Chapter 8

A solid transportation problems with non-linear transportation cost and type-2 fuzzy parameters

8.1 Introduction

Almost all TP and / or STP are formed as a linear programming problem (LPP), where objective function is the progressive sum of the product of transported quantity (variable) and unit transportation cost (constant) (cf. Haley [52], Shell [136], Basu et al. [11], Bit et al. [14], Ojha et al. [110], and many more). Ojha et al. [109], and Gupta et al. [51] developed the TP with a fixed cost and solved through different optimization process. In transportation allocation system entropy function plays an important role to search more number of allocation cell. But in reality, transportation cost not linearly depends on the quantity. In general, upto a certain limit of quantity, it does not matter how much quantity the managers transport and after that limit, it may varies exponentially. Till now, none has considered the conception of non-linear transportation cost, which is quite adequate in reality.

Liu and Mendel [84] proposed the concept of an interval type-2 fuzzy set for dealing with the impreciseness of quadratic order. Zeng and Liu [156] described the important advances concerning type-2 fuzzy sets for pattern recognition, and in [139] explored the calculation of the union and intersection of concave type-2 fuzzy sets. From the computational viewpoint, type-2 fuzziness is more difficult to deal with than type-1 fuzziness because the possibility of a type-2 fuzzy variable taking on a crisp value is a fuzzy number in [0, 1]. To avoid this difficulty, some type reduction approaches have been proposed in the literature for dealing with type-2 fuzziness, for example, Qin [122] proposed a defuzzification method with the concept of a centroid of a type-2 fuzzy set. Liu [89] employed a centroid type reduction strategy for a general type-2 fuzzy logic system and Qiu et al. [123] developed a statistical method for deciding on interval-valued fuzzy membership functions and a probability type reduction reasoning method for use with the interval-valued fuzzy logic system. Takac [138] discussed inclusion and subsethood measure for interval-valued fuzzy sets and for continuous type-2 fuzzy sets

After, a many research (cf Mendel and John [102], Mizumoto and Tanaka [105], Karnik and Mendel [68], Fazel et al. [35], Hidalgo et al. [55, 56]) studies the development of T2FSs theoretically. Now a day, T2FSs has been applied in different fields like group decision making system (neural network (Aliev et al. [3], Amiri [7], Chen and Wang [22])), portfolio selection problem (Hasuike and Ishi [53]), data envelopment analysis (Quin et al. [122]), pattern recognition (Mitchell [104]), data envelopment analysis (Quin et al. [122]), etc. Karnik and Mendel [68] is discussed the centroid of an interval type-2 fuzzy set (IT2FS) and they advanced a centroid type-reduction method to convert IT2FS into T1FS. But this method is very hard to apply the method to general T2 FS. The centroid type reduction strategies has been developed by Wu and Tan [145] for general type 2 fuzzy logic system. Liu and Liu [85] discussed a type-2 fuzzy variable (T2FV) for possibility theory, as a map from a fuzzy possibility space to the set of real numbers. Three kinds of reduction methods(pessimistic CV, optimistic CV and CV reduction) has been introduced by Qin et al. [122] for T2FVs based on critical values

(CVs) of regular fuzzy variables. Some uncertainty measures such as fuzziness (entropy), cardinality, skewness, variance of an IT2FS are presented by Wu and Mendel [145]

In this chapter, a solid transportation problem has been considered with non-linear unit transportation cost of the item depended on the amount of transportation. The unit transportation cost decreases at an inverse exponential rate if the transported amount increases. The availability of the sources, demand of the destinations, capacity of the conveyances are considered as a type-2 fuzzy parameters. The model is solved following CV-based reduction method, nearest interval approximation method and Chance constrained programming based credibility measures. Finally, a numerical example has beeen taken to illustrate the model and compared the results obtained through different methods.

8.2 Notations and Assumptions

8.2.1 Notations

In this solid transportation problem, instead of common notations the following additional notations have been used.

(i) $\widetilde{\widetilde{C'}}_{ijk}$ = initial type-2 fuzzy unit transportation cost of the solid transportation from i-th supply to j-th destination via k-th conveyance.

(ii) $\widetilde{\widetilde{a}}_i$ = type-2 fuzzy amount of a homogeneous product available at i-th source.

(iii) $\widetilde{\widetilde{b}}_j$ = type-2 fuzzy demand at j-th destination.

(iv) $\widetilde{\widetilde{e}}_k$ = type-2 fuzzy amount of product which can be transported/carried by k-th conveyance.

(v) d_{ijk} = maximum limit of transportation of the item upto which the initial unit cost is fixed.

8.2.2 Assumptions

To develop the proposed solid transportation model, the following assumptions have been made.

(i) In this model, the unit transportation cost has been taken into non-linear form by which unit cost is increase in decreasing order. The unit transformation costs $\tilde{\tilde{C}}_{ijk}$ is the following form

$$\tilde{\tilde{C}}_{ijk} = \begin{cases} \tilde{\tilde{C'}}_{ijk} e^{-\frac{x_{ijk}}{d_{ijk}}} & if\ x_{ijk} > d_{ijk} \\ \tilde{\tilde{C'}}_{ijk} & if\ x_{ijk} \leq d_{ijk} \end{cases} \tag{8.1}$$

8.3 Mathematical Formulation of Non-linear Solid Transportation Problem (NLSTP)

As stated in the above assumptions the proposed non-linear solid transportation problem with type-2 fuzzy parameters is

$$\text{Minimize } \tilde{\tilde{Z}} = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} \tilde{\tilde{C}}_{ijk}\, x_{ijk} \tag{8.2}$$

subject to the constraints

$$\begin{aligned} \sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} &\leq \tilde{\tilde{a}}_i & i = 1, 2, ...M \\ \sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} &\geq \tilde{\tilde{b}}_j & j = 1, 2, ...N \\ \sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} &\leq \tilde{\tilde{e}}_k & k = 1, 2, ...K \\ x_{ijk} &\geq 0 \quad \text{for all} \quad i, j, k. \end{aligned} \tag{8.3}$$

Where $\tilde{\tilde{C}}_{ijk}$ is given by equation number (8.1)

The first constraint in equation (8.3) represents the capacity of the sources, where as the second & third constraint of equation (8.3) represent the demand of the each destination

& capacity of the each conveyance respectively. The expression in equation number (8.1) indicates that unit transportation cost increase due to the increasing the transportation item in decreasing order.

8.4 Solution procedure

8.4.1 Chance constrained programming using generalized credibility

Now, we solve the problem (8.2) to construct a chance-constrained programming model with reduced these fuzzy parameters. The reduced fuzzy parameters are may not be normalized, so general credibility measure is not be used. Since the problem is minimization problem, the following chance-constrained programming model is constructed as the following (using generalized credibility)

$$\text{Minimize}_x(Minimize\ \tilde{\tilde{Z}}) \tag{8.4}$$

$$\widetilde{Cr}\{\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\tilde{\tilde{C}}_{ijk}x_{ijk} \leq \tilde{\tilde{Z}}\} \geq \alpha$$

subject to the constraints

$$\begin{aligned}
&\widetilde{Cr}\{\sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} \leq \tilde{\tilde{a}}_i\} \geq \alpha_i \quad \forall \quad i = 1, 2, ...M \\
&\widetilde{Cr}\{\sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} \geq \tilde{\tilde{b}}_j\} \geq \beta_j \quad \forall \quad j = 1, 2, ...N \\
&\widetilde{Cr}\{\sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} \leq \tilde{\tilde{e}}_k\} \geq \gamma_k \quad \forall \quad k = 1, 2, ...K \\
&x_{ijk} \geq 0 \quad \text{for all} \quad i, j, k.
\end{aligned} \tag{8.5}$$

where Minimize Z is the minimum possible crisp form that the objective function attains with generalized credibility at least $\alpha(0 < \alpha \leq 1)$. α_i, β_j and γ_k $(0 \leq \alpha_i, \beta_j, \gamma_k \leq 1)$ are predetermined generalized credibility levels of satisfaction of the respective constraints for all i, j, k.

8.4.2 Crisp equivalences

Suppose that the $\tilde{\tilde{C}}_{ijk}$, $\tilde{\tilde{a}}_i$, $\tilde{\tilde{b}}_j$ and $\tilde{\tilde{e}}_k$ are all mutually independent type-2 triangular fuzzy variables given by $\tilde{\tilde{C}}_{ijk} = (C^1_{ijk}, C^2_{ijk}, C^3_{ijk}, \theta^l_{ijk}, \theta^r_{ijk})$, $\tilde{\tilde{a}}_i = (a^1_i, a^2_i, a^3_i, \theta^l_i, \theta^r_i)$, $\tilde{\tilde{b}}_j = (b^1_j, b^2_j, b^3_j, \theta^l_j, \theta^r_j)$ and $\tilde{\tilde{e}}_k = (e^1_k, e^2_k, e^3_k, \theta^l_k, \theta^r_k)$. Also let $\tilde{C}_{ijk}$, $\tilde{a}_i$, $\tilde{b}_j$ and $\tilde{e}_k$ are the corresponding reductions by the CV reduction method. Using chance-constrained model formulation (8.4)-(8.5) of Model (i.e., (8.2)-(8.3)) can be converted to the following crisp equivalent (theorem 2.2 in§2.1.13) parametric programming problems:

Case-I: $0 < \alpha \leq 0.25$: Then the equivalent parametric programming problem for the model (8.4)-(8.5) is

$$\text{Minimize } Z = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} \frac{(1+2\alpha+(1+4\alpha)\theta^r_{ijk})C^1_{ijk}x_{ijk} + 2\alpha C^2_{ijk}x_{ijk}}{1+(1-4\alpha)\theta^r_{ijk}} \tag{8.6}$$

subject to the constraints

$$\begin{array}{lllll}
\sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} & \leq & F_{ai} & \forall & i = 1, 2, ...M \\
\sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} & \geq & F_{bj} & \forall & j = 1, 2, ...N \\
\sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} & \leq & F_{ek} & \forall & k = 1, 2, ...K \\
x_{ijk} & \geq 0 & \text{for all} & i, j, k. &
\end{array}$$

Case-II: $0.25 < \alpha \leq 0.50$: Then the equivalent parametric programming problem for the model representation (8.4)-(8.5) is

$$\text{Minimize } Z = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} \frac{(1-2\alpha)C^1_{ijk}x_{ijk} + (2\alpha+(4\alpha-1)\theta^l_{ijk})C^2_{ijk}x_{ijk}}{1+(4\alpha-1)\theta^l_{ijk}}$$

subject to the constraints

$$\sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} \leq F_{ai} \quad \forall \quad i = 1,2,......M$$
$$\sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} \geq F_{bj} \quad \forall \quad j = 1,2,......N$$
$$\sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} \leq F_{ek} \quad \forall \quad k = 1,2,......K$$
$$x_{ijk} \geq 0 \quad \text{for all} \quad i,j,k.$$

Case-III: $0.50 < \alpha \leq 0.75$: Then the equivalent parametric programming problem for the model representation (8.4)-(8.5) is

$$\text{Minimize} \quad Z = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} \frac{(2\alpha-1)C_{ijk}^{3}x_{ijk} + (2(1-\alpha)+(3-4\alpha)\theta_{ijk}^{l})C_{ijk}^{2}x_{ijk}}{1+(3-4\alpha)\theta_{ijk}^{l}}$$

subject to the constraints

$$\sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} \leq F_{ai} \quad \forall \quad i = 1,2,......M$$
$$\sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} \geq F_{bj} \quad \forall \quad j = 1,2,......N$$
$$\sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} \leq F_{ek} \quad \forall \quad k = 1,2,......K$$
$$x_{ijk} \geq 0 \quad \text{for all} \quad i,j,k.$$

Case-II: $0.75 < \alpha \leq 1$: Then the equivalent parametric programming problem for the model representation (8.4)-(8.5) is

$$\text{Minimize} \quad Z = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} \frac{(2\alpha-1+(4\alpha-3)\theta_{ijk}^{r})C_{ijk}^{3}x_{ijk} + 2(1-\alpha)C_{ijk}^{2}x_{ijk}}{1+(4\alpha-3)\theta_{ijk}^{r}}$$

subject to the constraints

$$\begin{array}{llll}
\sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} & \leq & F_{ai} & \forall \quad i=1,2,......M \\
\sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} & \geq & F_{bj} & \forall \quad j=1,2,......N \\
\sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} & \leq & F_{ek} & \forall \quad k=1,2,......K \\
x_{ijk} & \geq 0 & \text{for all} & i,j,k.
\end{array}$$

where,

$$F_{ai} = \begin{cases} \frac{(1-2\alpha_i+(1-4\alpha_i)\theta_i^l)a_i^3+2\alpha_i a_i^2}{1+(1-4\alpha_i)\theta_i^l} & \text{if } 0<\alpha_i\leq 0.25; \\ \frac{(1-2\alpha_i)a_i^3+(2\alpha_i+(4\alpha_i-1)\theta_i^r)a_i^2}{1+(4\alpha_i-1)\theta_i^r} & \text{if } 0.25<\alpha_i\leq 0.50; \\ \frac{(2\alpha_i-1)a_i^1+(2(1-\alpha_i)+(3-4\alpha_i)\theta_i^r)a_i^2}{1+(3-4\alpha_i)\theta_i^r} & \text{if } 0.50<\alpha_i\leq 0.75; \\ \frac{(2\alpha_i-1+(4\alpha_i-3)\theta_i^l)a_i^1+2(1-\alpha_i)a_i^2}{1+(4\alpha_i-3)\theta_i^l} & \text{if } 0.75<\alpha_i\leq 1; \end{cases} \tag{8.7}$$

$$F_{bj} = \begin{cases} \frac{(1-2\beta_j+(1-4\beta_j)\theta_i^l)b_j^3+2\beta_j b_j^2}{1+(1-4\beta_j)\theta_i^l} & \text{if } 0<\beta_j\leq 0.25; \\ \frac{(1-2\beta_j)b_j^3+(2\beta_j+(4\beta_j-1)\theta_i^r)b_j^2}{1+(4\beta_j-1)\theta_i^r} & \text{if } 0.25<\beta_j\leq 0.50; \\ \frac{(2\beta_j-1)b_j^1+(2(1-\beta_j)+(3-4\beta_j)\theta_i^r)b_j^2}{1+(3-4\beta_j)\theta_i^r} & \text{if } 0.50<\beta_j\leq 0.75; \\ \frac{(2\beta_j-1+(4\beta_j-3)\theta_i^l)b_j^1+2(1-\beta_j)b_j^2}{1+(4\beta_j-3)\theta_i^l} & \text{if } 0.75<\beta_j\leq 1; \end{cases} \tag{8.8}$$

$$F_{ek} = \begin{cases} \frac{(1-2\gamma_k+(1-4\gamma_k)\theta_i^l)e_k^3+2\gamma_k e_k^2}{1+(1-4\gamma_k)\theta_i^l} & \text{if } 0<\gamma_k\leq 0.25; \\ \frac{(1-2\gamma_k)e_k^3+(2\gamma_k+(4\gamma_k-1)\theta_i^r)e_k^2}{1+(4\gamma_k-1)\theta_i^r} & \text{if } 0.25<\gamma_k\leq 0.50; \\ \frac{(2\gamma_k-1)e_k^1+(2(1-\gamma_k)+(3-4\gamma_k)\theta_i^r)e_k^2}{1+(3-4\gamma_k)\theta_i^r} & \text{if } 0.50<\gamma_k\leq 0.75; \\ \frac{(2\gamma_k-1+(4\gamma_k-3)\theta_i^l)e_k^1+2(1-\gamma_k)e_k^2}{1+(4\gamma_k-3)\theta_i^l} & \text{if } 0.75<\gamma_k\leq 1; \end{cases} \tag{8.9}$$

8.4.3 Using nearest interval approximation

Suppose that the $\tilde{\tilde{C}}_{ijk}, \tilde{\tilde{a}}_i, \tilde{\tilde{b}}_j$ and $\tilde{\tilde{e}}_k$ are all mutually independent type-2 triangular fuzzy variables defined by $\tilde{\tilde{C}}_{ijk} = (C_{ijk}^1, C_{ijk}^2, C_{ijk}^3, \theta_{ijk}^l, \theta_{ijk}^r)$, $\tilde{\tilde{a}}_i = (a_i^1, a_i^2, a_i^3, \theta_i^l, \theta_i^r)$, $\tilde{\tilde{b}}_j = (b_j^1, b_j^2, b_j^3, \theta_j^l, \theta_j^r)$ and $\tilde{\tilde{e}}_k = (e_k^1, e_k^2, e_k^3, \theta_k^l, \theta_k^r)$. Then we find nearest interval approximations (credibilistic interval approximation, cf. §2.1.14) of $\tilde{\tilde{C}}_{ijk}, \tilde{\tilde{a}}_i, \tilde{\tilde{b}}_j$ and $\tilde{\tilde{e}}_k$ suppose these are $[C_{ijkL}, C_{ijkU}]$, $[a_{iL}, a_{iU}]$, $[b_{jL}, b_{jU}]$ and $[e_{kL}, e_{kU}]$ respectively. Then with these nearest interval approximations the Model (8.2-8.3) becomes

$$\text{Minimize } Z = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}[C_{ijkL}, C_{ijkU}]\ x_{ijk} \tag{8.10}$$

subject to the constraints

$$\begin{aligned}
\sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} &\leq [a_{iL}, a_{iU}] && i = 1, 2, ...M \\
\sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} &\geq [b_{jL}, b_{jU}] && j = 1, 2, ...N \\
\sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} &\leq [e_{kL}, e_{kU}] && k = 1, 2, ...K \\
x_{ijk} &\geq 0 \quad \text{for all} && i, j, k.
\end{aligned} \tag{8.11}$$

Where $\tilde{\tilde{C}}_{ijk}$ is given by equation number (8.1)

Now if the constraints are allowed to be gratified with some predetermined possibility degree level α_i, β_j and $\gamma_k (0 \leq \alpha_i, \beta_j, \gamma_k \leq 1)$respectively, then the equivalent deterministic inequalities of the respective constraints are given as follows:

$$\begin{aligned}
\sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} &\leq a_{iU} - \alpha_i(a_{iU} - a_{iL}) && i = 1, 2, ...M \\
\sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} &\geq b_{jL} + \beta_j(b_{jU} - b_{jL}) && j = 1, 2, ...N \\
\sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} &\leq e_{kU} - \gamma_k(e_{kU} - e_{kL}) && k = 1, 2, ...K
\end{aligned} \tag{8.12}$$

Now to deal with objective function we find minimum possible objective function value (say Z_*) and maximum possible objectivefunction value (say Z^*) for the interval costs $[C_{ijkL}, C_{ijkU}]$, by solving the following two problems:

$$Z_* = \underset{C_{ijkL} \leq C_{ijk} \leq C_{ijkU}}{\text{Minimize}} \left\{ \text{Minimize } \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} C_{ijk}\ x_{ijk} \right\} \tag{8.13}$$

$$Z^* = \underset{C_{ijkL} \leq C_{ijk} \leq C_{ijkU}}{\text{Maximize}} \left\{ \text{Minimize } \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} C_{ijk}\ x_{ijk} \right\} \tag{8.14}$$

subject to the above constraints (8.12) for both cases.

Now we search compromise optimal solution by using two objective problems (8.13) and (8.14) together and applying Zimmermann's [158] fuzzy linear programming as given by:

Let us consider $Z_1(x_{ijk}) = Z_*$ and $Z_2(x_{ijk}) = Z^*$.

We lower and upper interval for both the objective Z_1 and Z_2 as $[Z_{1L}, Z_{1U}]$ and $[Z_{2L}, Z_{2U}]$ respectively. For these objective functions, we construct the following two membership function asgiven by

$$\mu_1(Z_1) = \begin{cases} 1, & \text{if } Z_1 \leq Z_{1L}; \\ \frac{Z_{1U}-Z_1}{Z_{1U}-Z_{1L}}, & \text{if } Z_{1L} < Z_1 < Z_{1U}; \\ 0, & \text{if } Z_1 \geq Z_{1U}. \end{cases} \qquad \mu_2(Z_2) = \begin{cases} 1, & \text{if } Z_2 \leq Z_{2L}; \\ \frac{Z_{2U}-Z_2}{Z_{2U}-Z_{2L}}, & \text{if } Z_{2L} < Z_2 < Z_{2U}; \\ 0, & \text{if } Z_2 \geq Z_{2U}. \end{cases}$$

Now, we solve the problem

$$\text{Maximize} \quad \lambda$$

subject to

$$\mu_1(Z_1) \geq \lambda, \;\; \mu_2(Z_2) \geq \lambda \tag{8.15}$$

and the above constraints (8.12)

$$0 \leq \lambda \leq 1$$

Solving this we minimizes both the objectives Z_1 and Z_2 for the solution x^*_{ijk}(say) $\forall i, j, k$, with certain degree λ^* (say) and the range of the objective value $[\underline{Z}_*, \underline{Z}^*]$.

8.5 Numerical Illustration

In this section the proposed models and methods are illustrated numerically. Two different examples for the models are presented and solved to demonstrate the proposed methodologies numerically.

8.5.1 Illustration of discrete type-2 fuzzy problem

To illustrate the Model, we consider an example with two sources, three destinations and two conveyance, i.e., $i = 1, 2$; $j = 1, 2, 3$ and $k = 1, 2$. The unit transportation costs $\tilde{\tilde{C}}_{ijk}$, the availability of the source($\tilde{\tilde{a}}_i$), demand of the destinations($\tilde{\tilde{b}}_j$) and capacity of the conveyances ($\tilde{\tilde{e}}_k$) are the following discrete type-2 fuzzy variables.

$$\tilde{\tilde{C}}_{111} = \begin{cases} 2, \text{ with chance } (0.2, 0.4, 0.6, 0.8); \\ 4, \text{ with chance } (0.5, 0.7, 0.9); \\ 5, \text{ with chance } (0.3, 0.5, 0.7). \end{cases} \qquad \tilde{\tilde{C}}_{112} = \begin{cases} 7, \text{ with chance } (0.5, 0.7, 0.8); \\ 8, \text{ with chance } (0.3, 0.5, 0.7); \\ 9, \text{ with chance } (0.6, 0.8, 0.9). \end{cases}$$

$$\tilde{\tilde{C}}_{121} = \begin{cases} 7, \text{ with chance } (0.4, 0.6, 0.7); \\ 8, \text{ with chance } (0.5, 0.7, 0.8); \\ 9, \text{ with chance } (0.7, 0.9, 1.0). \end{cases} \qquad \tilde{\tilde{C}}_{122} = \begin{cases} 5, \text{ with chance } (0.2, 0.4, 0.6); \\ 6, \text{ with chance } (0.4, 0.6, 0.8); \\ 7, \text{ with chance } (0.1, 0.3, 0.5, 0.7). \end{cases}$$

$$\tilde{\tilde{C}}_{131} = \begin{cases} 1, \text{ with chance } (0.3, 0.5, 0.6); \\ 2, \text{ with chance } (0.3, 0.4, 0.6); \\ 3, \text{ with chance } (0.4, 0.6, 0.7). \end{cases} \qquad \tilde{\tilde{C}}_{132} = \begin{cases} 8, \text{ with chance } (0.3, 0.5, 0.6); \\ 9, \text{ with chance } (0.5, 0.7, 0.8, 0.9); \\ 10, \text{ with chance } (0.5, 0.6, 0.8). \end{cases}$$

$$\tilde{\tilde{C}}_{211} = \begin{cases} 4, \text{ with chance } \begin{pmatrix} 0.3 & 0.5 & 0.7 \\ 0.4 & 1 & 0.7 \end{pmatrix} \\ 5, \text{ with chance } \begin{pmatrix} 0.6 & 0.8 & 0.9 \\ 0.5 & 0.9 & 1 \end{pmatrix} \\ 7, \text{ with chance } \begin{pmatrix} 0.5 & 0.7 & 0.8 \\ 0.4 & 1 & 0.7 \end{pmatrix} \end{cases} \qquad \tilde{\tilde{C}}_{212} = \begin{cases} 5, \text{ with chance } (0.2, 0.6, 0.7); \\ 8, \text{ with chance } (0.3, 0.4, 0.8); \\ 10, \text{ with chance } (0.2, 0.7, 0.9). \end{cases}$$

$$\tilde{\tilde{C}}_{221} = \begin{cases} 4, \text{ with chance } \begin{pmatrix} 0.3 & 0.4 & 0.6 \\ 0.6 & 1 & 0.7 \end{pmatrix} \\ 5, \text{ with chance } \begin{pmatrix} 0.6 & 0.8 & 0.9 \\ 0.7 & 1 & 0.8 \end{pmatrix} \\ 6, \text{ with chance } \begin{pmatrix} 0.5 & 0.6 & 0.7 & 0.8 \\ 0.3 & 0.4 & 1 & 0.5 \end{pmatrix} \end{cases} \qquad \tilde{\tilde{C}}_{231} = \begin{cases} 3, \text{ with chance } (0.3, 0.4, 0.6); \\ 5, \text{ with chance } (0.7, 0.9, 1.0); \\ 6, \text{ with chance } (0.4, 0.6, 0.7). \end{cases}$$

$$\tilde{\tilde{C}}_{222} = \begin{cases} 6, \text{ with chance } (0.4, 0.5, 0.7, 0.8); \\ 7, \text{ with chance } (0.6, 0.8, 0.9); \\ 9, \text{ with chance } (0.4, 0.6, 0.7). \end{cases} \quad \tilde{\tilde{C}}_{232} = \begin{cases} 6, \text{ with chance } (0.5, 0.7, 0.9); \\ 8, \text{ with chance } (0.4, 0.5, 0.9); \\ 11, \text{ with chance } (0.5, 0.6, 0.7). \end{cases}$$

$$\tilde{\tilde{a}}_1 = \begin{cases} 30, \text{ with chance } (0.5, 0.7, 0.9); \\ 32, \text{ with chance } (0.4, 0.5, 0.9); \\ 34, \text{ with chance } (0.5, 0.6, 0.7). \end{cases} \quad \tilde{\tilde{a}}_2 = \begin{cases} 42, \text{ with chance } (0.3, 0.6, 0.7); \\ 43, \text{ with chance } (0.1, 0.2, 0.4); \\ 44, \text{ with chance } (0.5, 0.8, 0.9). \end{cases}$$

$$\tilde{\tilde{b}}_1 = \begin{cases} 25, \text{ with chance } (0.2, 0.4, 0.5); \\ 26, \text{ with chance } (0.5, 0.6, 0.7); \\ 27, \text{ with chance } (0.5, 0.6, 0.9). \end{cases} \quad \tilde{\tilde{b}}_2 = \begin{cases} 22, \text{ with chance } (0.3, 0.6, 0.7); \\ 23, \text{ with chance } (0.7, 0.8, 0.9); \\ 27, \text{ with chance } (0.5, 0.6, 0.9). \end{cases}$$

$$\tilde{\tilde{b}}_3 = \begin{cases} 23, \text{ with chance } (0.6, 0.8, 0.9); \\ 25, \text{ with chance } (0.3, 0.5, 0.6); \\ 27, \text{ with chance } (0.6, 0.7, 0.8). \end{cases}$$

$$\tilde{\tilde{e}}_1 = \begin{cases} 35, \text{ with chance } (0.6, 0.7, 0.9); \\ 36, \text{ with chance } (0.1, 0.4, 0.6); \\ 38, \text{ with chance } (0.2, 0.6, 0.7). \end{cases} \quad \tilde{\tilde{e}}_2 = \begin{cases} 35, \text{ with chance } (0.3, 0.6, 0.9); \\ 39, \text{ with chance } (0.1, 0.7, 0.8); \\ 42, \text{ with chance } (0.5, 0.6, 0.8). \end{cases}$$

To solve the above problem we first find corresponding defuzzified (crisp) values of the type-2 fuzzy cost parameters $\tilde{\tilde{C}}_{ijk}$, the availability of the source($\tilde{\tilde{a}}_i$), demand of the destinations($\tilde{\tilde{b}}_j$) and capacity of the conveyances ($\tilde{\tilde{e}}_k$). For this purpose we first apply CV reduction method to reduce type-2 fuzzy variables to type-1 fuzzy variables, then applying centroid method we get the corresponding crisp values. Now using these crisp costs, we optimized the objective function using the $Lingo-11.0$ toolbox. The optimum solution of the problem is obtained and given in $Table-8.1$.

Table-8.1: Optimum solution of the problem 8.5.1

Defuzzification value of $\tilde{\tilde{C}}_{ijk}$ $\tilde{\tilde{a}}_i$ $\tilde{\tilde{b}}_j$ $\tilde{\tilde{e}}_k$ using CV method								Transp-ortation Cost
C_{111} C_{211} C_{131}	C_{112} C_{212} C_{132}	C_{121} C_{221} C_{231}	C_{122} C_{222} C_{232}	a_1 b_1 e_1	b_2	a_2 b_3 e_2	Optimum Solution	
3.6956	8.0385	8.1071	6.0476	31.931		43.081	$x_{121} = 21.63$ $x_{231} = 14.64$	
5.4615	7.7181	5.121	7.36	26.105	23.913	24.949	$x_{132} = 10.31$ $x_{212} = 26.12$	29.7313
2.1	9.0482	4.8523	8.269	36.269		38.712	$x_{222} = 2.29$	

8.5.2 Illustration of continuous type-2 fuzzy problem

To illustrate the Model, we consider an example with two sources, three destinations and two conveyance, i.e., $i = 1, 2$; $j = 1, 2, 3$ and $k = 1, 2$. The unit transportation costs $\tilde{\tilde{C}}_{ijk}$, the availability of the source($\tilde{\tilde{a}}_i$), demand of the destinations($\tilde{\tilde{b}}_j$) and capacity of the conveyances ($\tilde{\tilde{e}}_k$) are the following continuous type-2 fuzzy variables. In this example the inputs are given in $Table - 8.2$.

Table-8.2: Continuous type-2 fuzzy input of the problem 8.5.2

$\tilde{\tilde{C}}_{111}$ $\tilde{\tilde{C}}_{131}$ $\tilde{\tilde{C}}_{221}$	$\tilde{\tilde{C}}_{112}$ $\tilde{\tilde{C}}_{132}$ $\tilde{\tilde{C}}_{222}$	$\tilde{\tilde{C}}_{121}$ $\tilde{\tilde{C}}_{211}$ $\tilde{\tilde{C}}_{231}$	$\tilde{\tilde{C}}_{122}$ $\tilde{\tilde{C}}_{212}$ $\tilde{\tilde{C}}_{232}$	$\tilde{\tilde{a}}_1$ $\tilde{\tilde{a}}_2$	$\tilde{\tilde{b}}_1$ $\tilde{\tilde{b}}_2$ $\tilde{\tilde{b}}_3$	$\tilde{\tilde{e}}_1$ $\tilde{\tilde{e}}_2$
(2,3,4,0.4,0.6)	(4,5,6,0.3,0.6)	(8,9,10,0.5,0.7)	(6,7,8,0.2,0.6)	(33,37,41,0.5,0.7)	(35,38,41,0.4,0.8)	(30,34,38,0.2,0.9)
(1,2,3,0.4,0.5)	(2,3,4,0.4,0.5)	(7,9,11,0.6,0.9)	(5,6,7,0.2,0.8)	(41,44,47,0.3,0.7)	(18,20,22,0.2,0.6)	(44,48,51,0.4,0.9)
(6,7,8,0.5,0.7)	(7,8,9,0.4,0.8)	(2,4,6,0.3,0.6)	(3,5,7,0.5,0.6)		(20,22,23,0.2,0.9)	

Solution using chance-constrained programming: The predetermined general credibility levels for the chance-constrained programming model as formulated to solve the Model are taken for various $\alpha, \alpha_i, \beta_j$ and γ_k , $i = 1, 2$; $j = 1, 2, 3$; $k = 1, 2$.

Table-8.3: Result of chance-constrained programming problem

α	α_i	β_j	γ_k	Optimum Solution	Transportation cost
0.80				$x_{132}=20$ $x_{112}=17$ $x_{211}=21$ $x_{231}=12$ $x_{232}=11$	22.7191
0.65	0.5	0.5	0.5	$x_{131}=21$ $x_{112}=21$ $x_{211}=18$ $x_{222}=20$ $x_{232}=6$	20.2748
0.50				$x_{131}=21$ $x_{112}=21$ $x_{211}=18$ $x_{222}=20$ $x_{232}=6$	19.2717
	0.6			$x_{131}=16.16$ $x_{112}=20.44$ $x_{211}=17.85$ $x_{222}=20$ $x_{232}=5.85$	19.5238
0.50	0.55	0.50	0.50	$x_{131}=16.08$ $x_{112}=20.70$ $x_{211}=17.92$ $x_{222}=20$ $x_{232}=5.92$	19.4065
	0.45			$x_{131}=15.90$ $x_{112}=21.35$ $x_{211}=18.10$ $x_{222}=20$ $x_{232}=6.10$	19.1153
		0.6		$x_{131}=16.27$ $x_{112}=20.73$ $x_{211}=17.73$ $x_{222}=20.31$ $x_{232}=5.96$	19.3453
0.50	0.50	0.55	0.50	$x_{131}=16.14$ $x_{112}=20.86$ $x_{211}=17.86$ $x_{222}=20.16$ $x_{232}=5.98$	19.3085
		0.50		$x_{131}=21$ $x_{112}=21$ $x_{211}=18$ $x_{222}=20$ $x_{232}=6$	19.2717
			0.55	$x_{131}=15.90$ $x_{112}=21.10$ $x_{211}=17.90$ $x_{222}=20$ $x_{232}=6.10$	19.2979
0.50	0.50	0.50	0.45	$x_{131}=16.12$ $x_{112}=20.88$ $x_{211}=18.12$ $x_{222}=20$ $x_{232}=5.88$	19.2386
			0.40	$x_{131}=16.26$ $x_{112}=20.74$ $x_{211}=18.25$ $x_{222}=20$ $x_{232}=5.74$	19.1956

Solution using nearest interval approximation: The nearest interval approximations (credibilistic) of the given triangular type-2 fuzzy parameters are calculated.The corresponding unit transportation costs, supplies, demands, and capacities are presented in $Table-8.4$ as follows:

Table-8.4: Result using nearest interval approximation

C_{111} C_{131} C_{221}	C_{112} C_{132} C_{222}	C_{121} C_{211} C_{231}	C_{122} C_{212} C_{232}	a_1 a_2	b_1 b_2 b_3	e_1 e_2
[3.49,4.51]	[4.49,5.51]	[8.49,9.51]	[6.49,7.52]	[34.98,39.02]	[36.47,39.53]	[31.92,36.08]
[1.50 , 2.50]	[2.50,3.50]	[7.99,10.01]	[5.48,6.51]	[42.47,45.53]	[18.98,21.02]	[45.95,49.53]
[6.49,7.50]	[7.49,8.51]	[2.98,5.01]	[3.99,6.01]		[20.98,22.51]	

Taking $\alpha_i = \beta_j = \gamma_k = 0.5$, we get these deterministic constraints and then solve the objective functions (8.13) and (8.14) and corresponding solutions are as follows: $Z_* = 18.7458$; $x_{111} = 23.48$, $x_{221} = 8.51$, $x_{122} = 12.51$, $x_{212} = 16.04$, $x_{232} = 20.98$ and $Z^* = 25.2377$; $x_{111} = 15.05$, $x_{221} = 21.03$, $x_{112} = 23.97$, $x_{232} = 22.51$.

Now, we apply the fuzzy linear programming (8.15) to get an unique optimum solution. The compromise optimal solution is given by as follows:

$Z_L = 18.8444$, $Z_U = 25.8811$ and the corresponding solution are $x_{121} = 2.4320$, $x_{112} = 10.0379$, $x_{132} = 22.5096$, $x_{211} = 29.4927$, $x_{222} = 16.0402$.

8.6 Conclusion

In this chapter, for the first time, non-linear cost coefficients has been considered where the unit transportation cost is depend on the amount of transportation. The unit cost coefficients, availability of the source, demand of the destination and capacity of conveyance are followed type-2 fuzzy nature. A defuzzification method of general type-2 fuzzy variable is outlined. A chance-constrained programming problem with triangular type-2 fuzzy variables has been formulated and solved.