Chapter 5

Solving a fuzzy solid transportation Problem with fuzzy ranking

5.1 Introduction

Vogel's approximation method (VAM) method is one of the oldest and widely used technique to calculate the initial feasible solution(IFS) of any Transportation problem. this is an itertive technique to findout IFS. This method also has been applied in solid transportation problem. Many researcher has been worked in STP using VAM method. Now, the cost values considered by the researcher are crisp values but in real world it is seen that the cost value are not fixed i.e, imprecise in nature. To discuss the impreciseness, fuzzy numbers are used by many researchers.When Vogel's approximation method used in solid transportation problem, then the allocation is made at minimum cost.If the cost is fuzzy in nature, then the minimum cost is not known precisely. So, what will be the minimum fuzzy cost value? Find the minimum value among the fuzzy number, ranking is very useful technique. In real-life, the ranking of a fuzzy number is an important criteria in many decision making problems. There are various ranking method proposed by the different researcher (cf Abbasbandy et al. [5], Asady and Zendehnam [10], Chen and Hsieh [23], Chu and Kwang [24], Lee and Li [75]) at different time. Since Jain [62] employed the concept of maximizing set to order the fuzzy numbers in 1976, many authors looked into various ranking methods. Some of these ranking methods have been compared and reviewed by Bortolan and Degani [17] and more recently by Choobineh and Li [25] explained an index for ordering fuzzy numbers. Dias [30] studied the method of ranking an alternative fuzzy number. Requena et al. [127] proposed the ranking of fuzzy numbers using artificial neural networks. Ranking and defuzzification methods based on area compensation presented by Fortemps and Roubens [36]. However, some these methods are counter-intuitive and difficult to implement, and others are counterintuitive and not discriminating. Furthermore, many of them produce different ranking outcomes for the same problem. Patra and Mondal [115] discussed a new approach of ranking of generalized trapezoidal fuzzy numbers. Jian et al. [63] and kin et al. [69] proposed interval-based fuzzy ranking approach. Recently, Gu and Xuan [50] discuss the ranking procedure of a fuzzy numbers based on possibility theory. As no one has consider fuzzy unit cost value in solid transportation problem

In the real life, the different types of cost are not certain enough, but varie with a range of uncertainty in the transportation cost considered by several researchers (cf. Ojha et al. [108], Jimenez and Verdegay [64], Pandian and Natarajan [114]). Vogel's approximation method(VAM) is one of the oldest methods for the solution of transportation problem. In 2011, Vasko and Nelya [140] discussed its reality. For simplicity and morern utility, Samuel and Venkatachalapathy [133] applied it for a fuzzy transportation problem with weighted mean. Juman and Hoque [66], almaatani et al. [6] have been modified Vogel's approximation method for an unbalanced transportation problem.

In this chapter, a solid transportation problem is considered with imprecise cost. The proposed STP is solved using modified vogal's approximation method. The obtained basic feasible solutions are converted into an optimal one by MODI method. The required operations and comparisons between the fuzzy numbers are executed by ranking method.

Finally, explain the proposed method is explained with an example. More over the result obtained through this method is compared with the result obtained by $Lingo - 9.0$.

5.2 Notations

5.2.1 Notations

In this solid transportation problem, all notations are mentioned in common notations.

5.3 Formulation of Fuzzy Solid Transportation Problem(FSTP)

In the fuzzy solid transportation problem, if x_{ijk}, the number of units, to be transported from i-th supply to j-th destination by k-th conveyance, then the optimization problem becomes.

$$\text{Minimize } \widetilde{Z} = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} \widetilde{C}_{ijk}\, x_{ijk} \tag{5.1}$$

subject to constraints

$$\begin{aligned}
&\sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} \leq a_i, \qquad i = 1, 2, ...M\\
&\sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} \geq b_j, \qquad j = 1, 2......N\\
&\sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} \leq e_k, \qquad k = 1, 2, ...K\\
&\sum_{i=1}^{M} a_i = \sum_{j=1}^{N} b_j = \sum_{k=1}^{K} e_j\\
&\text{and } x_{ijk} \geq 0, \qquad \forall i, j, k.
\end{aligned} \tag{5.2}$$

The objective function $\widetilde{Z}$ in equation (5.1) represents the total transportation cost. The first three constraints in equation (5.2) present the availability constraints, demand constraints and capacity constraints of the origins, destinations and conveyances respectively. The fourth constraint represents the balance criteria and last constraint represents the feasibility condition of the STP.

5.4 Algorithm for Proposed Method

Step 1: Check whetherthe given STP is balanced or not. If it is not balanced, change into a balance one by introducing one dummy source or destination or conveyance having zero transportation cost.

Step 2: Identify the boxes having minimum and next to minimum transportation cost in each source and write their difference (penalty)using equation (2.25) , along the side of the table against the corresponding source.(Identify the boxes having minimum and next to minimum transportation cost is done by comparison the rank of transportation cost)

Step 3: We apply the similar process Step 2 for demands and conveyance

Step 4: Identify the maximum penalty (through compairing the rank) from the side of the table, make maximum allotment to the box having minimum cost of transportation in that source or demand or conveyance. If in the table, two or more penalties are equal, you are at liberty to break the tie arbitrarily.

Step 5: Discard the fulfilment source or destination or conveyance (followed by Step 4) and repeat the above Steps (2-4) in the remaining problem until all restrictions are satisfied.

Step 6: The number of basic feasible solution(BFS) (obtained by followed steps 2-5) are $(M+N+K-2)$. If one of basic variables in the optimal solution of the problem is zero, that is, the number of nonzero basic variables is less than $(M+N+K-2)$, then the problem is called degenerate, then follow Step 7; Otherwise it is called non-degenerate and goto Step 8.

Step 7: For degenerate BFS, we assign a small positive quantity ϵ in a non allocated cell and goto step-8.

Step 8: Determine the set of $(M + N + K)$ numbers MODI-indices $\widetilde{p}_i(i = 1, 2, ...M)$,$\widetilde{q}_j(j = 1, 2, ...N)$ and $\widetilde{r}_k(j = 1, 2, ...K)$. By taking two MODI-indices values as zero and applying the conditions $\widetilde{C}_{ijk} = \widetilde{p}_i + \widetilde{q}_j + \widetilde{r}_k$, for all basic cells (i, j, k), we can compute the rest of the MODI-indices.

Step 9: For non basic cell (i, j, k), we compute $\widetilde{d}_{ijk}$ by using the formula $\widetilde{d}_{ijk} = \widetilde{C}_{ijk} - (\widetilde{p}_i + \widetilde{q}_j + \widetilde{r}_k)$.

Step 10: Examine the matrix of cell evaluation $\widetilde{d}_{ijk}$ for negative entries and conclude the following

(i) If all $\widetilde{d}_{ijk} > \widetilde{0}$, then the solution is optimal and unique.

(ii) If all $\widetilde{d}_{ijk} \geq \widetilde{0}$ and at least one $\widetilde{d}_{ijk} \sim \widetilde{0}$, then the solution is optimal and alternate solution also exists.

(iii) If at least one $\widetilde{d}_{ijk} < \widetilde{0}$ Solution is not optimal. If it is so, further improvement is required by following step - 11 onwards.

Step 11: Check and compute the following:

(i) See the most negative cell in the matrix [$\widetilde{d}_{ijk}$].

(ii) Allocate θ to this empty cell in the final allocation table. Then, subtract and add the amount of this allocation to other corners of the loop in order to restore feasibility.

(iii) The value of θ, in general is obtained by equating to zero the minimum of the allocations containing θ.

(iv) Substitute the value of θ and find a fresh allocation table.

Step 12: Again, apply the above test for optimality till you find all $\widetilde{d}_{ijk} \geq \widetilde{0}$

5.5 Numerical Illustration

A solid transportation problem with three suppliers, three destinations and three conveyances in fuzzy environment with their shipping cost is considered as given below

Table-5.1: Input data of fuzzy solid transportation problem

	D_1			D_2			D_3			Capacity
	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	
O_1	(1,3,9)	(2,8,10)	(3,9,11)	(1,2,7)	(6,8,14)	(3,8,9)	(1,5,9)	(2,6,11)	(1,2,3)	11
O_2	(3,4,5)	(1,2,3)	(0,6,8)	(0,1,2)	(1,3,5)	(6,8,10)	(2,7,16)	(3,3,7)	(3,6,9)	13
O_3	(3,9,11)	(0,1,2)	(2,3,4)	(2,4,6)	(5,7,9)	(3,3,3)	(4,5,7)	(5,6,8)	(3,4,5)	10
Conveyance	11	14	9	11	14	9	11	14	9	
Demand	7			15			12			

To compare the given fuzzy cost, rank of each costs are calculated following the definition-2.25

Table-5.2: Fuzzy STP with rank value

	D_1			D_2			D_3			Capacity
	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	
O_1	(1,3,9) R=6.89	(2,8,10) R=8.44	(3,9,11) R=9.11	(1,2,7) R=5.22	(6,8,14) R=10.22	(3,8,9) R=7.44	(1,5,9) R=7.33	(2,6,11) R=8.72	(1,2,3) R=2.33	11
O_2	(3,4,5) R=3.67	(1,2,3) R=2.33	(0,6,8) R=7.11	(0,1,2) R=1.67	(1,3,5) R=4	(6,8,10) R=7.33	(2,7,16) R=12.56	(3,3,7) R=4.89	(3,6,9) R=7	13
O_3	(3,9,11) R=9.11	(0,1,2) R=1.67	(2,3,4) R=3	(2,4,6) R=4.66	(5,7,9) R=6.67	(3,3,3) R=2	(4,5,7) R=5.05	(5,6,8) R=5.72	(3,4,5) R=4.44	10
Conveyance	11	14	9	11	14	9	11	14	9	
Demand	7			15			12			

Here, R denotes rank of respective fuzzy number.

Following Step-3, choose two minimum fuzzy costs for each origin, destination and conveyance, then calculate the corresponding penalties.

For example, along E_1, two minimum fuzzy costs are (0,1,2) and (3,4,5) (since their ranks are minimum) and corresponding penalty is (1,3,5).

Similarly other penalties are,

for E_2: (-1,1,3)=(1,2,3)-(0,1,2)

for E_3: (-2,-1,0)=(1,2,3)-(3,3,3)

for O_1: (-2,0,6)=(1,2,7)-(1,2,3)

for O_2: (-1,1,3)=(1,2,3)-(0,1,2)

for O_3:(1,2,3)=(3,3,3)-(0,1,2)

for D_1: (-1,1,3)=(1,2,3)-(0,1,2)

for D_2: (1,2,3)=(3,3,3)-(0,1,2)

for D_3: (0,2,4)=(3,4,5)-(1,2,3)

Next, select the maximum penalties (-2,0,6) corresponding to O_1 (following defination 2.25) and follow Step-5, allocate maximum possible amount to the cell (1,3,3). And proceeding in this way, we get all the allocation as follows:

Table-5.3: Initial BFS following VAM method

	D_1			D_2			D_3			Capacity
	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	
O_1	0 (1,3.9)	0 (2,8,10)	0 (3,9,11)	0 (1,2,7)	0 (6,8,14)	0 (3,8,9)	0 (1,5,9)	2 (2,6,11)	9 (1,2,3)	11
O_2	0 (3,4,5)	0 (1,2,3)	0 (0,6,8)	11 (0,1,2)	2 (1,3,5)	0 (6,8,10)	0 (2,7,16)	0 (3,3,7)	0 (3,6,9)	13
O_3	0 (3,9,11)	7 (0,1,2)	0 (2,3,4)	0 (2,4,6)	2 (5,7,9)	0 (3,3,3)	0 (4,5,7)	1 (5,6,8)	0 (3,4,5)	10
Conveyance	11	14	9	11	14	9	11	14	9	
Demand	7			15			12			

The number of BFS is 7 which is equal to $(M+N+L-2)$, so the solution is non-degenerate. Then, we test the optimality by computing the MODI indices $\widetilde{p_i}, \widetilde{q_j}$ and $\widetilde{r_k}$ such that for for all basic cells (i,j,k), $\widetilde{C_{ijk}} = \widetilde{p_i} + \widetilde{q_j} + \widetilde{r_k}$, and for non basic cell (i,j,k) we compute $\widetilde{d_{ijk}}$ (such fuzzy numbers are enclosed in [ ])by using the formula $\widetilde{d_{ijk}} = \widetilde{C_{ijk}}(\widetilde{p_i} + \widetilde{q_j} + \widetilde{r_k})$ we get the following $Table-5.4$.

Table-5.4: Optimulity test of the BFS

	D_1			D_2			D_3			$\widetilde{p_i}$
	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	
O_1	[-16,4,28] (1,3,9)	[-14,7,24] (2,8,10)	[-21,12,42] (3,9,11)	[-19,-3,17] (1,2,7)	[-13,1,19] (6,8,14)	[-24,5,31] (3,8,9)	[-22,1,23] (1,5,9)	(-20,0,20) (2,6,11)	(-29,0,29) (1,2,3)	(-10,0,10)
O_2	[-4,9,22] (3,4,5)	[-5,5,15] (1,2,3)	[-14,13,37] (0,6,8)	(-10,0,10) (0,1,2)	(-8,0,8) (1,3,5)	[-11,9,30] (6,8,10)	[-11,7,28] (2,7,16)	[-9,1,14] (3,3,7)	[-17,8,33] (3,6,9)	(-8,-4,0)
O_3	[-4,10,20] (3,9,11)	(-6,0,6) (0,1,2)	[-12,6,25] (2,3,4)	[-8,-1,6] (2,4,6)	(-4,0,4) (5,7,9)	[-14,0,15] (3,3,3)	[-9,1,11] (4,5,7)	(-7,0,7) (5,6,8)	[-19,3,21] (3,4,5)	(0,0,0)
$\widetilde{r_k}$	(0,5,10)	(5,7,9)	(-12,3,17)	(0,5,10)	(5,7,9)	(-12,3,17)	(0,5,10)	(5,7,9)	(-12,3,17)	
$\widetilde{q_j}$	$(-9,-6,-3)$			$(0,0,0)$			$(-4,-1,3)$			

Since all $\widetilde{d_{ijk}}$ are positive . Therefore this is the optimal table and optimal solution as $x_{132} = 2$, ,$x_{133} = 9$,$x_{221} = 11$, $x_{222} = 2$,$x_{312} = 7$,$x_{322} = 2$, $x_{332} = 1$ and the corresponding $\widetilde{Z}$=(30,74,121)

To compaire the results obtained through modified VAM using ranking method, we consider the same imprecise STP. In this method using 'extension principle', we convert the objective function into a weighted average function

$$Z(x_{ijk}) = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}(\frac{C_{ijk}^{L} + 2C_{ijk}^{C} + C_{ijk}^{R}}{4})\ x_{ijk}$$

and minimize $Z(x_{ijk})$ using Lingo-9.0 software. The obtained results are shown in the following $Table - 5.5$.

Table-5.5: Optimul solution of FSTP using $Lingo - 9.0$

	D_1			D_2			D_3			Capacity
	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	
O_1	0 (1,3,9)	0 (2,8,10)	0 (3,9,11)	2 (1,2,7)	0 (6,8,14)	0 (3,8,9)	0 (1,5,9)	0 (2,6,11)	9 (1,2,3)	11
O_2	0 (3,4,5)	0 (1,2,3)	0 (0,6,8)	6 (0,1,2)	4 (1,3,5)	0 (6,8,10)	0 (2,7,16)	3 (3,3,7)	0 (3,6,9)	13
O_3	0 (3,9,11)	7 (0,1,2)	0 (2,3,4)	3 (2,4,6)	0 (5,7,9)	0 (3,3,3)	0 (4,5,7)	0 (5,6,8)	0 (3,4,5)	10
Conveyance	11	14	9	11	14	9	11	14	9	
Demand	7			15			12			

The corresponding optimum value of Z is 75.25, which shows that the result obtained through modified VAM by using ranking method is better than the result obtained through Lingo-9.0 software by using extension principle and weighted average value.

5.6 Sensitivity Analysis

Proposition-5.1: Let (i, j, k)-th cell be a non-basic cell corresponding to an optimal solution of the STP with $\widetilde{d}_{ijk} = \widetilde{C}_{ijk} - \widetilde{p}_i - \widetilde{v}_j - \widetilde{r}_k (\geq 0)$. If $\widetilde{C}_{ijk} + \widetilde{\triangle}_{ijk}$ is the perturbed cost of $\widetilde{C}_{ijk}$, then the range of $\widetilde{C}_{ijk} = [\widetilde{\triangle}_{ijk}, \infty)$

Proof: Now, since (i, j, k)th cell is a non-basic cell and the perturbed cost $\widetilde{C}_{ijk} + \widetilde{\triangle}_{ijk}$ is not affected the current optimal solution to the problem, $\widetilde{C}_{ijk} - \widetilde{p}_i - \widetilde{v}_j - \widetilde{r}_k (\geq 0)$.

This implies that, $\widetilde{d}_{ijk} \leq -\widetilde{\triangle}_{ijk}$. Therefore, the range of $\widetilde{\triangle}_{ijk} = [-\widetilde{d}_{ijk}, \infty)$ Hence the theorem.

Proposition-5.2: Let (i,j,k)-th cell be basic cell corresponding to an optimal solution of the STP with $\widetilde{d}_{ijk} = \widetilde{C}_{ijk} - \widetilde{p}_i - \widetilde{v}_j - \widetilde{r}_k (= 0)$. If $\widetilde{C}_{ijk} + \widetilde{\triangle}_{ijk}$ is the perturbed cost of $\widetilde{C}_{ijk}$ and $\widetilde{pi}$ is the minimum value of $\widetilde{d}_{ijk}$ for all non-basic cells in the i-th origin, $\widetilde{q}_j$ is the minimum value of $\widetilde{d}_{ijk}$ for all non-basic cells in the j-th destination and $\widetilde{r}_k$ is the minimum value of $\widetilde{d}_{ijk}$ for all non-basic cells in the k-th conveyance, then the range of $\widetilde{\triangle}_{ijk} is (-\infty, \widetilde{Y}_{ijk}]$ where $\widetilde{Y}_{ijk}$ = the maximum $\{\widetilde{p}_i, \widetilde{q}_j, \widetilde{r}_k\}$;

Proof: Now, since $\widetilde{C}_{ijk} + \widetilde{\triangle}_{ijk}$ is the perturbed value of $\widetilde{C}_{ijk}$ and the current optimal solution remains optimal, $\widetilde{d}_{ijk} = \widetilde{C}_{ijk} - \widetilde{p}_i - \widetilde{v}_j - \widetilde{r}_k \geq \widetilde{0}$, for all non-basic cells (i,j,k) are positive. Now, attaching the $\widetilde{\triangle}_{ijk}$ to first u_i , then v_j and last w_k , we have the following:
$\widetilde{C}_{ifg} - (\widetilde{p}_i + \widetilde{\triangle}_{ijk}) - \widetilde{v}_f - \widetilde{w}_g \geq \widetilde{0}$,(i,f,g) is non - basic cells, for all f and g ;
$\widetilde{C}_{hjg} - \widetilde{p}_i - (\widetilde{v}_f + \widetilde{\triangle}_{ijk}) - \widetilde{w}_g \geq \widetilde{0}$, (h, j,g) is non - basic cells, for all h and g and
$\widetilde{C}_{hfk} - \widetilde{p}_i - \widetilde{v}_f - (\widetilde{w}_g + \widetilde{\triangle}_{ijk}) \geq \widetilde{0}$, (h,f,k) is non - basic cells, for all h and f . Then, we can conclude that is implies that
$\widetilde{\triangle}_{ijk} \leq \widetilde{p}_i$; $\widetilde{\triangle}_{ijk} \leq \widetilde{q}_j$ and $\widetilde{\triangle}_{ijk} \leq \widetilde{r}_k$. Now, since we attach any one of the MODI-indices $\widetilde{p}_i$, $\widetilde{v}_j$ and $\widetilde{r}_k$, we take, $\widetilde{M}_{ijk}$ = maximum $\widetilde{p}_i, \widetilde{q}_j, \widetilde{W}_k$ for getting better range. Therefore, the range of $\widetilde{\triangle}_{ijk} = (-\infty, \widetilde{M}_{ijk}]$. Hence the theorem.

5.6.1 Effect of change in the components of sources, destinations and conveyances

Any change of the source component or destination component or conveyance component, does not affect the optimality condition i.e, the allocation box does not change with the change of availability, resource or capacity of conveyance. But, it changes with the amount of transportation. Here, percentage of effect in total cost due to the change of a_i, b_j, e_k in $\widetilde{Z}$ are given below:

Table-5.6: Sensitivity Analysis

$a_1(\%)$	$a_2(\%)$	$a_3(\%)$	$b_1(\%)$	$b_2(\%)$	$b_3(\%)$	$e_1(\%)$	$e_2(\%)$	$e_3(\%)$	$\widetilde{Z}(\%)$
-40	-40	-40	-40	-40	-40	-40	-40	-40	-40
$-18 \cdot 18$	$-15 \cdot 38$	$-20 \cdot 00$	$-28 \cdot 57$	$-13 \cdot 33$	$-16 \cdot 66$	$-18 \cdot 18$	$-24 \cdot 29$	$-22 \cdot 22$	$-11 \cdot 43$
$-9 \cdot 09$	$-7 \cdot 69$	$-10 \cdot 00$	$-14 \cdot 3$	$-6 \cdot 67$	$-8 \cdot 33$	$-9 \cdot 09$	$-7 \cdot 14$	$-11 \cdot 1$	$-5 \cdot 71$
-5	-5	-5	-5	-5	-5	-5	-5	-5	-5
0	0	0	0	0	0	0	0	0	0
+5	+5	+5	+5	+5	+5	+5	+5	+5	+5
$+9 \cdot 09$	$+7 \cdot 69$	$+10 \cdot 00$	$+14 \cdot 3$	$+6 \cdot 67$	$+8 \cdot 33$	$+9 \cdot 09$	$+7 \cdot 14$	$+11 \cdot 1$	$+5 \cdot 71$
$+18 \cdot 18$	$+15 \cdot 38$	$+20 \cdot 00$	$+28 \cdot 57$	$+13 \cdot 33$	$+16 \cdot 66$	$+18 \cdot 18$	$+24 \cdot 29$	$+22 \cdot 22$	$+11 \cdot 43$
+40	+40	+40	+40	+40	+40	+40	+40	+40	+40

5.7 Conclusion

Here, for the first time, the Vogel's Approximation method has been introduced in a three-dimensional transportation problem. Here, the unit transportation costs are assumed imprecise in nature and their comparison, algebraic operations are performed by using the concept of rank of fuzzy number. The obtained solution is globally optimized, which is self explained. A sensitive analysis has been done for the availabilities, demands and capacities of the considerable illustration (shown in $Table - 5.6$), which shows the effects of such parameters on the total transportation cost.