2017

PHYSICS

[Honours]

(CBCS)

[First Semester]

PAPER —C1T

Full Marks : 40

Time : 2 hours

Answer any **five** questions from Group—A, **four** from Group—B and **one** from Group—C

The figures in the right hand margin indicate marks

GROUP—A

Answer any **five** questions : 5 × 2

1. Determine the value of a, so that the function $f(x)$ defined by :

(Turn Over)

$$f(x)=\begin{cases}\dfrac{a\cos x}{\pi-2x} & \text{for } x\neq\dfrac{\pi}{2}\\[2ex] 0 & \text{for } x=\dfrac{\pi}{2}\end{cases}$$

be continuous. 2

2. If $f(r)$ is differentiable then calculate curl $(\vec{r}\, f(r))$. 2

3. Prove that $\oint_C \phi \, d\,\vec{r} = \iint_S d\vec{S}\times\vec{\nabla}\phi$. 2

4. Find the integrating factor of the differential equation

$$\cos x\frac{dy}{dx}+y\sin x=\sec^2 x.$$ 2

5. Show that the area bounded by a simple closed curve C in a plane is given by

$$A=\frac{1}{2}\oint(xdy-ydx).$$ 2

6. A loaded dice has the probabilities $\frac{1}{21}, \frac{2}{21}, \frac{3}{21}, \frac{4}{21}, \frac{5}{21}$ and $\frac{6}{21}$ of turning up 1, 2, 3, 4, 5 and 6 respectively. If it is thrown twice, what is probability that the sum of the numbers that turn up is even ? 2

7. Prove that $x\,\delta'(x) = -\delta(x)$. 2

8. The mean and the variance of a binomial variable X are 2 and 1 respectively. Find the probability that X takes values greater than 1. 2

GROUP—B

Answer any **four** questions : 5 × 4

9. If $\vec{A}$ and $\vec{B}$ are constant vectors then prove that

$$\vec{\nabla}[\vec{A}\cdot(\vec{B}\times\vec{r})] = \vec{A}\times\vec{B}.$$ 5

10. Solve $\frac{d^2y}{dx^2} + y = \sec^2 x$. 5

11. Poisson distribution gives the probability that x events occur in unit time when the mean rate of occurence is m.

$$P_x = \frac{e^{-m} m^x}{x!}$$

Show that

$$P_{x-1} = \frac{x}{m} P_m \text{ and } P_{x+1} = \frac{m}{x+1} P_x.$$ 5

12. (*a*) Two dices are thrown simultaneously. What is the probability of getting faces whose sum will be 6 ? 2

(*b*) Two coordinate system have same origin but rotated coordinate axes. Unit vectors of the coordinate systems are respectively. $\hat{e}_1, \hat{e}_2, \hat{e}_3$ and $\hat{e}_1', \hat{e}_2', \hat{e}_3'$ respectively.Show that 3

$$\hat{e}_1' = l_{11}\hat{e}_1 + l_{12}\hat{e}_2 + l_{13}\hat{e}_3$$

$$\hat{e}_2' = l_{21}\hat{e}_1 + l_{22}\hat{e}_2 + l_{23}\hat{e}_3$$

$$\hat{e}_2' = l_{31}\hat{e}_1 + l_{32}\hat{e}_2 + l_{33}\hat{e}_3$$

13. If $f(x)$ is the probability density of x given by $f(x) = x\,e^{-x/\lambda}$ over the internal $0 < x < \infty$, find the mean and the most probable values of x. 5

14. Verify Green's theorem in the plane for

$$\int_C (x+y)dx + (x-y)dy,$$

where C is the closed curve of the region bounded by $y = x^2$ and $y^2 = 8x$. 5

GROUP—C

Answer any **one** questions : 10 × 1

15. Verify the Gauss' divergence theorem for

$$\vec{F} = 4x\,\hat{i} - 2y^2\hat{j} + z^2\hat{k}$$

over the region bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 3$. 10

16. (*i*) Find the unit normal vector at the point $\left(\frac{a}{\sqrt{3}}, \frac{b}{\sqrt{3}}, \frac{c}{\sqrt{3}},\right)$ on the surface of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

(*ii*) Solve : $x^2 \frac{d^2 y}{dx^2} - x\frac{dy}{dx} + y = 2\log x.$

(*iii*) Evaluate

$\left| \int_C \vec{r} \times d\vec{\theta} \right|$, for a circle C of radius r with centre at the origin. 3 + 4 + 3
