Chapter 3

A profit maximization solid transportation problem in fuzzy environment using genetic algorithm

3.1 Introduction

In reality, the decision making problem [54] becomes important in our daily life. Transportation problem is one of such important decision making problem. F. L. Hitchcock [57] first defined the transportation problems. The solid transportation problem (STP) which was stated by Shell [136] who suggested the situation where the STP would arise.

In many practical realistic applications, it is assumed that the transported amount which can be delivered on any specific route, bears an uncertain fixed cost for that route. Such fixed costs may also be applied to some production-planning models. Ojha et al. [107] look upon such type of fixed charge as well as vehicle cost in STP in deterministic forms. The use of vehicle cost in addition with conventional unit transportation cost is very much realistic, when the destinations belong in rural areas. There are also another existing literature in which the fixed charge has been considered in deterministic nature. Recently, Dalman et al. [26] suggested a Multi-objective Multi-item Solid Transportation Problem Under Uncertainty. Here an all unit discount policy has been introduced also. Till now, no one has considered fixed charge, AUD of transportational cost and vehicle cost together in imprecise nature.

Jimecncz and Verdegay [64] worked out both interval and fuzzy solid transportation problem by a prologation of auxiliary lincar program suggested by Chanas ct al. [19]. Pan et al. [116] discuss different fuzzy programming technique to solve different decision making problem. Li et al [76] took advantage of improved genetic algorithm to work out the fuzzy multi-objective solid transportation problems, where the cost coefficients objective function are fuzzy numbers. The approximate of the fuzzy number to its nearest interval discussed by Grzegorzewski [47]. Omar and Samir [112] and Chanas and Kuchta [19] discussed the solution algorithm for solving the transportation problem in fuzzy environment. Very recently, Huang et al. [60] discussed the Signed distance and Centroid method to defuzzify the fuzzy parameters.

In this chapter, a fuzzy solid transportation problem is considered as profit maximization problcm following Sakawa ct al. [131] . Thc path from sourcc(s) to dcstination(s) is divided into two parts on the basis of the permit of road capacity of the convenance. Here, a discount is given to the unit cost of transportation which depends on the amount of transportation, AUD cost is also imprecise in nature. All traditional constraints are also fuzzy. In addition, a budget constraint is taken on the transportation cost. There are several non-analytic processes to find the solution of a linear / non-linear programing problem such as neural network [117], analytic hierarchy process [111], etc. The proposed model has been optimized using Genetic Algorithm(GA). Finally, a numerical example is illustrated. The basic differences of the proposed model from other four existing models such as Ojha et al. [107], Sakawa et al. [131], Liu et al. [82] and Bit et al. [14] have been given below:

Table-3.1: Comparison table anong the existing models with proposed model 3.3

References(s)	Unit Trans. Cost	Fixed Charge	Vehicle Cost	Profit	Solving Method
Bit et al.	Fuzzy	No	No	No	Fuzzy programming
Sakawa et al.	Yes, fuzzy	No	No	Yes	Fuzzy programming
Liu et al.	Yes, fuzzy	Yes, Crisp	No	No	GA
Ojha et al.	Crisp following AUD	Yes, Crisp	Yes	No	GA
This model	Fuzzy following AUD	Yes, Fuzzy	Yes with two different capacities	Yes	GA

3.2 Notations and Assumptions

3.2.1 Notations

In place of common notations the following superfluous notations have been used in the solid transportation problem

(i) $\widetilde{f}_{ijk}$ = fuzzy fixed charge of the transportation problem.

(ii) V_{ijk} = transportation vehicle cost per unit item.

(iii) $\widetilde{S}_j$ = fuzzy unit selling price of the item at the $j-th$ destination.

(iv) $\widetilde{R}_i$ = fuzzy unit perchasing cost of the item at the $i-th$ source..

(v) $\widetilde{D}$ = maximum available total direct cost.

3.2.2 Assumptions

To develop the proposed solid transportation model, the following assumptions have been made.

(i) In the traditional transportational problem, it is seen that the quantity is transported from a source to a destination by one convenance only. But, in practical business system, it is not always true. There exist some transportation problems in which the materials required can not be transported directly from the source to the destination by one convenance only. In such situation, the materials are transported to a station near by the destination by one convenance only and then from this near by station, it is transported to the exact destination by another convenance.

(ii) For transportation the material from the near by station to the exact destination, a vehicle cost,V has been assumed. Here, two types of vehicles such as one having large loading capacity to be G_c and another having small loading capacity to be $G_c^{'}$ i.e., $G_c > G_c^{'}$ have been considered and the corresponding vehicle cost for large and small vehicles are G and G' respectively. Therefore, the total vehicle cost $V(x_{ijk})$ for transportation the items of amount x_{ijk} has been proposed in the following form

$$V(x_{ijk}) = \begin{cases} h.G \quad if \;\; h.G_c = x_{ijk} \\ h.G + G' \quad if \;\; x_{ijk} - h.G_c > 0 \end{cases} \tag{3.1}$$

where $h = [x_{ijk}/G_c]$ and $[x]$ denotes the greatest integer but less then equal to x .

(iii) For transportation of the materials from $i-th$ source to the near by place of $j-th$ destination by $k-th$ convenance, the unit transportation costs $\widetilde{C}_{ijk}$ have been discounted on the basis of transported amount according to AUD policy. Again, it is seen that due to the various factors the unit transportation costs may also vary i.e., realistically, it may be uncertain. In this model, it has been considered as fuzzy numbers. Therefore, the fuzzy unit transformation costs $\widetilde{C}_{ijk}$ have been taken in the following form

$$\widetilde{C}_{ijk} = \begin{cases} \widetilde{C}_{1ijk} \quad if \;\; 0 < x_{ijk} < R_1 \\ \widetilde{C}_{2ijk} \quad if \;\; R_1 \leq x_{ijk} < R_2 \\ .... \\ .... \\ \widetilde{C}_{tijk} \quad if \;\; R_{(t-1)} \leq x_{ijk} < R_t \\ \widetilde{C}_{(t+1)ijk} \quad if \;\; R_t \leq x_{ijk} \end{cases} \tag{3.2}$$

where $\widetilde{C}_{1ijk} > \widetilde{C}_{2ijk} > \widetilde{C}_{3ijk} > ..... > \widetilde{C}_{tijk} > \widetilde{C}_{(t+1)ijk}$ are all unit costs.

(iv) A fixed charge was taken into consideration at the time transportation like, road taxes, etc. Assume that a uniform product is to be transported to N destinations from M sources by K different transport modes. Furthermore, a fixed charge $\widetilde{f}_{ijk}$, is related to the transportation that a unit of this product is carried with $k-th$ transport mode from the $i-th$ source to the $j-th$ destination. Assume that y is function of x_{ijk} which takes the values 0 and 1 to describe the transportation activity from $i-th$ source to $j-th$ destination through $k-th$ transport mode and it is defined as

$$y(x_{ijk}) = \begin{cases} 1, & if \ \ 0 < x_{ijk} \\ 0, & \text{otherwise} \end{cases}$$

Also fixed charges $\widetilde{f}_{ijk}$ are taken as fuzzy then the the total fixed charge $\widetilde{d}_{ijk}$ is in the form :

$$\widetilde{d}_{ijk} = \widetilde{f}_{ijk} \ \ y(x_{ijk}) \tag{3.3}$$

(v) Direct costs (such as material, labour, power or fuel) vary with the rate of output but are uniform for each unit of production and are usually under the control and responsibility of the department manager. As a general rule, most costs are fixed in the short run and variable in the long run. So, we consider a boundedness of direct transportation cost.

(vi) In traditional transportation problem, we determine the minimum transportation cost. Here, we assume that the items are selling at destination. In this case a profit function has been taken.

3.3 Mathematical Formulation of a Solid Transportation Problem

In the traditional STP, here $\widetilde{C}_{ijk}$ be the unit cost under AUD systems connected with transportation of a unit product from $i-th$ source to $j-th$ destination by means of the

$k-th$ conveyance which is imprecise in nature. we consider the unit purchasing price for each item at $i-th$ source to be $\widetilde{R}_i$ $(i=1,2,....M)$; Also we consider the unit selling price for each item at $j-th$ destination to be $\widetilde{S}_j$ $(j=1,2,....N)$. Then the problem is converted into a profit maximization problem subject to traditional transportation constraints and boundedness of direct transportation cost.

$$Max\ \tilde{P}(x_{ijk}) = \sum_{j=1}^{N}\left\{\widetilde{S}_j\sum_{i=1}^{M}\sum_{k=1}^{K}\ x_{ijk}\right\} - \left\{\sum_{i=1}^{M}(\widetilde{R}_i\sum_{j=1}^{N}\sum_{k=1}^{K}\ x_{ijk})\right.$$
$$\left.\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\widetilde{C}_{ijk}\ x_{ijk}\ +\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\widetilde{d}_{ijk}\ +\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}V(x_{ijk})\right\} \quad (3.4)$$

subject to

$$\sum_{j=1}^{N}\sum_{k=1}^{K}x_{ijk} = \widetilde{a}_i \qquad i=1,2,3,...,M$$
$$\sum_{i=1}^{M}\sum_{k=1}^{K}x_{ijk} = \widetilde{b}_j \qquad j=1,2,3,...,N$$
$$\sum_{i=1}^{M}\sum_{j=1}^{N}x_{ijk} = \widetilde{e}_k \qquad k=1,2,3,...,K$$
$$\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\widetilde{C}_{ijk}\ x_{ijk} \le \widetilde{D} \quad (3.5)$$
$$x_{ijk} \ge 0 \qquad \forall i,j,k$$

where $\widetilde{C}_{ijk}$ and $\widetilde{d}_{ijk}$ are given by equation 3.2 and 3.3 respectively.

Defuzzification of the Model:

In the model with equations 3.4-3.56-10, $\widetilde{a}_i, \widetilde{b}_j, \widetilde{e}_k, \widetilde{C}_{ijk}, \widetilde{d}_{ijk}$ and $\widetilde{S}_i$ have been taken as fuzzy numbers with α-cuts are $[a_{iL}, a_{iU}]$, $[b_{jL}, b_{jU}]$, $[e_{kL}, e_{kU}]$, $[C_{ijkL}, C_{ijkU}]$, $[d_{ijkL}, d_{ijkU}]$ and $[S_{iL}, S_{iU}]$ respectively. Let $\tilde{A} = (a_1, a_2, a_3)$ be a triangular fuzzy number then left and right cut of $\widetilde{A}$ are $A_L(\alpha) = a_1 + \alpha(a_2 - a_1)$ and $A_U(\alpha) = a_3 - \alpha(a_3 - a_2)$.Also let $\tilde{B} = (b_1, b_2, b_3, b_4)$ be a trapezoidal fuzzy number then left and right cut of $\widetilde{B}$ are $B_L(\alpha) = b_1 + \alpha(b_2 - b_1)$ and $B_U(\alpha) = b_4 - \alpha(b_4 - b_3)$.

Therefore using interval arithmetic, α-cut of the profit is given by

$$\widetilde{P}(x_{ijk},\alpha) = \Big[P_L(x_{ijk},\alpha), P_U(x_{ijk},\alpha)\Big] \tag{3.6}$$

where

$$\begin{aligned} P_L(x_{ijk},\alpha) = \quad & \sum_{j=1}^{N}\{S_{jL}\sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk}\} - \sum_{i=1}^{M}\{R_{iU}\sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk}\} \\ & -\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} C_{ijkU}\ x_{ijk} - \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} d_{ijkU} - \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} V(x_{ijk}), \\ P_U(x_{ijk},\alpha) = \quad & \sum_{j=1}^{N}\{S_{jU}\sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk}\} - \sum_{i=1}^{M}\{R_{iL}\sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk}\} \\ & -\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} C_{ijkL}\ x_{ijk} - \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} d_{ijkL} - \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} V(x_{ijk}) \end{aligned}$$

subject to constrains

$$\begin{aligned} &\sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} \in [a_{iL}, a_{iU}] \quad \text{equivalent to} \quad a_{iL} \le \sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} \le a_{iU} \\ &\sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} \in [b_{jL}, b_{jU}] \quad \text{equivalent to} \quad b_{jL} \le \sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} \le b_{jU} \\ &\sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} \in [e_{kL}, e_{kU}] \quad \text{equivalent to} \quad e_{kL} \le \sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} \le e_{kU} \\ &D_L \le \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}[\frac{C_{ijkL}+C_{ijkU}}{2}]x_{ijk} \le D_U \end{aligned} \tag{3.7}$$

So, the problem is reduced to determine x_{ijk} so that $\widetilde{P}(x_{ijk},\alpha)$ is maximum for fixed α subject to the constraint (3.7).

3.4 Solution Procedures

To find the optimal solution of above constrainted solid transportation problem, we use the following Genetic Algorithm.

3.4.1 Algorithm of Proposed Model

To get optimal solution of the proposed model (3.6) with equations(3.7), the following algorithm has been developed.

Step-1: In order to find the optimum value of the Equation (3.6) subject to constraints (3.7), the objective functions are converted into a single objective function $P_C(x_{ijk}, \alpha) = [P_L(x_{ijk}, \alpha) + P_U(x_{ijk}, \alpha)]/2$.

Step-2: $P_C(x_{ijk}, \alpha)$ is maximized through Step-3, with the following constraints (3.7).

Step-3: Then the objective function $P_C(x_{ijk}, \alpha)$ is maximized with variables x_{ijk}, $i = 1, 2..., M$ for fixed α by the following GA algorithm.

```
begin
p ← 0
initialize Population(p)
evaluate Population(p)
while(not terminate-condition)
{
p ← p + 1
select Population(p) from Population(p-1)
alter(crossover and mutation) Population(p)
evaluate Population(p)
}
Print Optimum Result
end.
```

3.5 Numerical Illustration

Let us consider a manufacturing system which is situated in two different lacation and the system is conducted by two retailers in two different cities. Also there two different modes of transportation are available. The unit transportation cost from a source to a destination through a conveyance is given in the following $Table-3.2$.

Table-3.2: Input Values of $\widetilde{C}_{ijk}$ (in \$) under AUD

$\widetilde{C}_{ijk}$	$\widetilde{p}_{ijk}$	AUD	$\widetilde{C}_{ijk}$	$\widetilde{p}_{ijk}$	AUD
	$(4,5,6)$	$0 < x_{ijk} < 10$		$(7,8,9)$	$0 < x_{ijk} < 10$
$\widetilde{C}_{111}$	$(2,3,4)$	$10 \le x_{ijk} < 20$	$\widetilde{C}_{111}$	$(5,6,7)$	$10 \le x_{ijk} < 20$
	$(1,2,3)$	$x_{ijk} \ge 20$		$(3,4,5)$	$x_{ijk} \ge 20$
	$(5,6,7)$	$0 < x_{ijk} < 10$		$(6,7,8)$	$0 < x_{ijk} < 10$
$\widetilde{C}_{121}$	$(3,4,5)$	$10 \le x_{ijk} < 20$	$\widetilde{C}_{121}$	$(4,5,6)$	$10 \le x_{ijk} < 20$
	$(1,2,3)$	$x_{ijk} \ge 20$		$(2,3,4)$	$x_{ijk} \ge 20$
	$(7,8,9)$	$0 < x_{ijk} < 10$		$(8,9,40)$	$0 < x_{ijk} < 10$
$\widetilde{C}_{112}$	$(5,6,7)$	$10 \le x_{ijk} < 20$	$\widetilde{C}_{112}$	$(6,7,8)$	$10 \le x_{ijk} < 20$
	$(3,4,5)$	$x_{ijk} \ge 20$		$(4,5,6)$	$x_{ijk} \ge 20$
	$(4,5,6)$	$0 < x_{ijk} < 10$		$(6,7,8)$	$0 < x_{ijk} < 10$
$\widetilde{C}_{122}$	$(2,3,4)$	$10 \le x_{ijk} < 20$	$\widetilde{C}_{122}$	$(4,5,6)$	$10 \le x_{ijk} < 20$
	$(1,2,3)$	$x_{ijk} \ge 20$		$(2,3,4)$	$x_{ijk} \ge 20$

and the capacities of two different type vehicles are 4 lb , 2 lb with their costs 5 \$ and 3 \$ respectively. The fixed charges in the roots are $\widetilde{f}_{111} = (1,2,3)\$$; $\widetilde{f}_{121} = (2,3,4)\$$; $\widetilde{f}_{112} = (2,4,5)\$$; $\widetilde{f}_{122} = (1,3,4)\$$; $\widetilde{f}_{211} = (1,2,5)\$$; $\widetilde{f}_{221} = (3,5,6)\$$; $\widetilde{f}_{212} = (1,2,4)\$$; $\widetilde{f}_{222} = (2,3,5)\$$.

The unit purchase prices of two items are (4,5,6) \$ and (5,7,9) \$. If the unit selling price of two items are (10,12,14) \$ and (14,15,18) \$ and the maximum total budget cost does not exceed $\widetilde{D} = (500, 540, 580)$ \$. The values of the corresponding origins(i.e. sources), destination (i.e. demands) and conveyance(i.e. capacity) are also taken as fuzzy amount

$\widetilde{a}_1 = (48, 49, 50, 51)$, $\widetilde{a}_2 = (27, 29, 31, 33)$, $\widetilde{b}_1 = (52, 54, 56, 58)$, $\widetilde{b}_2 = (23, 24, 25, 26)$, $\widetilde{e}_1 = (22, 27, 39, 44)$, $\widetilde{e}_2 = (33, 38, 50, 55)$.The problem is to determine the optimal transportation policy of the decision maker for the maximum profit of the manufacturing system.

At first all the fuzzy inputs are converted into an interval following §2.1.7, then the amount of the origins, destination and conveyance become

$$[a_{1L}, a_{1U}] = [49 - \alpha, 50 + \alpha], \quad [a_{2L}, a_{2U}] = [29 - 2\alpha, 31 + 2\alpha],$$
$$[b_{1L}, b_{1U}] = [54 - 2\alpha, 56 + 2\alpha], \quad [b_{2L}, b_{2U}] = [24 - \alpha, 25 + \alpha],$$
$$[e_{1L}, e_{1U}] = [27 - 5\alpha, 39 + 5\alpha], \quad [e_{2L}, e_{2U}] = [38 - 5\alpha, 50 + 5\alpha],$$

With the above input data, GA is performed for the objective function given in equation(3.6) subject to the constraints (3.7) as stated earlier using above mentioned GAs. The optimum results are presented below:

Table-3.3 : Optimum Result of the model

Value of α	Transportable Amounts x_{111} x_{121} x_{112} x_{122} x_{211} x_{221} x_{212} x_{222}	$P_L(\alpha)$	$P_U(\alpha)$	P_C
1.0	8.3, 12.5, 27.5, 1.5, 5.1, 0.3 12.0, 10.7	535.6	535.6	535.6
0.9	7.2, 12.2, 27.2, 2.3, 5.0, 0.2 13.0, 10.1	529.2	542.3	535.7
0.8	7.7, 10.2, 27.3, 2.9, 3.9, 0.1 13.5, 10.9	520.7	546.8	533.8
0.7	7.8, 14.5, 26.2, 0.0, 5.0, 10.1 13.7, 0.0	529.1	570.3	549.7
0.6	10.5, 14.7, 24.0, 0.4, 6.6, 11.1 11.0, 0.0	543.2	597.8	570.5
0.5	10.4, 13.0, 25.5, 0.0, 5.6, 0.4 12.2, 11.8	548.0	616.8	582.4
0.4	11.2, 11.5, 25.6, 0.3, 4.5, 0.3 12.0, 12.3	535.5	614.9	575.2
0.3	11.8, 11.6, 23.4, 2.0, 4.4, 0.0 13.9, 10.9	533.5	628.5	581.0
0.2	11.8, 11.1, 26.7, 0.4, 4.3, 14.3 11.2, 0.0	554.2	666.1	610.1
0.1	0.0, 12.2, 26.5, 10.4, 15.7, 2.6 11.6, 0.0	535.4	661.5	598.4
0.0	3.6, 11.9, 23.2, 10.3, 17.2, 2.2 10.6, 0.0	526.9	665.1	596.0

Here the total transportation amount is almost equal (since $\sum \widetilde{a}_i = \sum \widetilde{b}_j = \sum \widetilde{e}_k$). Here GA produces a realistic scenario of most of the nonzero solution. Alao it is seen that, the total profit is not similar with the different values of α. For the different values of

α, optimal transportable amount(x_{ijk}) is different as result, different unit transportation cost has been taken due to the advantage of AUD of unit transportation cost.

3.6 Discussion

Here, the final output(optimum amount of transportation and optimum profit) of the proposed model for different discrete values of $\alpha-$Cut is shown in the $Table-3.3$. From $Table-3.3$ it is observed that

(i) The total amount of transportation $\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}x_{ijk}$ lies between Max$\{(\widetilde{a}_1+\widetilde{a}_2)_L,(\widetilde{b}_1+\widetilde{b}_2)_L,(\widetilde{e}_1+\widetilde{e}_2)_L\}$ and Min$\{(\widetilde{a}_1+\widetilde{a}_2)_U,(\widetilde{b}_1+\widetilde{b}_2)_U,(\widetilde{e}_1+\widetilde{e}_2)_U\}$.

(ii) Total amount of transportation does not tally with total cost or profit due to the existence of discount on unit of transportation.

3.7 Conclusion

Here for the first time, different types of vehicle costs are considered in a STP. The fixed charge, the unit transportation cost (under AUD) all are treated as fuzzy parameters. The problem is formulated as a profit maximization problem. The budget constraint has been incorporated taking fuzzy unit transportation cost as constraint. To find the optimal solution, genetic algorithm has been considered.