Chapter 1

Introduction

1.1 Operations Research

Operations Research (OR) is a branch of science which deals with any real life decision making problem – gives direction for the mathematical formulation of the problem, suggests solution approaches and finally helps for appropriate and efficient decision making. It can be applied to the management of any business sector, industry, defense, health etc. According to Churchman et al. [35], OR is an application of scientific methods, techniques and tools to find an optimal solution of a decision making problem.

The main origin of OR was during the Second World-War. At that time, the military management in England called upon a team of scientists to study the strategic and tactical problems related to air and land defence of the country with very limited military resources, it was necessary to decide upon the most effective utilization of them, e.g., the efficient ocean transport, effective bombing etc. As the team was dealing with research on (military) operations, the work of this team of scientists was named as '*Operational Research*' in England.

After the end of war, the team of scientists (researchers) was attracted by the industrial managers, who were seeking optimal solutions for their complex executive-type problems. George Bernard Dantzig, an American mathematician, first invented a mathematical technique in the field of OR in 1947. This technique is called Simplex Method (SM) of Linear Programming Problem (LPP). After few years, the extended version of this research work was again published by

Dantzig [37]. Since that time, operational research has expanded into a field widely used in industries ranging from petrochemicals to airlines, finance, logistics, and government, moving to a focus on the development of mathematical models that can be used to analyse and optimize complex systems, and has become an area of active academic and industrial research.

1.1.1 Inventory Control System in Operations Research

Nowadays, the impact of OR can be felt in many areas. Apart from military and business applications, the OR activities include transportation system, libraries, hospitals, city planning, financial institution etc. For example, in real life, it is observed that a small retailer knows roughly the demand of his/her customer in a month or a week and accordingly places order to the wholesaler to meet the demand of his/her customer. But this is not the case with a manager of big departmental store or a big retailer, because the stocking in such cases depends on several factors, e.g., demand, time of ordering, time lag between the orders and actual receipt, deterioration, time value of money, inflation etc. and the impreciseness of these factors. So the problem for the managers/retailers is to have a compromise between over-stocking and under-stocking. The study of such type of problem is known by the term '*Inventory Control*'.

Inventory control is concerned with the flow of materials from supplier to production and the subsequent flow of products through distribution centers to the customers. It is responsible for the planning, acquisition, storage, movement and control of materials and final products. It attempts to get the right goods at the right price at the right time to maintain desired service level at minimum cost.

In broad sense, inventory is defined as an idle resource of an enterprize/company/manufacturing firm. It can be defined as a stock of physical goods, commodities or other economic resources which are used to meet up the customer's demand or requirement of production. This means that the inventory acts as a buffer stock between a supplier and a customer. The inventory or stock of goods may be kept in any one of the following forms:

- Raw materials.
- Semi-finished goods (work-in-process inventory).
- Finished (or produced) goods.

- Spare parts, etc.

The control and maintenance of inventory is a problem common to all organizations in any sector of economy. Inventories of physical goods are maintained in government and non-government establishments, e.g., Agriculture, Industry, Military, Business, Health, Education etc. Some reasons for maintaining inventories of business organizations are:

- to conduct smooth and efficient running of business.
- to provide the customers' service by meeting their demands from stock without delay.
- to earn price discount for bulk purchasing.
- to take the financial advantage of transporting/shipping economics.
- to maintain more stable operating and/or workforce levels.
- to plan overall operating strategy through decoupling of successive stages in the chain of acquiring goods, preparing products, shipping to branch warehouses and finally serving the customers.
- to motivate the customers to purchase more by displaying large number of goods in the show-room/shop.
- to take the advantages in purchasing of some raw materials and some commonly used physical goods (such as paddy, wheat etc.) whose prices seasonally fluctuate.

For similar reasons, inventories of other sectors are used. The problem of inventory control is primarily concerned with the following fundamental questions:

- Which items should be carried in stock or which items should be produced?
- How much of each of these items should be ordered/produced?
- When should an order be placed or when to produce?
- What type of inventory control system should be used?

So, an inventory problem is a problem of making optimal decisions regarding the above questions. In other words, an inventory problem deals with decisions that improve customer service, goodwill of the organization etc. and optimize either the cost function (total or average cost) or the profit function (total or average profit) of the inventory system. In practice, it is a formidable task to determine a suitable inventory policy to deal with the above mentioned questions in a real life situation. While making use of the techniques of OR for inventory control problems, a mathematical model of the problem is formulated. The model is actually a simplified representation of the problems in which only the most important features are considered for reasons of simplicity. Then, an optimal or most favourable solution is found.

1.1.2 Supply Chain Management in Operations Research

Another part of OR is '*Supply Chain Management*' (SCM), which is basically similar to the '*Inventory Control System*'. SCM can be defined as a set of approaches, utilized to efficiently integrate suppliers, manufacturers, warehouses/stores, distribution centres, wholesalers, retailers and customers so that merchandise is produced and distributed the right quantities, to the right locations at the right time to minimize/maximize the system-wide costs/profits satisfying the service level requirements.

A SCM can be defined as an extension of inventory control system. In an inventory control system, normally the decisions are made from the view of a single decision maker/player involved in system, whereas in a SCM, the decisions are made from the supply chain point of view, where all the players involved in the system are benefited. According to the researchers, if the number of players of a business chain is grater than two or three, then the system known as SCM. The researchers and the practitioners formulate a prototype of the problems related to a supply chain mathematically, normally known as '*Supply Chain*' (SC) model and then solve it for a feasible/real-life solution for the decision makers (DMs).

Traditionally, SCM has been defined as the management of physical, logical and financial flows in networks of intra- and inter- organizational relationships/co-ordinations jointly adding value and achieving customer satisfaction [101]. In the literature, there are different types of SC models such as

- 2/3-stages SC model: These types of SC involves two or three participants excluding the customers (i.e., wholesaler–retailer-customer/supplier–wholesaler–retailer-customer).
- Closed loop SC model: These types of SC explicitly taken into account of product returns in addition to the downstream flow of materials [95]. A supply chain is termed as closed loop if it recovers a product from a customer in exchange for value and recycles the product for profit and gain by putting it through quality check and refurbishment. On behalf of the Company,

this type of collection is done by the retailer and the retailer enjoys some incentives for the collection and shipping.

One of the key features of a SC model is the information sharing among the system participants. This information sharing may be in the form of (i) promotional cost sharing between the retailer and the wholesaler, (ii) demand estimation passed on to the manufacturer by the retailer, etc.

In this research work, our aim is to formulate mathematical models of the different realistic inventory control systems/supply chain management and to solve those using different suitable mathematical techniques.

1.2 Basic Concepts and Terminologies in Supply Chain/Inventory models

The inventory control/SCM models deals with the several factors like demand, lead time, replenishment, various types of costs, constraints etc. The detailed descriptions of these factors are found in the existing literature (cf. Hadley and Whitin [71], Naddor [131], Silver and Peterson [155], Tersine [182] etc.). However, for the advantage of new readers, some well known definitions related to inventory control/SCM model are briefly described as follows.

1.2.1 Definitions and Terminologies

To describe inventory control/SCM model, some definitions and terminologies are presented in this subsection.

Demand: The term '*Demand*' refers to the quantities taken from an inventory to meet the customers requirement at any instant. This is the most important factor for any type of inventory control/SCM model. The demand or market demand usually depends on the decisions of the people outside of the organization. Demand can be categorized according to its size, rate and pattern.

- Deterministic demand, which may be constant, may depends on promotional effort, time, customers' credit period, frequency of advertisement, displayed inventory level, selling price etc.

- Stochastic demand with known/unknown probability distribution.
- Imprecise demand, which occurs due to the insufficient past data for the determination of the parameters involved in the demand.

Planning Horizon: The duration of time for which decisions are made for an inventory control system or SCM is called *planning horizon* or *time horizon* of the considered system. In other words, the *planning horizon* is a period of time for which a manager decides to make his/her business decisions. Depending upon the nature of the commodity involved in the inventory control system or SCM, the planning horizon may be *finite* or *infinite.*

Lead Time: *Lead time* is the length of time interval between the making of a decision to replenish an item and its actual addition to stock, i.e., the time gap between the time of placing an order or starting of the production and the time of actual arrival or delivery of goods to the inventory is called *lead time.* It can be dependent or independent of order-size, may be crisp (i.e., a finite number), probabilistic or imprecise.

Replenishment: *Replenishment* can be categorized according to size, pattern and lead time. Replenishment size refers to the quantity or size of the order to be received into inventory. The size may be constant or variable, depending on the type of inventory system. Replenishment patterns refers to how the units are added to the inventory. The replenishment patterns are usually instantaneous, uniform or in batch.

Deterioration: It is defined as decay, evaporation, obsolescence, loss of utility or marginal value of a commodity that results in the decreasing usefulness from the original condition. Medicines, vegetables, food grains, gasoline and semiconductor chips etc., are the examples of such products. The rate of deterioration is measured by the fraction deteriorated per unit quantity per unit time and it may be constant or vary with time or stocked units.

Trade Credit: In recent competitive market, manufacturer/wholesaler/retailer frequently offers delay period for settling the account on purchasing amount of units (greater than or equal to a certain amount fixed by the wholesaler/manufacturer). This is termed as trade credit period. Depending upon the credit period, demand of an item increases or decreases. Different types of credit policy are observed in reality:

- If credit period is offered to the retailers only by the wholesaler, or if the credit period is offered to the customers only by the retailer, it is called **one level trade credit**.
- If both the wholesaler and the retailer offer credit period to his/her retailer and the customers respectively, it is called **two level trade credit**.
- On the other hand, if the supplier, the wholesaler and the retailer offer credit period to his/her wholesaler, retailer and customers respectively, it is called **three level trade credit**.
- Again, if credit period is offered depending upon some conditions (like amount of purchase should exceed some label, frequency of order etc.), it is called **conditionally delay in payment** or **conditional credit period**.

Price Discount: Nowadays, it is very frequently observed that to boost the demand, some manufacturers offer price discount in the form of putting additional materials in every unit pack, bringing down the unit price for a certain period of time. Obviously demand increases due to low price. After that specified period of time, the manufacturer withdraws the additional amount and thus unit price increases. By this process demand increases due to the fact that some customers have already accustomed with the product during the price discount period and do not switch over to other products though price discount is withdrawals. This process of boosting a product is commonly practiced by different manufacturers, specially when a product is newly launched in the market.

Promotional Effort: In the recent competitive market, it is often noticed that the supplier/wholesaler/retailer adopts various types of promotional activities to boost the market demand. Promotional effort refers to the set of marketing activities undertaken by the parties of a SC to boost the sales of a product. This effort is made to attract the new customers, to hold the existing customers, to nullify competition and to take advantage of opportunities that are revealed by the market research. It is made up of activities, both outside and inside activities, to increase company sales. Outside sales promotion activities include advertising, publicity, public relations activities, special sales events etc. Inside sales promotion activities includes window displays, product and promotional material display and promotional programs such as premium awards and contests. Some of the devices used in sales promotion include freebies, coupons, samples, premiums, Point-of-Purchase displays, discounts, demonstrations, contests, rebates, sweepstakes, product samples, exhibitions or trade shows, giveaways etc. The most important and effective activities in this directions are listed below:

- **Trade credit policy -** In the competitive business environment, nowadays it is observed that the supplier/wholesaler/retailer offers some credit period to the wholesaler/retailer/customer for settling the account. This strategy is very useful in any field of business as the credit opportunity for the customers increases the market demand of any item.

- **Price Discount -** Price discount facility is another approach applied to the buyers for more buying, which basically increases the market demand of an item. Basically price discount facility offered by a wholesaler/retailer to the retailer/customer depending upon the order quantity, i.e., if a buyer purchases more units of an item or purchasing amount exceeds a level, predefined by the vendor.

- **Advertisement -** Advertisement is another effective promotional tool which basically used to focus about the product specifications/utilities/opportunities to the customers through various communication media such as television, daily newspaper, different social sites etc. Though this type of promotional activity involves significant cost, it is very useful approach to run a business.

Inflation and Time Value of Money: Inflation is a present increase in the level of consumer price or a persistent decline in the purchasing power of money, caused by an increase in available currency and credit beyond the proportion of available goods and service. It is the rate at which the general prices for goods and services are rising and subsequently, purchasing power is falling. With the increase of inflation rate, more amount of money is to be paid for the same quantity of commodity. As for example, if the inflation rate is 1%, a \$5 of pen will cost \$5.05 in a year. Mathematically, Buzacott [20] assumed that cost at time t, $\phi(t)$, becomes $\phi(t+\delta t) = \phi(t) + i\phi(t)\delta t$ at time $(t+\delta t)$ (where, δt is sufficiently small) when a constant inflation rate i (\$/unit) exists in the market, i.e.,

$$\begin{aligned}
& \phi(t+\delta t) = \phi(t) + i\phi(t)\delta t \text{ as } \delta t \to 0 \\
\Rightarrow\ & \frac{\phi(t+\delta t) - \phi(t)}{\delta t} \to i\phi(t) \text{ as } \delta t \to 0 \\
\Rightarrow\ & \frac{d\phi(t)}{dt} = i\phi(t) \\
\Rightarrow\ & \frac{d\phi(t)}{\phi(t)} = idt
\end{aligned}$$

which yields a solution as $\phi(t) = \phi(0)e^{it}$, where, $\phi(0)$ is the cost at time $t = 0$.

On the other hand, time value of money is one of the basic concept of finance. We know that if we deposit money in a bank we will receive interest. For example, \$1 today invested for one year at 7% return would be worth \$1.07 in a year. Because of this, we prefer to receive money today rather than the same amount in the future. Money we receive today is more valuable to us than money received in the future by the amount of interest we can earn with the money. It is the changes in purchasing power of money over time.

So, if $i\%$ and $r\%$ are the annual inflation and interest rate respectively, resultant effect of inflation and time value of money (i.e., increased rate of cost) on purchasing a unit of item in future is $(i-r)\%$. So if $\phi(0)$ is the cost of an item at time $t = 0$, its cost at time t, $\phi(t) = \phi(0)e^{-Rt}$, where, $R = (r - i)$ is called discount rate of cash flow.

Fully/Partially Back-logged Shortages: During stock-out period, the sales and/or good-will may be lost either by a delay or complete refusal in meeting the demand of the customers. In the case where the unfulfilled demand for the goods can be satisfied completely at a later date, then it is a case of fully back-logged shortages, i.e., it is assumed that no customer balk away during this period and the demand of all these waiting customers is met at the beginning of the next period. Again, it is normally observed that during the stock-out period, some of the customers wait for the product and others balk away. When this happens, the phenomenon is called partially backlogged shortages.

Constraints: Constraints in inventory system/SCM deal with various properties that some way place limitations imposed on the inventory system. Constraints may be imposed on the amount of investment, available space, resources and finance, the amount of inventory held, average inventory expenditure, number of orders etc. These constraints can also be fuzzy/rough in nature, i.e., data for constraints, goals for the objectives, resources etc., may be imprecise and vague.

1.2.2 Costs and Revenue

Different types of costs involved in any inventory control/SCM models are described briefly in this subsection.

Holding Cost or Carrying Cost: It is the cost associated with the storage of the inventory until its use or sale. It is directly proportional to the quantity in inventory and the time for which the stocks are held. This cost generally includes the costs such as rent for storage space, interest on the money locked-up, insurance, taxes, handling etc.

Ordering Cost or Set-up Cost: It is the cost associated with the expense of issuing a purchase order to an out-side supplier or setting up machines before internal production. These costs also include clerical and administrative costs, telephone charges, telegram, transportation costs, loading and unloading costs, watch and ward costs etc. Generally, this cost is assumed to be independent of the quantity ordered for or produced. In the costs like transportation cost etc., some part of it may be quantity dependent.

Purchase Cost: It is the unit purchase price to obtain the item from an external source or the unit production cost for internal production. It may also depend upon the demand when production is done in large quantities as it results in reduction of production cost per unit. Also, when quantity discounts are allowed for bulk orders, unit price is reduced and depends on the quantity purchased or ordered. Unit production cost is also production dependent. For example, if one worker is needed to tend the machine, then as more units are produced per unit time, the wages of the worker spread over more units.

Shortage Cost or Stock-out Cost or Penalty Cost: It is the penalty incurred when the stock proves inadequate to meet the demand of the customers. This cost parameter does not depend upon the source of replenishment of stock but upon the amount of inventory not supplied to the customer.

Lost Sale Cost: When an inventory is out of stock then a portion of the customers wait for the product, but, others find alternative source to purchase it. As a result the company losses its goodwill/reputation as well as it has some revenue loss due to unmet demand. The total cost associated with the stock-out period is termed as lost sale cost.

Advertisement Cost: The advertisement cost is the cost associated with advertisement of an item in popular media like Newspaper, Magazine, TV, Radio etc. and through the sales representative to boost the sale of that item.

Transportation Cost: The transportation cost is the cost associated in moving products or assets to a different place. It needs to bring the products to the retailer for sale to consumers.

Promotional Cost: Promotional cost is the cost which is used to promote the item(s) in order to increase the market demand of the item(s) involved in the SCM or inventory control system.

Learning Effect: In many realistic situations, because the firms and employees perform the same task repeatedly, they learn how to perform more efficiently. Therefore, the actual production and/or set-up cost of a job is less rather than earlier in the production process. This phenomenon is known as the '*learning effect*' in the literature. Different types of learning effects have been demonstrated and extensively studied in a number of areas. One of them is processing/set-up time. Production process of some jobs to be faster than those of others, i.e., the learning is some time job-dependent.

Sales Revenue: The amount earned from selling goods/services by a business is called sale revenue.

1.3 Different Environments

The parameters, like demand, inventory costs (holding cost, ordering cost, shortage cost, advertisement cost etc.), lead time, budget, order quantity, available resources, goals etc., involved in the inventory/SCM model may be precise in nature (crisp) or some of these parameters may be imprecise in nature. Depending on these type of parameters, the models can classified in different environments.

Crisp Environment: When all the parameters and the resources used in the inventory/SCM model are deterministic and precisely defined, then the environment is known as crisp environment.

Fuzzy Environment: When some or all of the parameters and the resources used in the inventory/SCM model and/or goal(s) of the objectives(s) are imprecise in nature and these parameters or quantities are characterized by valid membership functions, then the the environment is called fuzzy environment.

Rough Environment: When some or all of the parameters and the resources used in the inventory/SCM model and/or goal(s) of the objectives(s) are rough in nature, i.e., cannot be defined properly due to insufficient past data or vagueness, then the environment is known as rough environment.

1.4 Historical Review on Inventory/SC Models

From the beginning of 20th century, a lot of research articles have been developed by several researchers of inventory control problems/supply chain management. Some simple EOQ (Economic Order Quantity) model has been developed at the beginning of the 20th century. After that EPQ (Economic Production Quantity) model or EMQ (Economic Manufacturing Quantity) model has been developed. These models are modified and extended to improve the business strategy by considering different assumptions to make it more realistic.

1.4.1 Historical review on models in crisp environment

In any business sector, the control and maintenance of inventory is a common problem. Some researchers have been studied on inventory control problem in deterministic environment since early twentieth century. The earliest work on EOQ model was developed by Ford Harris [74] of Westinghouse Corporation, USA. After few years, the same formula was also developed by R.H. Wilson [192] independently and this formula was named as Harris-Wilson formula or Wilson's formula. In this model, the sum of inventory carrying and set-up costs was minimized, where the demand rate was known and constant, the shortages were not allowed, the replenishment rate was infinite and the lead time was negligible. Since then, the several researchers developed various types of inventory control/SC models by modifying and extending the above mentioned model to make the models more and more realistic. Many full length books was published on inventory control system [5, 71, 72, 157, 158]. Also many research articles was published on inventory control/SC models [4, 6, 12, 13, 23, 32, 33, 38, 39, 49, 50, 52, 56, 58, 63, 77, 94, 100, 102, 125].

1.4.2 Historical review on models in imprecise environment

Due to the rapid increasing complexities of the environment, it is difficult to define different inventory parameters precisely. As a result, it may not be possible to define the different inventory costs as well as the constraints precisely. For example, production of an item in any manufacturing organization deeply depends on efficiency, effectiveness of the system, i.e., quality of the process output, inventory turnover ratio and so many factors related to the production process, which leads to uncertainty/impreciseness in any production process.

The philosopher, Max Black [15] was first introduced the concept of vague set in 1937, which is an important step towards dealing with vagueness. In 1965, the first publication accommodating the uncertainty in non-stochastic sense was made by Prof. Zadeh [208]. The concept of '*Fuzzy Set Theory*' was first introduced by him. Bellman and Zadeh [9], first used the concept of fuzzy set theory in decision making problem. Since then, extensive research works have been done by the researchers in this area [42, 46–48, 209, 213].

But applications of fuzzy sets in inventory control problems [21, 66, 69, 118, 119, 191] are around 20-25 years. Kao and Hsu [88] discussed the inventory problem with fuzzy demands where back-orders are permitted. Maiti and Maiti [113] developed a fuzzy inventory model with two warehouses under possibility constraints. Maiti and Maiti [115] developed a two-storage inventory model with lot-size dependent fuzzy lead-time under possibility constraints via genetic algorithm. Bera et al. [10] proposed an inventory model with fuzzy lead-time and dynamic demand over finite time horizon using a multi-objective genetic algorithm. Kumar and Rajput [98] developed a fuzzy inventory model for deteriorating items with time dependent demand rate and partial backlogged shortages. Kundu et al. [99] proposed a production inventory model with price discounted fuzzy demand using an interval compared hybrid algorithm.

Till now, fuzzy differential equation (FDE) and fuzzy integration are little used to solve fuzzy inventory models [68, 168], though the topics on fuzzy differential equations have been rapidly growing in the recent years. Solving FDE was first motivated by Kandel and Byatt [86]. After two years, an extended version of their work has been published by them [87]. Some notable papers in this direction are due to Petrovic et al. [148], Buckley and Feuring [19], Chalco-Cano and Román-Flores [25, 26] etc. On the other hand, study on fuzzy integration was initiated by Sugeno [178]. Dubois and Prade [43, 44] presented two most valuable research paper on fuzzy integration. Sims and Wang [174] gave a good review of this subject. After that several researchers investigated different procedure for fuzzy integration. Wu [194] introduced the concept of fuzzy Riemann integration (FRI) and its numerical integration. Using FDE and FRI, some inventory models with fuzzy demand/production rate have been published by some authors [68, 99].

Along with fuzzy set theory, rough set theory is another tool to formulate the above mentioned problems with more accuracy. Z. Pawlak [144] was introduced the concept of '*Rough Set Theory*'. Then several research works have been done in this area [105, 107, 145–147]. Santana-Quintero et al. [164] have been presented a multi-objective optimization using PSO and rough sets theory. Xu and Zhao [199] developed a fuzzy rough multi-objective decision-making model, according to fuzzy rough theory. Shi et al. [173] proposed a probability maximization model based on rough approximation and its application to the inventory control problem. Mondal et al. [126] developed a production-repairing inventory model in fuzzy rough environment incorporating inflationary effects. Guchhait et al. [69] developed an inventory model for a deteriorating item with time dependent deterioration in imprecise environment. An excellent survey on rough set theory and its applications was made by Zhang et al. [210]. Recently, Garai et al. [54] developed a multi-objective multi-item inventory model with fuzzy rough coefficients.

1.4.3 Historical review on demand pattern

Demand is the most important factor for any type of SC/inventory control system. In real-life problems, demand depends on several factors, like displayed inventory, time, selling price, promotional effort, trade credit, price discount, advertisement frequency etc. Some researchers developed their models considering constant demand [3, 63, 134]. Influence of displayed inventory level on the demand of any item is a well established phenomenon [38, 56, 58, 65, 66, 127, 160]. Demand also can changes with time. Many researchers have been considered time dependent demand [17, 27, 32, 39, 73, 94, 98, 100, 125, 152, 167]. Selling price is also influence the demand. So selling price dependent demand [4, 68, 83, 99, 112, 113, 129, 136, 138, 143, 154, 162, 187] is also considered by many researchers. Some research articles have already been published incorporating promotional cost sharing in supply chain [23, 136, 150, 153, 193]. In all these studies, it is assumed that a promotional effort influences the demand of an item and promotional cost is a function of this promotional effort. Other important features which influences the demand are credit period [66, 69, 75, 118, 137], price discount [99, 138], advertisement frequency [113, 120] etc.

1.4.4 Historical review on trade credit policy

The inventory models are normally developed on the basic assumption that the retailer is paid for the units of the item immediately after the units are received. However it may not be true for today's competitive business transactions. Nowadays, it is normally found that the supplier allows a certain fixed time period (termed as credit period) to its retailers for settling the amount that the retailer owes to the supplier for the item supplied. The trade credit is a supplier's short term loan to the retailer, allowing the retailer to delay payment of an invoice. Also the retailer can allow trade credit for the customers to increase his/her demand of the items. Goyal [63] first developed an EOQ model under the conditions of permissible delay in payment, where a supplier offers a fixed credit period to a retailer to settle his/her account. Aggarwal and Jaggi [3] extended Goyal's model for a deteriorating item.

With the novel invention of Goyal [63], several researchers have been developed various models under single level trade credit policy considering different realistic assumptions [3, 33, 34, 102, 134]. In the above studies, it is observed that a supplier offers a delay period in payment to a retailer but in turn the retailer does not offer any credit period to his/her customers, it is an unrealistic phenomenon. To overcome this unrealistic phenomenon, Huang [80] first extended an EOQ model under two levels of trade credit policy, where a supplier offers a delay period in payments for settling his/her account to the retailer and in turn the retailer also offers a trade credit period to the customers. Huang [81] investigated the optimal retailer's replenishment decisions under two levels of trade credit policy in the EPQ model. Teng [181] extended the Huang's model [81] to complement the shortcoming of the model. Ho [76] proposed a generalized, integrated, supplier–retailer inventory model under two-level trade credit policy. After that many researchers have been developed the inventory models under trade credit policy [66, 67, 69, 78, 124]. In real practice, trade credit policy is observed at different levels. Recently, three level trade credit policy is considered by some authors [135, 150, 151].

1.4.5 Historical review on price discount policy

Any item is supposed to be sold in maximum retail price printed on the packet, but in reality, it is observed that different retailers give different discounts to attract their customers. Sometime packaging is made with some extra amount which basically decreases the unit price. Free gift/extra amount with a purchase above a predefined minimum amount is another approach of reducing the selling price. Viswanathan and Wang [187] discussed the effectiveness of quantity discounts and volume discounts as coordination mechanisms in distribution channels with price-sensitive demand. Yang [203] developed an optimal pricing and ordering policy for a deteriorating item with price sensitive demand. Shah et al. [172] developed a mathematical model for an inventory system that considers a temporary price discount for an time dependent deteriorating items. After that many inventory/SC models have been developed by the researchers with price discounted demand [29, 41, 99, 138, 163, 166].

1.4.6 Historical review on deteriorating item

Deterioration is an important factor for inventory decisions of deteriorating items. In any business, it is too difficult to preserve the deteriorating items. There are so many products such as fruits, vegetables, medicines, electronic goods etc., deteriorate continuously due to evaporation, spoilage, obsolescence etc. Ghare and Schrader [57] developed an inventory model first time for deteriorating items. After that a large number of research articles found, considering constant deterioration [33, 34, 58, 95, 100, 125, 201]. But in reality, there are so many items such as medicines, bloods in blood bank, volatile liquids, high-tech products etc., those are not only deteriorate continuously with time, but also each of these items have an expiration time. From this point of view, there are several research works developed under time dependent deterioration [36, 69, 149] and time dependent deterioration with an expiration time [109, 171, 180]. An excellent review on inventory control models of deteriorating items since 2001, is made by Bakker et al. [7].

1.4.7 Historical review on displayed inventory

The displayed inventory has significant role in drawing attention to the customers. Due to this reason, it is normally observed that the retailers decoratively displayed their items in the market outlet. In the market place, it is very difficult to acquire sufficient space to store and display the units. Normally, the retailer distributes the display area among different items depending upon the amount of dependency of the demands on displayed units of different items [16, 114, 117]. Due to insufficient size of the market outlet, they hire a warehouse with sufficiently large capacity, little away from the market. Items are initially stored in this warehouse and transferred to the market outlet for sell in a regular time interval. So the retailer uses two warehouses to run his/her business smoothly. There are some inventory control models incorporating this phenomenon [12, 139, 140, 200, 212].

1.4.8 Historical review on inflation and time value of money

Sometimes it is observed that the price of an item changes very frequently. This phenomenon is basically occurs due to the fluctuating rate of interests offered by the bank (discount rate) and the rate of inflation in the market. Fluctuation in the price of an item basically depends on the resultant effect of these two parameters (difference of discount rate from inflation rate). In 1975, Buzacott [20] was first introduced the effect of inflation and time value of money in inventory control problem. He dealt with an EOQ model under inflation with different types of pricing policies. Bierman and Thomas [14] has taken inflation in inventory decisions. Some authors have shown the effects of inflation and time-value of money on an inventory model with time-dependent demand rate and shortages [39, 73]. Bose et al. [17] have been formulated an EOQ model for deteriorating items with linear time-dependent demand rate and shortages under inflation and time discounting. After that the several research works in inventory/SC models have been done under inflation [24, 38, 77, 102, 126, 139, 140, 156, 160, 167, 183, 200–202].

1.4.9 Historical review on promotional effort strategy

Promotional activity is very essential in inventory control/SC models to reach the products to the customers efficiently in competitive business market. To improve the demand, the retailer uses some promotional activities, like, local advertisement, offering price discount, free gift etc. and the cost due to these activities is known as promotional cost. Some researchers introduced the promotional effort by the retailer or the supplier and they optimize their goal independently [118, 138, 153]. During last few decade, several research papers have been published reflecting some portions of the above mentioned real life phenomenon [2, 23, 78, 136, 150, 184, 185, 193]. In all these studies, it is assumed that due to promotional activity, the demand of an item is increased by the promotional effort and cost of promotional activity is a function of the promotional effort. But all these studies does not outline how demand actually increases with the promotional activity and what is the actual cost due to this activity. Moreover, most of these studies considered two level supply chain involving wholesaler, retailer and customers.

1.5 Motivation and Objectives of the Thesis

In any supply chain of an item, the units are despatched to the retailer through different agents and are finally distributed to the customers by the retailer. The output of the chain mostly depends on the customers' demand of the item. As a result retailer tries to boost the demand using different promotional activities. Among different promotional activities, decoratively displaying the units is most effective to draw attention of the customers. Due to scarcity of places at the heart of the market place, the retailer rents a sufficiently large warehouse little away from the market place to store the units and market outlet is decorated properly to display the units appealingly. Though this is a normal practice, not much attention has been paid to develop supply chain model with retailer's two warehouse facility. Moreover, the retailer also introduces other additional promotional activities, like, price discount, free gift, advertisement etc., to boost the base demand of the item. As the retailer bears the total cost of these promotional activities, he plays as the key decision maker or leader of the supply chain and other parties act as the followers. But performance of a supply chain primarily depends on the proper coordination among different parties of the chain. Promotional cost sharing among different parties may be treated as a coordination mechanism for such a supply chain. Studies of these supply chain become more complicated when different parameters of the problem are not precisely defined, i.e., supply chain in imprecise environment, like, fuzzy environment, rough environment etc. There is a significant gap in the existing literature of such supply chain models, specially in imprecise environment. This situation forces us to develop a supply chain model with stock dependent demand and retailer's two warehouse facility in both precise and imprecise environments under promotional cost sharing. As optimisation in imprecise environment is not properly defined and due to the absence of any classical optimisation approach for the objective with imprecise parameters, here some approaches are proposed to deal with such problems and a soft computing algorithm, PSO is used to solve the model. The model is mentioned as Model 3.1 and discussed in Chapter 3.

Customers' demand is the most important factor in any supply chain, as total revenue from the chain mostly depends on it. For a seasonal product, demand normally depends on time. At the beginning of the season, the demand normally increases with time and reaches a maximum limit at peak season time. Then it gradually decreases and at the end of the season, demand comes down to off-season level. So seasonal demand curve of such an item is parabolic type. A considerable number of research papers have been published by several researchers on time dependent demand but none has considered this phenomenon specially in supply chain model. This phenomenon motivated us to develop a supply chain model of a seasonal deteriorating item with time, price, and promotional activity dependent demand in a finite time horizon. The model is mentioned as Model 3.2 and discussed in Chapter 3.

In any production system, the production rate of an item is normally imprecise in nature. On the other hand, though demand of an item depends on several factors, no precise form can be defined to model the demand, i.e., demand parameters are normally imprecise in nature and can be estimated as fuzzy parameters using expert's opinion. But existing literature on the manufacturer-supplier supply chain does reflect these real life phenomena. Also considerable amount of research works have been done to improve the channel performance of a SC, but very few have outlined the actual process of utilization of promotional cost and its sharing in the chain. To overcome this research gap, in Chapter 3, a supply chain model under inflation is considered incorporating the above mentioned real life phenomenon and named it as Model 3.3.

Tsao [184] developed a multi-item supply chain under trade credit and Huang et al. [78] made some corrections to that study. But still there are some limitations and errors in their studies which are summarised below:

- The supplier does not hold any product, but the holding cost of the supplier is considered.
- The interest earned and the interest paid by the retailer and the supplier are not properly calculated.
- They did not consider the uncertain situation due to volatile market, i.e., changing bank interests at regular intervals, uncertain available resource (i.e., budget).

Extensions of Tsao's [184] and Huang et al.'s [78] investigations incorporating higher level of trade credit and imprecise constraints on resources after correcting the mistakes in their formulations are discussed in Model 4.1 in Chapter 4. Again, to boost the demand, some manufacturers offer price discount in the form of putting additional materials in every unit pack, bringing down the unit price for a certain period of time. Obviously, the demand increases due to low price. After that specified period of time, the manufacturer withdraws the additional amount and thus unit price increases. By this process, the demand increases due to the fact that some customers have been already accustomed with the product during the price discount period and do not switch over to other products though price discount is withdrawn. This phenomenon is considered in another model, Model 4.2, in Chapter 4.

In any supply chain, profit of each party mostly depends on the market demand of the items involved in the chain. Though every item has some base demand in the market, goal of every supply chain is to improve this base demand to survive in the market. Displayed inventory level always influences the customers and accordingly retailers normally hire a showroom in the market place to attract the customers. This investment is mainly done at the retailer level. Two other promotional activities which highly influence the demand are – advertisement and selling price. Again for multi-item supply chain, significant cost reduction can be done using joint ordering and joint replenishment. All these phenomenon are incorporated together to develop a supply model, Model 4.3, in Chapter 4.

Normally, a manufacturer produces several types of items and a supplier/distributor also supplies/distributes several items together to a wholesaler and so on. A retailer orders several items from its wholesaler together at the time of replenishment to reduce the ordering cost, transportation cost etc. The retailer usually orders different items at a regular time interval, may be termed as basic period and cycle length of any item is an integer multiple of this basic period. Though joint replenishment policy is reflected in few two level supply chain models, it has not been reflected in multi-level supply chains under trade credit and promotional cost sharing. Moreover, most of the existing supply chain models are developed in crisp environment, though impreciseness of different parameters of any supply chain is a well established phenomenon. To overcome these shortcomings, a multi-item supplier-wholesaler-retailer-customers supply chain model is proposed incorporating inflationary effects where each party offers a partial trade credit period to his/her purchasers. The model is named as Model 4.4 and is discussed in Chapter 4.

1.6 Organization of the Thesis

In this thesis, the above mentioned real life supply chain problems are formulated in crisp, fuzzy and rough environments and the problems are solved following different soft computing techniques. The proposed thesis has been divided into following four parts containing five chapters. The organization of the thesis is presented below.

Part I: Introduction and Solution Methodologies

Chapter 1
Introduction

This chapter contains an introduction giving an overview of the development on inventory control system/SCM models in different environments (precise and imprecise). The motivation behind this research work is also briefly presented in this section.

Chapter 2
Solution Methodologies

In this chapter, preliminary ideas and some important deductions on crisp set, fuzzy set and rough set are presented. Also in this chapter, some methodologies like linear/non-linear optimization technique in different environments (crisp, fuzzy, rough), Generalized Reduced Gradient (GRG) technique, Particle Swarm Optimization (PSO) technique, Multi-choice Artificial Bee Colony (MCABC) algorithm, Mixed-mode Multi-choice Artificial Bee Colony (MMCABC) algorithm are developed/used to solve the proposed supply chain models which are presented in the next part, i.e., in Part II of this thesis.

Part II: Some Supply Chain Management Models

Chapter 3
Single-item Supply Chain Management models

In this Chapter, three single-item supply chain models have been developed in different environments.

Model 3.1: Two-Level Supply Chain for a Deteriorating Item with Stock and Promotional Cost Dependent Demand Under Shortages

In this research work, a wholesaler-retailer-customer supply chain model for a deteriorating item is considered, where the retailer's warehouse in the market place has a limited capacity. The retailer can rent an additional warehouse (rented warehouse) if needed, with a higher rent compared to the existing warehouse (own warehouse). The customers' demand of the item is linearly influenced by the stock level and in case of shortages the base demand is partially backlogged. Being the leader of the supply chain, the retailer introduces some promotional cost to boost the base demand of the item. To participate in joint marketing decision, the wholesaler shares a compromise part of this promotional cost. Goal of this research work is to maximize the individual profits (when the retailer is the leader and the wholesaler is the follower) as well as the channel profit (when the retailer and the wholesaler jointly take marketing decision) of the system. It is established that if the wholesaler shares a part of the promotional cost then the channel profit as well as the individual profits increase. The supply chain model is also considered in imprecise environment, where different inventory parameters are fuzzy/rough in nature. In this case, the individual profits as well as the channel profit become fuzzy/rough in nature. As optimization of fuzzy/rough objective is not well defined, following credibility (cf. § 2.1.2.5)/trust measure (cf. § 2.1.3) of fuzzy/rough event, an approach is followed for comparison of fuzzy/rough objectives and a Particle Swarm Optimization algorithm (cf. § 2.2.2.1) is implemented to find the marketing decisions. Efficiency of the algorithm in solving the problem is statistically established. The existence of the joint marketing decision is established analytically and numerically (with illustration) in crisp as well as in imprecise environments.

Model 3.2: Two-Level Supply Chain of a Seasonal Deteriorating Item with Time, Price and Promotional Cost Dependent Demand Under Finite Time Horizon

In this research work, a wholesaler-retailer supply chain model is developed for a seasonal deteriorating item, where the demand of the item increases with time at the beginning of the season, reaches a maximum level and then decreases gradually to normal demand at the end of the season. Retailer introduces some promotional cost to boost the base demand of the item. The demand of the item also depends on unit selling price. In case of any shortages, the demand is partially backlogged. It is found that if wholesaler contributes a portion of promotional cost, then joint profit as well as individual profit increases. The supply chain model is also analyzed in imprecise environment, when different inventory parameters are fuzzy in nature. In this case, individual profits as well as channel profit become fuzzy in nature. To find the marketing decision, PSO (cf. § 2.2.2.1) is used as in Model 3.1.

Model 3.3: A Supply Chain with Fuzzy Production Rate and Demand Under Inflation and Promotional Cost Sharing Using ABC

In this model, a supplier-manufacturer supply chain is considered under inflation and time value of money in a fuzzy planning horizon. Here, the supplier supplies raw material to the manufacturer and the manufacturer produces the item in a production rate which is fuzzy in nature. Demand of the item also fuzzy in nature which depends on selling price. As the model is developed incorporating inflation, the demand also depends on inflation and time value of money. At the beginning of each production cycle, the manufacturer introduces some promotional cost by offering some price discount for a fixed period of time to increase the demand of the item. It is established that if the supplier shares some percentage of this promotional cost, then the profits of both the parties improve. As the production rate and the demand of the item are fuzzy in nature, fuzzy differential equation is used to formulate the model, which gives α-cut of on hand inventory level at any instant. Fuzzy-Riemann integration approach is used to find α-cuts of the total profit from the planning horizon. A Multi-choice Artificial Bee Colony (MCABC) algorithm (cf. § 2.2.2.3) is proposed where improvement of a solution by an employee bee or an onlooker bee is done using different rules. The algorithm is capable of solving optimization problems in crisp and imprecise environments. The efficiency of the algorithm is tested with a set of benchmark test problems available in the literature. It is found that the proposed MCABC algorithm is better than any existing algorithm of continuous optimization problems in the literature with respect to accuracy and consistency. Considering the α-cut of the fuzzy profit as an interval number and using fuzzy preference ordering (FPO) of interval comparison (cf. § 2.1.5), the proposed MCABC algorithm is used to find marketing decision of the model.

Chapter 4
Multi-item Supply Chain Management models

In this Chapter, four multi-item supply chain models have been developed in different environments.

Model 4.1: Uncertain Multi-item Supply Chain with Two Level Trade Credit Under Promotional Cost Sharing

In this model, we have extended the Tsao's [184] [Managing multi-echelon multi-item channels with trade allowances under credit period. International Journal of Production Economics, 127, 226-237] and Huang et al.'s [78] [Note on: Managing multi-echelon multi-item channels with trade allowances under credit period. International Journal of Production Economics, 138, 117-124] investigations incorporating higher level of trade credit and imprecise constraints on resources after correcting the mistakes in their formulations. Here, the multi-item two level supply chain model is formulated without/with budget constraint of the retailer. Customers' credit period is introduced to boost the base demand of the items. It is established that if the supplier shares a part of the promotional cost, then the channel profit as well as the individual profits increase. It is also established that the customers' credit period has sufficient significance in the supply chain. In their formulation, Tsao [184] as well as Huang et al. [78] considered some holding costs for the supplier. But according to the model, the supplier does not hold any product, i.e., the supplier's holding cost should be zero. Also the interest earned and the interest paid by the retailer and the supplier are not properly calculated in their study. In the present study, these mistakes are corrected. In addition, when the available budget of the retailer is uncertain in the sense of fuzzy or rough, the optimum profits of the said models are evaluated and presented. Models with imprecise constraints are transferred to equivalent crisp constraints following suitable techniques. Existence of the optimal solution of the unconstrained crisp model is established analytically. These models are solved by Generalized Reduced Gradient (GRG) method (cf. § 2.2.1.1) using LINGO 14.0 software and/or Particle Swarm Optimization (PSO) technique (cf. § 2.2.2.1). The models with imprecise inventory costs involve imprecise objectives and in those cases marketing decisions are made using PSO in two approaches – direct approach, i.e., no crisp equivalents of the imprecise objectives are used for solving the models and another is expected value (of the objectives) optimization approach.

Model 4.2: Fuzzy Optimization for Multi-item Supply Chain with Trade Credit and Two-Level Price Discount Under Promotional Cost Sharing

In this model, a multi-item two level supply chain is considered where the supplier offers a cash discount and a credit period to its retailer to boost the demand of the items. Due to this facility, the retailer also offers a cash discount to its customers to increase the base demand of the items. Retailer also introduces some promotional costs to boost the base demand of the items. It is established theoretically that if the supplier shares a part of the promotional cost, then the channel profit as well as the individual profit increase. It is also established that the cash discount given to the customers has sufficient significance in a supply chain. The supply chain model is also considered in imprecise environment, when different inventory costs are fuzzy in nature. In this case, the individual profits as well as the channel profit become fuzzy in nature. As optimization of fuzzy objective is not well defined, following credibility measure (cf. § 2.1.2.5) of fuzzy event, an approach is followed for comparison of fuzzy objectives and a Particle Swarm Optimization (PSO) algorithm (cf. § 2.2.2.1) is used to find marketing decisions. In another approach, Graded Mean Integration Values (GMIV) (cf. § 2.1.2.4) of the fuzzy objectives are optimized to find marketing decisions.

Model 4.3: A Two-warehouse Multi-item Supply Chain with Stock Dependent Promotional Demand Under Joint Replenishment Policy: A Mixed-mode ABC Approach

In this investigation, a multi-item two-level supply chain model under promotional cost sharing is proposed and analysed. Here, it is assumed that a retailer purchases different items from a wholesaler under joint replenishment policy and sells the items to its customers. Due to the scarcity of the market place, the retailer uses two rented warehouses to run the business – one with moderate capacity situated at the heart of the market place, namely RW_1 and another with sufficiently large capacity, a little away from the market place, namely RW_2. Items are ordered jointly using basic period (BP) policy, initially stored at RW_2 and transferred jointly to RW_1 for sell, following another BP policy. Demand of the items depend on displayed inventory levels, selling prices as well as the frequencies of the advertisements. Total cost due to the reduced selling prices and the advertisements is considered as the promotional cost. It is established that if the wholesaler and the retailer both share this promotional cost, then the profits of both the parties enhance. Here, selling price, frequency of advertisement, cycle length, space for RW_1 for each item, BP for ordering and shipment are considered as decision variables for both the wholesaler and the retailer to find the marketing decision to maximize their individual profits as well as the joint profit from the planning horizon. The problem is formulated as a mixed integer optimization problem in crisp as well in imprecise environments. To solve such complicated problems, here, a Mixed-mode

Multi-choice Artificial Bee Colony (MMCABC) algorithm (cf. § 2.2.2.4) is proposed for mixed integer optimization problems. The algorithm is tested against a set of benchmark test problems available in the literature and its efficiency to solve such problems is well established.

Model 4.4: A Multi-item Supply Chain with Multi-level Trade Credit Policy Under Inflation: A Mixed-mode ABC Approach

In this section, a multi-item supplier-wholesaler-retailer-customers supply chain with partial trade credit policy at each level under inflationary effect for a fixed planning horizon is developed and analysed. Here, the wholesaler receives a partial credit period from the supplier, i.e., a credit period on a portion of the amount of units purchased. Wholesaler also offers a partial credit period to its retailer and in turn the retailer also offers a partial credit period to its customers to boost the base demand of any item. Here, the credit period induced base demand of any item decreases linearly with time. Demand of the items are also influenced by the respective selling prices. The retailer introduces some promotional cost against advertisement and price discount to improve the demand of the items. Here, it is established that if the wholesaler shares a portion of this promotional cost, then the profits of both the retailer and the wholesaler improve. Model is formulated as a mixed integer profit maximization problem and is analysed in crisp as well as in imprecise (fuzzy/rough) environment and some managerial insights are outlined. To find the marketing decision of such a real-life supply chain model, here, also MMCABC algorithm (cf. § 2.2.2.4) is used as in Model 4.3.

Part III: Summary of the Thesis and Future Research Directions

Chapter 5

Summary of the Thesis and Future Research Directions

In this chapter, an overall summary of the different models developed in this dissertation has been presented. Also, some guidelines are presented to develop further better inventory/SC models for the future researchers in this field.

Part-IV: Bibliography and Index