2018

CBCS

3rd Semester

STATISTICS

PAPER—C5T

(Honours)

Full Marks : 60

Time : 3 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Linear Algebra and Numerical Analysis

1. Answer any *ten* questions : 10×2

(a) Define symmetric and skew-symmetric matrix.

(Turn Over)

(b) For what real values of k, the set of vectors

$\{(k, 1, 1), (1, k, 1), (1, 1, k)\}$ is linearly independent in R^3.

(c) Prove that transpose of an orthogonal matrix is orthogonal.

(d) Show that the set W of ordered triad $(a_1, a_2, 0)$ where $a_1, a_2 \in F$, a field, is a subspace of V_3 over R.

(e) Show that a spanning set of vectors may not be unique.

(f) If A be an idempotent matrix of n,

show that $(I_n - A)$ is an idempotent matrix.

(g) If $A = \begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix}$, show that $A^{-1} = \frac{1}{19} A$.

(h) For what values of m, the vector $(m, 3, 1)$ is a linear combination of vectors $(3, 2, 1)$, $(2, 1, 0)$.

(i) Prove that $(1 + \Delta)(1 - \nabla) = 1$, where symbols are usual meaning.

(j) Find the relative error in computation of $(x - y)$ for $x = 9.05$ and $y = 6.56$ having absolute error $\Delta x = 0.001$ and $\Delta y = 0.003$ respectively.

(k) Evaluate $\left(\frac{\Delta^2}{E}\right)x^3$, where symbols are usual meaning.

(l) Find the condition of convergence of Newton-Raphson method.

(m) Show that the second order differences are zero for the function $f(x) = 2x + 5$.

(n) Find the position of a positive real root of

$3x - \cos x - 1 = 0$.

(o) State the dimension theorem of vector.

2. Answer any *four* questions : 4×5

(a) Estimate the missing term of the following table :

x	0	1	2	3	4
$f(x)$	1	3	9	—	81

(b) Find a basis of the real vector space R^3 containing the vectors (1, 1, 2) and (3, 5, 2).

(c) Show that

$$\begin{vmatrix} 2a & a-b-c & 2a \\ 2b & 2b & b-c-a \\ c-a-b & 2c & 2c \end{vmatrix} = (a+b+c)^3.$$

(d) If $A=\begin{pmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{pmatrix}$, show that $A^2 - 4A - 5I_3 = 0$. Hence obtain a matrix B such that $AB = I_3$.

(e) Using Trapezoidal rule, evaluate $\int_0^1 \left(4x - 3x^2\right) dx$ taking $n = 5$.

(f) Let V be a real vector space with $\{\alpha, \beta, \gamma\}$ as a basis. Prove that the set $\{\alpha + \beta + \gamma, \beta + \gamma, \gamma\}$ is also a basis of V.

3. Answer any *two* questions : 2×10

(a) Describes Newton-Raphson method to find a real root of the equation $f(x) = 0$, where $f(x)$ is continuous function of x. Give geometrically interpretation of this method. Also, write advantage and disadvantage of this method. 5+2+3

(Turn Over)

(b) Determine the condition for which the system

$$x + y + z = 1$$

$$x + 2y - z = b$$

$$5x + 7y + az = b^2$$

admits of only one solution, no solution and many solutions. Determine the rank of A, where

$$A = \begin{pmatrix} 2 & 1 & 4 & 3 \\ 3 & 2 & 6 & 9 \\ 1 & 1 & 2 & 6 \end{pmatrix}.$$

6+4

(c) What is the degree of the approximating polynomials corresponding to the trapezoidal formula? Justify.

Using Simpson $^1/_3$ rule, evaluate $\int_0^1 \frac{x}{1+x} dx$ taking $n = 6$.

3+7

(d) Prove that a skew symmetric determinant of odd order is zero. Find the inverse of the matrix $\begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 0 \\ 2 & 1 & -1 \end{pmatrix}$ and using it solve the following system of equations:

$2x + y + z = 5,\ x - y = 0,\ 2x + y - z = 1.$

3+4+3
