2.15 to 2.18 using C++ programming language are placed in Table 2.19.

Table 2.19: Computational results for Tables 2.15 to 2.18.

Initial BFS	Location of D_1	Location of D_2	Location of D_3	Value of Z
Table 2.7	(1.000000,1.000000)	(0.000598,0.000393)	(1.000000,0.000000)	9.660450
Table 2.8	(0.000000,0.000000)	(0.999790,0.000000)	(1.000000,1.000000)	9.001470
Table 2.9	(1.000000,0.000000)	(0.869809,0.476981)	(0.000000,0.000000)	17.869660
Table 2.10	(1.000000,1.000000)	(0.999997,0.000003)	(0.000000,0.000000)	9.414249

2.5 Computational results and discussion

Here, first, we present the optimal solutions of the experimental study, obtained by two approaches. Second, we compare the performances of the proposed solution procedures for

T-LP, based on our experiment analysis.

Exact approach: We obtain the following nearest optimal solution by our iterative procedure, utilizing Table 2.6, as shown in Table 2.20. The convergence performance of the iterative procedure is delineated in Figure 2.2.

Table 2.20: The optimal solution of the proposed T-LP.

Decimal places	Location of D_1	Location of D_2	Location of D_3	Value of Z
4 decimal	$(0.0000, 0.0000)$	$(0.9997, 0.0000)$	$(1.0000, 1.0000)$	9.8299
6 decimal	$(0.000000, 0.000066)$	$(0.999790, 0.000000)$	$(1.000000, 1.000000)$	9.829962

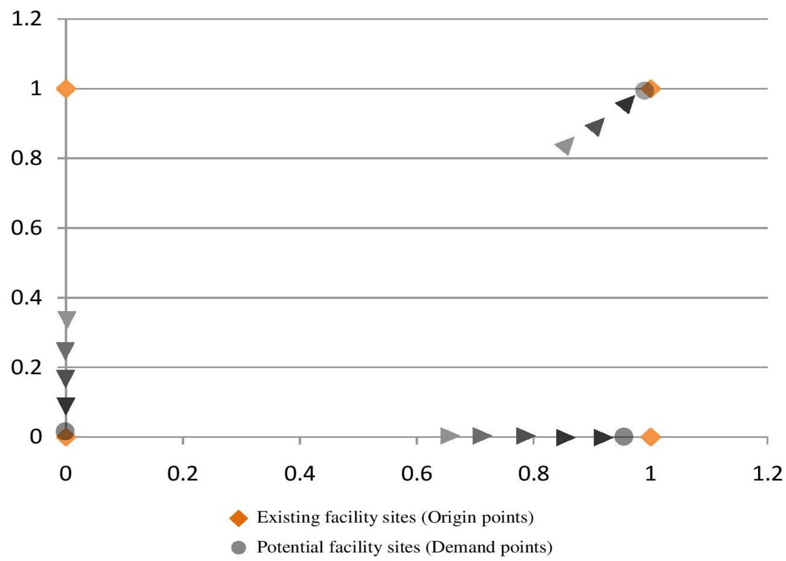

Fig. 2.2: Performance of Exact approach.

Loc-Alloc heuristic: We derive the sub-optimal solution by Loc-Alloc heuristic, employing Table 2.19, which is displayed in Table 2.21. Figure 2.3 shows the convergence performance of the heuristic.

Table 2.21: The sub-optimal solution of the proposed T-LP.

Decimal places	Location of D_1	Location of D_2	Location of D_3	Value of Z
4 decimal	$(0.0000, 0.0000)$	$(0.9997, 0.0000)$	$(1.0000, 1.0000)$	9.0014
6 decimal	$(0.000000, 0.000000)$	$(0.999790, 0.000000)$	$(1.000000, 1.000000)$	9.001470

2.5.1 Comparison of the obtained results

Here, we confront the computational results obtained by our two approaches. From Tables 20 and 21, the following conclusions are made and offered to further consideration and research.

- No difference exists between solutions (correct up to 4 decimal places).

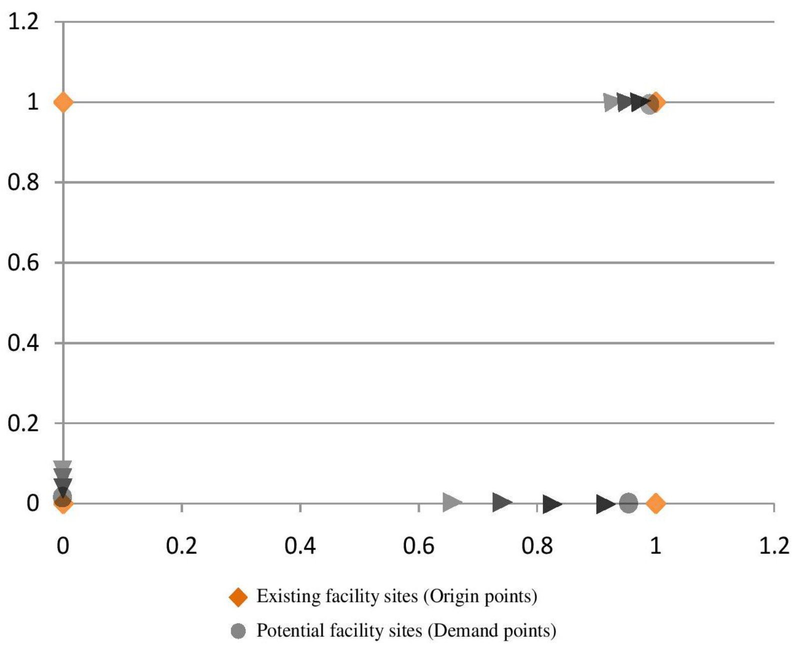

Fig. 2.3: Performance of Loc-Alloc heuristic.

- When we consider correct up to 6 decimal places, then the solution of the Loc-Alloc heuristic is slightly sub-optimal, compare to the exact solution which is depicted in Figure 2.4.

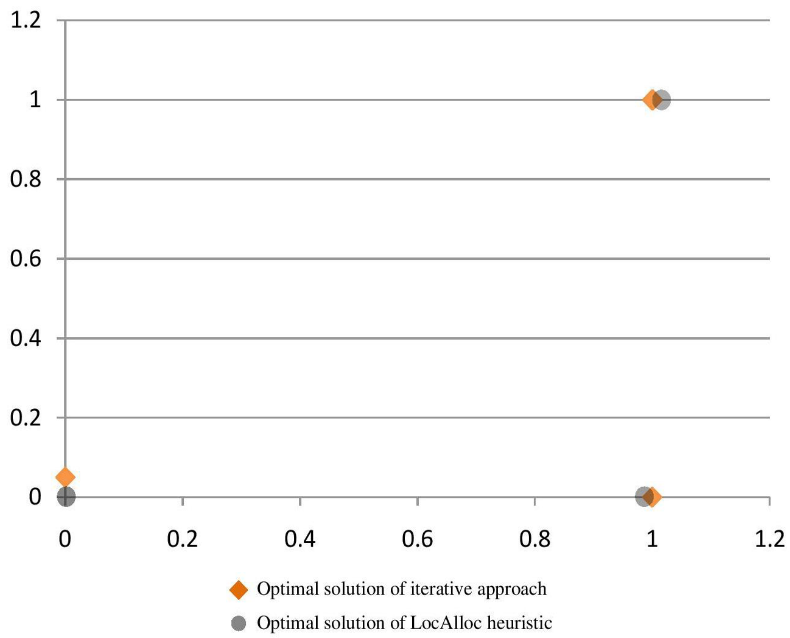

Fig. 2.4: Comparison between the obtained results.

2.6 Conclusion

This study has been introduced a practical problem for a transportation network that objects to reduce the overall transportation cost along the entire supply chain and to select potential facility sites for different plants. To the best of our knowledge, for the first time in research, we have provided a way of analyzing the connection between the FLP and TP. Thereafter, some fundamental propositions and a theorem on T-LP have been introduced to investigate the nature of T-LP. In addition to the aforementioned achievements, the development of novel versions of two approaches is analyzed to solve the proposed problem in an efficient manner. The studied model and developed procedures have been tested by a real-life example. Finally, the obtained computational results from our two approaches have been compared with suggestions for selecting the potential facility sites. In comparison, the iterative approach is more appropriate to solve the T-LP with small sizes. The Loc-Alloc heuristic is more suitable for T-LP of larger size since it can generate comparable solutions in less computational time. In fact, the formulation presented here can be employed in large-scale industrial applications such as the manufacturing of plants, transportation systems, emergency services, and online-shopping systems.