Journal of Physical Sciences, Vol. 25, 2020, 11-20
ISSN: 2350-0352 (print), www.vidyasagar.ac.in/publication/journal
Published on 16 October 2020

A Similarity Measure Between Generalized Fuzzy Numbers

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Received 12 April 2020; accepted 22 August 2020

ABSTRACT

Chen 2008, proposed a method for handling fuzzy risk analysis problems based on measures of similarity between interval-valued fuzzy numbers. In this paper, we proposed a method based on measures of similarity between generalized fuzzy numbers. In this method, the horizontal center-of-gravity, the perimeter, the height and the area of the two fuzzy numbers are considered and also the proposed approach is very simple and easy to apply in the real life problems.

Keywords: Fuzzy risk analysis, Generalized fuzzy numbers, similarity measures

Mathematical Subject Classification (2010): 94D05

1. Introduction

The similarity measure of fuzzy numbers is very important in many research fields such as pattern recognition and risk analysis in fuzzy environment [5, 8]. Some methods have been presented to calculate the degree of similarity between fuzzy numbers Chen [1-4, 6]. Also, [7,9] proposed method with modified similarity measure of generalized fuzzy numbers. Rezvani [10] proposed a new similarity measure of generalized fuzzy numbers based on left and right apex angles. Moreover Rezvani [11-15] proposed ranking approach based on values and ambiguities of the membership degree and the non- membership degree for trapezoidal intuitionistic fuzzy number.

Chen 2008 proposed a method for handling fuzzy risk analysis problems based on measures of similarity between interval-valued fuzzy numbers. This paper proposed a method based on measures of similarity between generalized fuzzy numbers. In this method, the horizontal center-of-gravity, the perimeter, the height and the area of the two fuzzy numbers are considered and also the proposed approach is very simple and easy to apply in the real life problems. Applications of different operators are discussed in [16-20].

2. Preliminaries

Generally, a generalized fuzzy number A is described as any fuzzy subset of the real line R, whose membership function μ_A satisfies the following conditions,

(i) μ_A is a continuous mapping from R to the closed interval [0,1],

(ii) $\mu_A(x) = 0, -\infty < u \le a,$

(iii) $\mu_A(x) = L(x)$ is strictly increasing on $[a, b]$,

(iv) $\mu_A(x) = w, b \le x \le c,$

(v) $\mu_A(x) = R(x)$ is strictly decreasing on $[c, d]$,
(vi) $\mu_A(x) = 0$, $d \leq x < \infty$
where $0 < w \leq 1$ and a, b, c, and d are real numbers. We call this type of generalized fuzzy number a trapezoidal fuzzy number, and it is denoted bye $A = (a, b, c, d; w)$.

When $w = 1$, this type of generalized fuzzy number is called normal fuzzy number and is represented by $A = (a, b, c, d)$.

The membership function μ_A: $R \rightarrow [0, 1]$ is defined as follows:

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a} & x \in (a,b) \\ w & x \in (b,c) \\ \frac{x-d}{c-d} & x \in (c,d) \\ 0 & x \in (-\infty, a) \cup (d, \infty) \end{cases} \tag{1}$$

where $a \leq b \leq c \leq d$ *and* $w \in [0, 1]$.

2.1. Arithmetic operations

In this section, addition and subtraction between two trapezoidal fuzzy numbers, defined on universal set of real numbers R. Let $A = (a_1, b_1, c_1, d_1; w_1)$ and $B = (a_2, b_2, c_2, d_2; w_2)$be two trapezoidal fuzzy number, then

i) $A \oplus B = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2; \min\{w_1, w_2\})$,

ii) $A\ B = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2; \min\{w_1, w_2\})$,

iii) $A\ B = (a_1/d_2, b_1/c_2, c_1/b_2, d_1/a_2; \min\{w_1, w_2\})$,

iv) $A \otimes B = (a_1 \times a_2, b_1 \times b_2, c_1 \times c_2, d_1 \times d_2; \min\{w_1, w_2\})$.

3. Approach

In this section some important results, that are useful for the proposed approach, are proved. Jiang Wen 2011 proposed the concept of the method to calculate the degree of similarity between generalized fuzzy numbers. In this method, the horizontal center-of-gravity, the perimeter, the height and the area of the two fuzzy numbers are considered.

Suppose that $A_1 = (a_1, b_1, c_1, d_1; w_1)$ and $A_2 = (a_2, b_2, c_2, d_2; w_2)$ be the generalized trapezoidal fuzzy numbers, where $0 \leq a_1 \leq b_1 \leq c_1 \leq d_1 \leq 1$ and $0 \leq a_2 \leq b_2 \leq c_2 \leq d_2 \leq 1$. Then the degree of similarity $S(A_1, A_2)$ between the generalized trapezoidal fuzzy numbers A_1 and A_2 is calculated as follows:

$$S(A_1,A_2) = [\![1 - |x^*_{A_1} - x^*_{A_2}|]\!] \times [\![1 - |w_{A_1} - w_{A_2}|]\!] \times \frac{\min(P(A1),P(A2)) + \min(A(A1),A(A2))}{\max(P(A1),P(A2)) + \max(A(A1),A(A2))} \tag{2}$$

where $x^*_{A_1}$ is the horizontal center-of-gravity of the generalized trapezoidal fuzzy numbers $A1$ is calculated as follows:

$$x^*_{A_1} = \frac{y^*_{A_1}(c_1+b_1)+(d_1+a_1)\left(w_{A_1}-y^*_{A_1}\right)}{2w_{A_1}} \tag{3}$$

$P(A_1)$ is the perimeters of two generalized trapezoidal fuzzy numbers which are calculated as follows:

$$P(A_1) = \sqrt{(a_1-b_1)^2 + {w_{A_1}}^2} + \sqrt{(c_1-d_1)^2 + {w_{A_1}}^2} + (c_1-b_1) + (d_1-a_1) \tag{5}$$

$A(A_1)$is the areas of two generalized trapezoidal fuzzy numbers which are calculated as follows:

$$A(A_1) = \frac{1}{2} w_{A_1}(c_1 - b_1 + d_1 - a_1) \tag{6}$$

The larger the value of S (A_1, A_2), the more the similarity measure between two generalized trapezoidal fuzzy numbers A_1 and A_2.

3. A similarity measure between generalized fuzzy numbers

In this section, we propose a similarity measure to calculate the degree of similarity between generalized fuzzy numbers. Suppose that $A_1 = (a_1, b_1, c_1, d_1; w_1)$ and $A_2 = (a_2, b_2, c_2, d_2; w2)$ be the generalized trapezoidal fuzzy numbers. where $0 \le a_1 \le b_1 \le c_1 \le d_1 \le 1$ and$0 \le a_2 \le b_2 \le c_2 \le d_2 \le 1$.

Step 1:

$x^*_{A_1}$ and $x^*_{A_2}$ are the horizontal center-of-gravity of the generalized trapezoidal fuzzy numbers A_1 and A_2 is calculated as follows:

$$x^*_{A_1} = \frac{y^*_{A_1}(c_1+b_1)+(d_1+a_1)\left(w_{A_1}-y^*_{A_1}\right)}{2w_{A_1}} \tag{7}$$

$$y^*_{A_1} = \begin{cases} \dfrac{w_{A_1}\left(\frac{c_1-b_1}{d_1-a_1}+2\right)}{6} & \text{if} \quad a_1 ? d_1 \text{ and } 0 < w_{A_1} = 1 \\ \dfrac{w_{A_1}}{2} & \text{if} \quad a_1 = d_1 \text{ and } 0 < w_{A_1} = 1 \end{cases} \tag{8}$$

and

$$x^*_{A_2} = \frac{y^*_{A_2}(c_1+b_2)+(d_2+a_2)\left(w_{A_2}-y^*_{A_2}\right)}{2w_{A_2}} \tag{9}$$

$$y^*_{A_2} = \begin{cases} \dfrac{w_{A_2}\left(\frac{c_2-b_2}{d_2-a_2}+2\right)}{6} & \text{if} \quad a_2 ? d_2 \text{ and } 0 < w_{A_2} = 1 \\ \dfrac{w_{A_2}}{2} & \text{if} \quad a_2 = d_2 \text{ and } 0 < w_{A_2} = 1 \end{cases} \tag{10}$$

$P(A_1)$ and $P(A_2)$ are the perimeters of two generalized trapezoidal fuzzy numbers which are calculated as follows:

$$P(A_1) = \sqrt{(a_1-b_1)^2 + {w_{A_1}}^2} + \sqrt{(c_1-d_1)^2 + {w_{A_1}}^2} + (c_1-b_1) + (d_1-a_1) \tag{11}$$

$$P(A_2) = \sqrt{(a_2 - b_2)^2 + {w_{A_2}}^2} + \sqrt{(c_2 - d_2)^2 + {w_{A_2}}^2} + (c_2 - b_2) + (d_2 - a_2) \quad (12)$$

$A(A_1)$ and $A(A_2)$ are the areas of two generalized trapezoidal fuzzy numbers which are calculated as follows:

$$A(A_1) = \frac{1}{2} w_{A_1}(c_1 - b_1 + d_1 - a_1) \quad (13)$$

$$A(A_2) = \frac{1}{2} w_{A_2}(c_2 - b_2 + d_2 - a_2) \quad (14)$$

Step 2:
The degree of similarity $S(A_1, A_2)$ between the generalized trapezoidal fuzzy numbers A_1 and A_2 is calculated as follows:

$$S(A_1, A_2) = [\![1 - |x^*_{A_1} - x^*_{A_2}|]\!] \times [\![1 - |w_{A_1} - w_{A_2}|]\!] \times \frac{\min(P(A1), P(A2)) + \min(A(A1), A(A2))}{\max(P(A1), P(A2)) + \max(A(A1), A(A2))} \quad (15)$$

The larger the value of $S(A_1, A_2)$, the more the similarity measure between two generalized trapezoidal fuzzy numbers A_1 and A_2.The proposed similarity measure between generalized fuzzy numbers has the following properties.

Theorem 1. Two generalized fuzzy numbers $A_1 = (a_1, b_1, c_1, d_1; w_1)$ and $A_2 = (a_2, b_2, c_2, d_2; w_2)$ are identical if and only if $S(A_1, A_2) = 1$.

Proof: If A_1 and A_2 are identical, then

$$a_1 = a_2, b_1 = b_2, c_1 = c_2, d_1 = d_2; w_1 = w_2.$$

We have

$$x^*_{A_1} = \frac{y^*_{A_1}(c_1 + b_1) + (d_1 + a_1)\left(w_{A_1} - y^*_{A_1}\right)}{2w_{A_1}}$$

$$y^*_{A_1} = \begin{cases} \dfrac{w_{A_1}\left(\dfrac{c_1 - b_1}{d_1 - a_1} + 2\right)}{6} & \text{if} \quad a_1 \neq d_1 \text{ and } 0 < w_{A_1} \leq 1 \\ \dfrac{w_{A_1}}{2} & \text{if} \quad a_1 = d_1 \text{ and } 0 < w_{A_1} \leq 1 \end{cases}$$

$$x^*_{A_2} = \frac{y^*_{A_2}(c_1 + b_2) + (d_2 + a_2)\left(w_{A_2} - y^*_{A_2}\right)}{2w_{A_2}}$$

$$y^*_{A_2} = \begin{cases} \dfrac{w_{A_2}\left(\dfrac{c_2 - b_2}{d_2 - a_2} + 2\right)}{6} & \text{if} \quad a_2 \neq d_2 \text{ and } 0 < w_{A_2} \leq 1 \\ \dfrac{w_{A_2}}{2} & \text{if} \quad a_2 = d_2 \text{ and } 0 < w_{A_2} \leq 1 \end{cases}$$

and

$$P(A_1) = \sqrt{(a_1 - b_1)^2 + {w_{A_1}}^2} + \sqrt{(c_1 - d_1)^2 + {w_{A_1}}^2} + (c_1 - b_1) + (d_1 - a_1)$$

$$P(A_2) = \sqrt{(a_2 - b_2)^2 + {w_{A_2}}^2} + \sqrt{(c_2 - d_2)^2 + {w_{A_2}}^2} + (c_2 - b_2) + (d_2 - a_2)$$

and

$$A(A_1) = \frac{1}{2} w_{A_1}(c_1 - b_1 + d_1 - a_1)$$
$$A(A_2) = \frac{1}{2} w_{A_2}(c_2 - b_2 + d_2 - a_2)$$

So
$S\,(A_1, A_2)$ = [1- | 0 |] × [1- | 0 |] × 1 = 1.

Overhand

If $S\,(A_1, A_2) = 1$, we have of Eq (15)

$$\mathrm{S}(\mathrm{A}_1, \mathrm{A}_2) = [\![1 - |\mathrm{x}^*{}_{\mathrm{A}_1} - \mathrm{x}^*{}_{\mathrm{A}_2}|]\!] \times [\![1 - |\mathrm{w}_{\mathrm{A}_1} - \mathrm{w}_{\mathrm{A}_2}|]\!] \times \frac{\min(\mathrm{P(A1), P(A2)}) + \min(\mathrm{A(A1), A(A2)})}{\max(\mathrm{P(A1), P(A2)}) + \max(\mathrm{A(A1), A(A2)})} = 1$$

It implies that

$$a_1 = a_2,\ b_1 = b_2,\ c_1 = c_2,\ d_1 = d_2;\ w_1 = w_2.$$

Therefore, A_1 and A_2 are identical.

Theorem 2. $S\,(A_1, A_2) = S\,(A_2, A_1)$.
Proof:

$$\mathrm{S}(\mathrm{A}_1, \mathrm{A}_2) = [\![1 - |\mathrm{x}^*{}_{\mathrm{A}_1} - \mathrm{x}^*{}_{\mathrm{A}_2}|]\!] \times [\![1 - |\mathrm{w}_{\mathrm{A}_1} - \mathrm{w}_{\mathrm{A}_2}|]\!] \times \frac{\min(\mathrm{P(A1), P(A2)}) + \min(\mathrm{A(A1), A(A2)})}{\max(\mathrm{P(A1), P(A2)}) + \max(\mathrm{A(A1), A(A2)})} = 1$$
$$\mathrm{S}(\mathrm{A}_2, \mathrm{A}_1) = [\![1 - |\mathrm{x}^*{}_{\mathrm{A}_2} - \mathrm{x}^*{}_{\mathrm{A}_1}|]\!] \times [\![1 - |\mathrm{w}_{\mathrm{A}_2} - \mathrm{w}_{\mathrm{A}_1}|]\!] \times \frac{\min(\mathrm{P(A2), P(A1)}) + \min(\mathrm{A(A2), A(A1)})}{\max(\mathrm{P(A2), P(A1)}) + \max(\mathrm{A(A2), A(A1)})} = 1$$

where

$$|\mathrm{x}^*{}_{\mathrm{A}_1} - \mathrm{x}^*{}_{\mathrm{A}_2}| = |\mathrm{x}^*{}_{\mathrm{A}_2} - \mathrm{x}^*{}_{\mathrm{A}_1}|,$$
$$|\mathrm{w}_{\mathrm{A}_1} - \mathrm{w}_{\mathrm{A}_2}| = |\mathrm{w}_{\mathrm{A}_2} - \mathrm{w}_{\mathrm{A}_1}|,$$
$$min(P(A_1), P(A_2)) = min(P(A_2), P(A_1))\ ,$$
$$min(A(A_1), A(A_2)) = min(A(A_2), A(A_1))\ ,$$
$$max(P(A_1), P(A_2)) = max(P(A_2), P(A_1))\ ,$$
$$max(A(A_1), A(A_2)) = max(A(A_2), A(A_1))\ ,$$

Therefore, we can see that $S\,(A_1, A_2) = S\,(A_2, A_1)$.

Theorem 3. If A_1 and A_2 are two real numbers, then $S(A_1, A_2) = 1- |a_1 - a_2|$.
Proof: If A_1 and A_2 are two real numbers, then we can see that

$$A_1 = (a_1, b_1, c_1, d_1; w_1) = (a_1, a_1, a_1, a_1; 1) = a_1$$

and

$$A_2 = (a_2, b_2, c_2, d_2; w_1) = (a_2, a_2, a_2, a_2; 1) = a_2$$

Based on formulas (8) and (10), we can see that

If $a_1 = d_1$ and $a_2 = d_2$, so $y^*_{A_1} = y^*_{A_2} = \frac{1}{2}$.
So

$$x^*_{A_1} = \frac{\frac{1}{2}(2a_1) + (2a_1)\left(1-\frac{1}{2}\right)}{2} = a_1 ,$$
$$x^*_{A_2} = \frac{\frac{1}{2}(2a_2) + (2a_2)\left(1-\frac{1}{2}\right)}{2} = a_2 ,$$

And

$$P(A_1) = \sqrt{(a_1-a_1)^2+1} + \sqrt{(a_1-a_1)^2+1} + (a_1-a_1) + (a_1-a_1) = 2 ,$$
$$P(A_2) = \sqrt{(a_2-a_2)^2+1} + \sqrt{(a_2-a_2)^2+1} + (a_2-a_2) + (a_2-a_2) = 2 ,$$

And

$$A(A_1) = \frac{1}{2}(a_1 - a_1 + a_1 - a_1) = 0 ,$$
$$A(A_2) = \frac{1}{2}(a_2 - a_2 + a_2 - a_2) = 0 ,$$

Based on formulas (15), we can see that

$$S(A_1, A_2) = [\![1 - |a_1 - a_2|]\!] \times [\![1 - |1-1|]\!] \times \frac{\min(2,2) + \min(0,0)}{\max(2,2) + \max(0,0)} = 1 - |a_1 - a_2| .$$

4. Results
In this section, we use three examples to illustrate the process of calculating the degrees of similarity between generalized fuzzy numbers.

Example 1. Assume that there are two generalized fuzzy numbers A_1 and A_2, where

$$A_1 = (0.1, 0.2, 0.3, 0.4; 0.5),$$

$$A_2 = (0.4, 0.5, 0.6, 0.7; 0.5).$$

Step 1:

$$because\ a_1 \neq d_1 \Longrightarrow y^*_{A_1} = \frac{0.5\left(\frac{0.1}{0.3}+2\right)}{6} = 0.194$$

$$because\ a_2 \neq d_2 \Longrightarrow \mathrm{y^*}_{\mathrm{A_2}} = \frac{0.5\left(\frac{0.1}{0.3}+2\right)}{6} = 0.194$$

and

$$\mathrm{x^*}_{\mathrm{A_1}} = \frac{0.194(0.5) + (0.5)(0.5-0.194)}{2(0.5)} = 0.25\,,$$
$$\mathrm{x^*}_{\mathrm{A_2}} = \frac{0.194(1.1) + (1.1)(0.5-0.194)}{2(0.5)} = 0.55\,,$$

and

$$P(A_1) = \sqrt{(-0.1)^2 + (0.5)^2} + \sqrt{(-0.1)^2 + (0.5)^2} + (0.1) + (0.3) = 1.42\,,$$
$$P(A_2) = \sqrt{(-0.1)^2 + (0.5)^2} + \sqrt{(-0.1)^2 + (0.5)^2} + (0.1) + (0.3) = 1.42$$

And

$$A(A_1) = \frac{1}{2}(0.5)(0.3-0.2+0.4-0.1) = 0.1\,,$$
$$A(A_2) = \frac{1}{2}(0.5)(0.6-0.5+0.7-0.4) = 0.1$$

Step 2; The degree of similarity S (A_1, A_2) between the generalized trapezoidal fuzzy numbers A_1 and A_2 is calculated as follows:

$$\mathrm{S(A_1, A_2)} = [\![1 - |0.25 - 0.55|]\!] \times [\![1 - |0.5 - 0.5|]\!] \times \frac{\min(1.42,1.42) + \min(0.1,0.1)}{\max(1.42,1.42) + \max(0.1,0.1)}$$
$$= 0.7\ .$$

Example 2. Assume that there are two generalized fuzzy numbers A_1 andA_2, where

A_1 = (0.1, 0.2, 0.3, 0.4; 1),

A_2 = (0.4, 0.5, 0.6, 0.7; 1).

*Step 1

$$because\ a_1 \neq d_1 \Longrightarrow \mathrm{y^*}_{\mathrm{A_1}} = \frac{\left(\frac{0.1}{0.3}+2\right)}{6} = 0.39$$
$$because\ a_2 \neq d_2 \Longrightarrow \mathrm{y^*}_{\mathrm{A_2}} = \frac{\left(\frac{0.1}{0.3}+2\right)}{6} = 0.39$$

And

$$\mathrm{x^*}_{\mathrm{A_1}} = \frac{0.39(0.5) + (0.5)(1-0.39)}{2} = 0.25\,,$$
$$\mathrm{x^*}_{\mathrm{A_2}} = \frac{0.39(1.1) + (1.1)(1-0.39)}{2} = 0.55\,,$$

And

$$P(A_1) = \sqrt{(-0.1)^2 + 1} + \sqrt{(-0.1)^2 + 1} + (0.1) + (0.3) = 2.41\ ,$$
$$P(A_2) = \sqrt{(-0.1)^2 + 1} + \sqrt{(-0.1)^2 + 1} + (0.1) + (0.3) = 2.41$$

And

$$A(A_1) = \frac{1}{2}(0.3 - 0.2 + 0.4 - 0.1) = 0.2\ ,$$
$$A(A_2) = \frac{1}{2}(0.6 - 0.5 + 0.7 - 0.4) = 0.2$$

Step 2: The degree of similarity S (A_1, A_2) between the generalized trapezoidalfuzzy numbers A_1 and A_2 is calculated as follows:

$$\mathrm{S}(\mathrm{A}_1, \mathrm{A}_2) = [\![1 - |0.25 - 0.55|]\!] \times [\![1 - |1 - 1|]\!] \times \frac{\min(2.41{,}2.41) + \min(0.2{,}0.2)}{\max(2.41{,}2.41) + \max(0.2{,}0.2)}$$
$$= 0.7\ .$$

Example 3. Assume that there are two generalized fuzzy numbers A_1 andA_2, where

A_1 = (0.1, 0.2, 0.3, 0.4; 1),

A_2 = (0.4, 0.5, 0.6, 0.7; 0.5).

Step 1:

$$because\ a_1 \neq d_1 \Longrightarrow \mathrm{y}^*{}_{\mathrm{A}_1} = \frac{\left(\frac{0.1}{0.3} + 2\right)}{6} = 0.39$$
$$because\ a_2 \neq d_2 \Longrightarrow \mathrm{y}^*{}_{\mathrm{A}_2} = \frac{(0.5)\left(\frac{0.1}{0.3} + 2\right)}{6} = 0.194$$

And

$$\mathrm{x}^*{}_{\mathrm{A}_1} = \frac{0.39(0.5) + (0.5)(1 - 0.39)}{2} = 0.25\ ,$$
$$\mathrm{x}^*{}_{\mathrm{A}_2} = \frac{0.194(1.1) + (1.1)(1 - 0.194)}{2(0.5)} = 0.55\ ,$$

And

$$P(A_1) = \sqrt{(-0.1)^2 + 1} + \sqrt{(-0.1)^2 + 1} + (0.1) + (0.3) = 2.41\ ,$$
$$P(A_2) = \sqrt{(-0.1)^2 + (0.5)^2} + \sqrt{(-0.1)^2 + (0.5)^2} + (0.1) + (0.3) = 1.42$$

And

$$A(A_1) = \frac{1}{2}(0.3 - 0.2 + 0.4 - 0.1) = 0.2\,,$$
$$A(A_2) = \frac{1}{2}(0.5)(0.6 - 0.5 + 0.7 - 0.4) = 0.1$$

Step 2: the degree of similarity S (A_1, A_2) between the generalized trapezoidal fuzzy numbers A_1 and A_2 is calculated as follows:

$$\begin{aligned} S(A_1, A_2) &= [\![1 - |0.25 - 0.55|]\!] \times [\![1 - |1 - 0.5|]\!] \times \frac{\min(2.41,1.42) + \min(0.2,0.1)}{\max(2.41,1.42) + \max(0.2,0.1)} \\ &= 0.7\ . \end{aligned}$$

6. Conclusion

In this paper, a method based on measures of similarity between generalized fuzzy numbers is proposed. In this method, the horizontal center-of-gravity, the perimeter, the height and the area of the two fuzzy numbers are considered and also the proposed approach is very simple and easy to apply in the real life problems.

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