Abstract

Graph theory has undergone a powerful development, since its beginning in the 18th century. Many real-life problems can be modelled by graph-theoretic problems. The graph problems are usually NP-hard and hence there is no efficient algorithm for solving them, unless P = NP. One way to overcome this hardness is to solve the problems when restricted to special classes of graphs. The intersection graphs are the most useful graph classes and interval, circular-arc, permutation and trapezoid graphs are major subclasses of intersection graphs for which several NP-complete problems can be solved in polynomial time. The main focus of this work is the study of some important problems on *interval, permutation, trapezoid and circular-arc graphs* which are the very important subclasses of the intersection graphs.

In Chapter 1, basic definitions, recognitions, applications, survey, etc. of the interval, permutation, trapezoid graphs, circular-arc graphs and related graphs are discussed.

In Chapter 2, computation of inverse 1-center location problem on the weighted trees is studied. Let T be a tree with $(n + 1)$ vertices and n edges with positive edge weights. The inverse 1-center problem on an edge weighted tree consists in changing edge weights at minimum cost so that a pre-specified vertex becomes the 1-center. In this chapter, an optimal algorithm is designed to find an inverse 1-center location on the weighted trees whose number of vertices and edges are $(n+1)$ and n, and the edge weights can be changed within certain bounds.

Chapter 3 is devoted for solving inverse 1-center location problem on the weighted interval graphs and construction of minimum average distance tree on fuzzy interval graphs. Let T_{IG} be the tree corresponding to the weighted interval graph. In an inverse 1-center location problem the parameter of an interval tree T_{IG} corresponding to the weighted interval graph, like vertex weights have to be modified at minimum total cost such that a pre-specified vertex $s \in V$ becomes the 1-center of the interval graph G. Here an $O(n)$ time algorithm is designed to find an inverse 1-center location problem on the weighted tree corresponding to the weighted interval graph, where the vertex weights can be changed within certain bounds and n is the number of vertices of the graph G. Besides these, an $O(n)$-time algorithm is also presented to compute the average distance of a graph G (with finite number of nodes and edges). This is the average of the distances over all unordered pairs of nodes. A minimum average distance spanning tree (MADST, in short) of G is a spanning tree of G having minimum average distance. Further, an $O(n^2)$-time algorithm is designed to determine a MADST on the fuzzy interval graph, where n is the cardinality of the node set of given

graph.

In Chapter 4, the methods of computation of minimum average distance tree and inverse 1-center location problem on the circular-arc graphs are described. The average distance of a finite graph G is the average of the distances over all unordered pairs of vertices. A minimum average distance spanning tree of G is a spanning tree of G with minimum average distance. Such a tree is sometimes referred to as a minimum routing cost spanning tree. In this chapter, an efficient algorithm is designed to compute a minimum average distance spanning tree on circular-arc graph which takes $O(n^2)$ time, where n is the number of vertices of the graph. Also, in this chapter, an optimal algorithm is designed to find an inverse 1-center location on the weighted tree corresponding to the weighted circular-arc graph, where the node weights can be modified within certain bounds. The time complexity of the proposed algorithm is $O(n)$, where n is the number of vertices.

Chapter 5 discusses methods of computation of a minimum average distance tree and inverse 1-center location problem on permutation graphs and on weighted permutation graphs, respectively. The average distance $\mu(G)$ of a finite graph G is the average of the distances over all unordered pairs of vertices. A minimum average distance spanning tree of G is a spanning tree of G with minimum average distance. In this chapter, an efficient algorithm is designed to compute a minimum average distance spanning tree on permutation graphs in $O(n^2)$ time, where n is the number of vertices of the graph. Also, an $O(n^2)$-time algorithm is presented to compute an inverse 1-center of the weighted permutation graph, where the node weights can be modified under some certain restrictions. The time complexity of the proposed algorithm is $O(n)$.

The sixth chapter presents an optimal algorithm for computation of inverse 1-center location problem on the weighted trapezoid graphs. Let T_{TRP} be the tree corresponding to the weighted trapezoid graph $G = (V, E)$. The eccentricity $e(v)$ of the vertex v is defined as the sum of the weights of the vertices from v to the vertex farthest from $v \in T_{TRP}$. A vertex with minimum eccentricity in the tree T_{TRP} is called the 1-center of that tree. In an inverse 1-center location problem the parameter of the tree T_{TRP} corresponding to the weighted trapezoid graph, like vertex weights have to be modified at minimum total cost such that a pre-specified vertex $s \in V$ becomes the 1-center of the trapezoid graph G. In this chapter, an optimal algorithm is presented to find an inverse 1-center location on the weighted tree T_{TRP} corresponding to the weighted trapezoid graph, where the vertex weights can be changed within certain bounds. The time complexity of the proposed algorithm is $O(n)$.

Finally, Chapter 7 contains some concluding remarks and scopes of further research on the problems that have been studied in the thesis.