

List of Common Notations

- (i) M = number of sources/origins of the transportation problem.
- (ii) N = number of destinations/location of the transportation problem.
- (iii) K = number of conveyances i.e., different modes of the transportation problem.
- (iv) O_i = i -th origin of the transportation problem.
- (v) D_j = j -th destination of the transportation problem.
- (vi) E_k = k -th conveyance of the transportation problem.
- (vii) a_i, \tilde{a}_i = crisp and fuzzy amount of a homogeneous product available at i -th source.
- (viii) b_j, \tilde{b}_j = crisp and fuzzy demand at j -th destination.
- (ix) e_k, \tilde{e}_k = crisp and fuzzy amount of product which can be transported/carried by k -th conveyance.
- (x) C_{ij}, \tilde{C}_{ij} = crisp, fuzzy unit transportation costs in (traditional) TP (2-dimensional).
- (xi) C_{ijk}, \tilde{C}_{ijk} = crisp, fuzzy unit transportation costs in STP (3-dimensional).
- (xii) x_{ijk} = the amount to be transported from i -th origin to j -th destination by means of k -th conveyance (decision variables).
- (xiii) Z, \tilde{Z} = crisp and fuzzy objective function.

1.1 Transportation Problem (TP) in Operations Research

Transportation maintains social and economic activity. It is the center of management science and operations research. During the second world war, when operations research was issued as a structured field, some problems which were arose from the need of optimization of the transportation activities and military logistics, were investigated. After the second world war put an end, the scope of operations research applications widened but transportation problems possessed a central place all the time. It is now unrestrainedly recognized that some of the most successful applications of operations research are envisged in transportation, most significantly in the commercial air shuttle industry where they underly almost every attitude of tactical, strategic and operational planning. The economic importance of transportation is one of the key factors of this success story. Also, the large scale and complexity of transportation problems demanded the powerful analytical techniques and the involved high volumes which imply that through the use of optimization, substantial savings can often be earned. Besides, it is noted that transportation problems are highly structured which makes them manageable to the use of efficient solution methods based on mathematical programming and network optimization techniques.

What is a Transportation Problem?

- A transportation problem is worried with transportation methods or identifying paths in a distribution network of product among distribution warehouses and the manufacturing units situated in local counters or different zones.

- In applying the transportation management, method is looking for a distribution path, which can guide to maximization of profit or minimization of transportation cost.

Balanced transportation problem

Balanced transportation problem is a transportation problem in which the demand and

supply is the same i.e, the availables for the columns must equal to requirements for the rows.

Unbalanced transportation problem

It is such type of transportation problem in which demand and supply are unequal. The two probable reasons of unbalanced transportation problems are given below:

- (i) Total demand exceeds the total supply or
- (ii) Total supply exceeds the total demand.

Conversion of Unbalanced TP to Balanced TP

(i) **Where aggregate demand exceeds aggregate supply in transportation problem**

In a transportation problem, if total demand surpasses total supply then a dummy row is added to convert the problem as balanced one. The excess demand is equal to the availability of dummy row and the unit transportation cost of each cell of this dummy row is zero. When the problem is balanced, it can be solved by using the usual transportation methods.

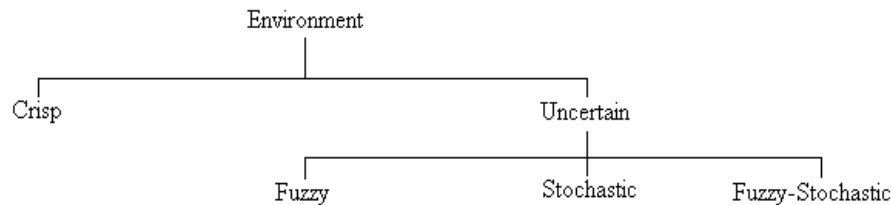
(ii) **Where aggregate supply exceeds aggregate demand in transportation problem**

When total supply exceeds the total demand, then this type of problem is converted as balanced transportation by creating a fictitious destination. A dummy column is introduced to the transportation tableau by which the requirement of dummy destination is equal to the amount of excess supply and the unit transportation cost equal to zero. Once the TP is balanced, it can be worked out by the normal procedures.

1.2 Basic Concepts and Terminologies

1.2.1 Different Environments

The parameters, like demand, quantity, transportation cost, goals, available resources, etc., involved in the transportation system may be deterministic (precise/crisp) or some of these may be non-deterministic (i.e., imprecise or stochastic or both imprecise and stochastic). So the environments in which transportation models are developed can be classified as follows



Crisp Environment: Crisp environment is the environment where all the the resources, system parameters etc., are deterministic and precisely defined.

Fuzzy Environment: In case of newly launched system, for example if a management launches a new product then in that case a newly launched system has no knowledge about demand and the other factors related to the product. Then management needs to collect the demand and the others information from experts. If the experts opinion are imprecise, demand or other factors related to the expert opinion is to be taken as a fuzzy and the corresponding environment is known as Fuzzy Environment.

Type-2 Fuzzy Environment: In this environment, uncertain nature also arises under another uncertainty. For example, if there is possibility levels $\mu_1(\tilde{A}(x)), \mu_2(\tilde{A}(x))\dots\dots\dots$ for an imprecise event $\tilde{A}(x)$. More precisely, let the demand of a commodity may be

either 4 or 5 or 6 units, i.e. the demand let $\tilde{A} = \{4, 5, 6\}$ having possibility membership function $J_4 = \{0.3, 0.4, 0.5\}$, $J_5 = \{0.6, 0.7, 0.9\}$, $J_6 = \{0.4, 0.6, 0.7, 0.9\}$. Again if there exist another secondary membership function of the demand 4 unit as $\tilde{\mu}_{\tilde{A}}(4) = \begin{pmatrix} 0.3 & 0.4 & 0.5 \\ 0.5 & 1.0 & 0.8 \end{pmatrix}$ $\tilde{\mu}_{\tilde{A}}(5) = \begin{pmatrix} 0.6 & 0.7 & 0.9 \\ 0.7 & 1.0 & 0.8 \end{pmatrix}$ $\tilde{\mu}_{\tilde{A}}(6) = \begin{pmatrix} 0.4 & 0.6 & 0.7 & 0.9 \\ 0.5 & 0.6 & 1.0 & 0.7 \end{pmatrix}$

Stochastic Environment: In this environment, it may happen that any factor of a commodity or the demand in the society is not certain, not precisely known, but some past data is available about it. From the probability distribution of demand, the available records or any other factor of the commodity can be determined briefly and acknowledged with that distribution the transportation problem can be solved and determined.

Fuzzy-Stochastic Environment: It is an environment, which is the combination of both stochastic and fuzzy environments. For example, the statement - "the probability of having large demand of cricket world cup ticket is 99%" involves randomness and impreciseness together. Here, 'probability' represents randomness and 'large' is fuzzy.

Difference Between Randomness and Fuzziness Randomness and fuzziness differ conceptually and mathematically though both systems use the unit interval $[0,1]$ as their measure. Fuzziness describes equivocal events. It measures the degree to which an event occurs. Randomness measures the certainty of event's occurrence. An event may occur at random, but to what degree it occurs, is fuzzy. Probability of a fuzzy event involves in the measure of the occurrence of equivocal events. As for example when we say "there is 80% chance of lower price of products for next month".

Table-1.1 Difference between randomness and fuzziness

Randomness	Fuzziness
(i) It is represented as probability function in a probability space i.e., $P(x \in A) = a$. $P(A) = a$. It implies that probability of x belonging to crisp set A is 'a'. Here, a is a real number in [0,1].	(i) It is represented as possibility function i.e., $\mu_{\tilde{A}}(x) = a$. It implies that membership grade of element x in a fuzzy set \tilde{A} is a. Here, a is a real number in [0,1].
(ii) $\sum_{x \in A} P(x \in A) = 1$	(ii) $\sum_{x \in \tilde{A}} \mu_{\tilde{A}}(x) \neq 1$
(iii) It is statistical inexactness due to random events. Example: There is a 50% chance that the selling price of an item is \$20.	(iii) It is imprecise and inexact due to the human perception process. Example: The selling price of an item may be about \$20.

1.2.2 Basic Concepts and Terminologies in Transportation

A transportation system depends on several parameters- such as resources, destinations, demand, availability, conveyance, capacities of the conveyance, transportation costs and times, constraints, etc. The Details of these parameters are available in the literature on transportation problems.

Resources:

The places of production units or warehouse of goods are called the sources or origins.

Destinations:

The places where the goods are to be supplied (warehouses, customer etc.) are called destination.

Demand:

Demand means to the amount of a product needed at a certain time. It generally depends on the people outside the society which has the inventory problem. Demand can be classified on the basis of its pattern, rate and size. In some cases, demand may be

represented by vague, imprecise and uncertain data. This type of demand is termed as fuzzy demand. Demand also can be treated as fuzzy-stochastic in nature.

Availability:

The amount/ quantity of the goods which can be possible to transport from the respective resource is referred as availability of that resource.

Unit Transportation Cost:

The cost by which one unit product is transported from a source to a destination.

Transportation Time:

The total duration of time for the transportation of all products/ resource from the source points to destination points.

Transportation Problem (TP):

The conventional transportation problem is one of the best known linear programming problems, where there are two constraints in terms of demand constraints and availability constraints, in which all the constraints are of equality type. From the warehouses or the supply points, a particular item should be transferred to the retailers through particular freight or by particular vehicles carriages in industrial problems. The conventional transportation problem is to carry required products from the warehouses to the demand points so that it can minimize the total carrying expenditures. In some cases, the amount of total availability is more than the total demands. Under this condition, the TP is reduced to an unbalanced TP. Suppose, M sources (or origins) O_i ($i = 1, 2, \dots, M$) and N demands (i.e. destinations) D_j ($j = 1, 2, \dots, N$). Let a_i be the amount of a homogeneous production found at i -th origin and consider b_j as the demand at j -th destination. The unit transportation cost from i -th origin to the wanting point j -th destination is C_{ij} . The unknown quantity to be transported from origin O_i to destination D_j is presented as the variable x_{ij} . The mathematical form of TP is

$$\text{Min } Z = \sum_{i=1}^M \sum_{j=1}^N C_{ij} x_{ij} \quad (1.1)$$

subject to

$$\sum_{j=1}^N x_{ij} = a_i, \quad i = 1, 2, 3, \dots, M \quad (1.2)$$

$$\sum_{i=1}^M x_{ij} = b_j, \quad j = 1, 2, 3, \dots, N \quad (1.3)$$

$$x_{ij} \geq 0 \quad \text{for all } i, j. \quad (1.4)$$

Constraints

Constraints in transportation system have sometimes limitations imposed on the system when they deal with various properties. Constraints depend on the necessary place, required/available space of the destination/resources (space constraint), amount of investment, finance (budget constraint) resources, and , etc. These constraints may be fuzzy in nature ,stochastic or crisp i.e., resources, aim for the objectives,constraints data, etc., may be imprecise or random, vague and deterministic.

Solid Transportation Problem(STP):

The solid transportation problem is a generalization of the popular transportation problem(TP). Where instead of destination and source, the three dimensional properties are considered into account in the constraint set and objective function. At first Shell [136] said about the STP. An identical product is transformed from an source to a destination by means of various modes of transport i.e, conveyances, such as ships, goods trains, cargo flights, trucks, etc. These conveyance help us to make the problem as the three dimension. When we consider only a single type of conveyance, then the STP can be transformed to a classical transportation problem.

We consider M sources (or origins) O_i ($i=1,2,\dots,M$) , N destinations D_j ($j=1,2,\dots,N$) and K conveyances E_k ($k=1,2,\dots,K$). Let a_i be the amount of a identical product available at i-th source, b_j be the demand at j-th destination and e_k represents the amount of product which can be transported by the k-th conveyance. The C_{ijk} and the variable

x_{ijk} is the unit transportation cost and unknown quantity to be transported from i -th source to j -th destination by means of the k -th conveyance respectively.

The mathematical form of STP is given by

$$\text{Min } Z = \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K C_{ijk} x_{ijk} \quad (1.5)$$

subject to

$$\sum_{j=1}^N \sum_{k=1}^K x_{ijk} = a_i \quad i = 1, 2, 3, \dots, M \quad (1.6)$$

$$\sum_{i=1}^M \sum_{k=1}^K x_{ijk} = b_j \quad j = 1, 2, 3, \dots, N \quad (1.7)$$

$$\sum_{i=1}^M \sum_{j=1}^N x_{ijk} = e_k \quad k = 1, 2, 3, \dots, K \quad (1.8)$$

$$x_{ijk} \geq 0 \quad \text{for all } i, j, k. \quad (1.9)$$

Multi-Objective Transportation Problem (MOTP):

The multi-objective transportation problem is a traditional transportation problem, to which some objective functions are added and all the constraints are of equality type. The C_{ij}^p are the unit exchange values by p -th criteria such as time, distance, transportation cost etc., from i -th origin to j -th demand point. x_{ij} be the transported quantity from origin O_i to destination D_j . The mathematical expression of MOTP is given by

$$\text{Min } Z^p = \sum_{i=1}^M \sum_{j=1}^N C_{ij}^p x_{ij}, \quad p = 1, 2, \dots, P \quad (1.10)$$

subject to

$$\sum_{j=1}^N x_{ij} = a_i \quad i = 1, 2, 3, \dots, M \quad (1.11)$$

$$\sum_{i=1}^M x_{ij} = b_j \quad j = 1, 2, 3, \dots, N \quad (1.12)$$

$$x_{ij} \geq 0 \quad \text{for all } i, j. \quad (1.13)$$

Multi-Objective Solid Transportation Problem (MOSTP):

The multi-objective solid transportation problem, to which some objective functions are added in the single-objective STP. C_{ijk}^p are the unit exchange values by p-th criteria such as time, distance, transportation cost etc., to j-th destination from i-th source by means of the k-th conveyance. The mathematical form of MOSTP is

$$\text{Min } Z^p = \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K C_{ijk}^p x_{ijk}, \quad p = 1, 2, \dots, P \quad (1.14)$$

subject to

$$\sum_{j=1}^N \sum_{k=1}^K x_{ijk} = a_i \quad i = 1, 2, 3, \dots, M \quad (1.15)$$

$$\sum_{i=1}^M \sum_{k=1}^K x_{ijk} = b_j \quad j = 1, 2, 3, \dots, N \quad (1.16)$$

$$\sum_{i=1}^M \sum_{j=1}^N x_{ijk} = e_k \quad k = 1, 2, 3, \dots, K \quad (1.17)$$

$$x_{ijk} \geq 0 \quad \text{for all } i, j, k. \quad (1.18)$$

Fixed Charge:

In many practical applications, it is realistic to assume that the product which can be dispatched on any identical path bears a fixed charge such as taxes, subscription etc. for that path. Further, when a path is entirely excluded, that can be expressed by its limiting value to zero. Such fixed costs of transportation may also be applied to some production-planning models also.

Breakable Items:

In the transportation business, different types of materials / items including fragile items such as glass, ceramics, plastics, mud, etc are transported from sources to destinations. In a transportation system, breakable amount of an item normally depends on the size of the consignment to be transported and the type of conveyance used for transportation. He/ she is tempted to transport a large size of consignment from sources to destinations to invites the loss of units at destination points due to breakability.

Discount Policy:

In order to introduce larger purchases, a supplier often offers a reduced price if amounts greater than some minimum amount are ordered. This means the price per unit is lower if a large enough is placed. Normally, two types of quantity price structure are considered: (i) All Unit Discount (AUD) and (ii) Incremental Quantity Discount (IQD).

1.3 Historical Review of Transportation Models

1.3.1 Historical Review of Crisp Transportation Problem

Transportation problems (TP) are one of the most common special type linear programming problems that concern constraints. TP is one of the optimization problems which are most widely used by Public and Private Sectors. The basic transportation problem was originally advanced by F. L. Hitchcock [57] and after Koopmans [71] discussed in detail. The problem is called transportation problem because of its involvement with the physical distribution or transportation of products from different supply points to a number of demand points. To solve a common TP, one needs the determination of the quantity of units of physical products, one should ship from each origin to each destination, gratifying destination demand and source availability while minimizing the total transportation cost.

The transportation problem was conceived by Dantzig [27] as a spacial class of linear programming problems and then advanced a special form of simplex technique (Dantzig [27]) taking advantage of the spacial nature of the coefficient matrix. For gaining an initial solution for the transportation problem, Kirca and Satir [70] have presented a heuristic algorithm. Vogel's approximation method usually gives a better initial basic feasible solution compared to the north-west corner rule, the column minimum-cost method and the row minimum-cost method.

Appa [1] argued about the several variations in transportation problem. For search-

ing an initial feasible solution to the transportation problem, Ramakrishnan [126] has illustrated a variation of Vogel's approximation method. Shafaat and Goyal [135] have advanced a procedure to ensure an improved solution for a problem with a single degenerate basic feasible solution. Arsham and Khan [4] have advanced an algorithm that is simpler than both in working out general transportation problem, more general than stepping-stone and faster than simplex. Gass [39] has described the different aspects of TP computational and methodologies results. Palekar et al. [119] and Adlakha et al. [2] reviewed inbrief the fixed charge transportation problem. Patel and Tripathy [120] discussed multi-index TP, where as in 1955, Shell [136] first introduced the concept of multi-dimensional TP (ie, STP). Vignaux et al. [142] developed the linear transportation problem using genetic algorithm. The fixed charge transportation problem was advanced by Kowalski and Lev [72] developed as a nonlinear programming problem of practical interest in industry and business. Gen et al. [40] discussed the bicriteria solid transportation problem of equality constants solved by genetic algorithms.

1.3.2 Historical Review of Fuzzy Transportation Problem

Depending upon different aspects, unit transportation costs, resources, demands, transportation times, available budget, etc., fluctuate due to uncertainty in judgement, lack of evidence, insufficient information, etc. i.e., it is not possible to get relevant precise data, which are assumed by several researchers (cf. Shell [136], Yang and Liu [147], Waiel and Waiel et al. [143, 144]). So, a transportation model becomes more realistic if these parameters are assumed to be flexible / imprecise in nature i.e., uncertain in non-stochastic sense and may be represented by fuzzy numbers.

Based on Das et al. [28], the interval number transportation problems were converted into deterministic multi-objective problems. Grzegorzewski [47] and Grzegorzewski et al. [48] approximated the fuzzy number and general fuzzy number to its nearest interval. Omar and Samir [112] and Chanas and Kuchta [19] discussed the solution algorithm for solving the transportation problems in fuzzy environment. Sakawa and Yano [130]

proposed an interactive fuzzy decision making method using linear and non-linear membership functions to solve the multi-objective linear programming problem. Verma et al. [141], Bit et al. [14–16], Jimenez and Verdegay [64], Li and Lai [79] and Waicl [143] presented the fuzzy compromise programming approach to multi-objective transportation problem. Recently, some fuzzy STPs have been reported in the literature. Yang and Liu [147] investigated a fixed charge STP under fuzzy environment with fuzzy direct costs, fuzzy supplies, fuzzy demands and fuzzy conveyance capacities as a expected value model / chance- constrained programming model / dependent -chance programming model and solved using fuzzy simulation and a evolutionary method- tabu search algorithm. Gen et. al. [40], Jimenez and Verdegay [64], Li et. al. [77, 78] and others formulated some fuzzy STPs and solved using Genetic Algorithms. Gao and Liu [38] developed the two-phase fuzzy algorithms for multi-objective transportation problem. But, till now, none has considered the man-machine interaction for imprecise parameters connected with fuzzy STPs.

Recently Yang and Liu [150] investigated the fixed charge solid transportation problem under fuzzy environments. Based on Liu et al. [81] and Liu [83], using the extension principle, the fuzzy STP has been transformed into a pair of mathematical programs that is employed to calculate the lower and upper bounds of the fuzzy total transportation cost. Liu and Lin [82] associated the fuzzy fixed charge solid transportation problem with chance constrained programming by using credibility measure.

1.3.3 Historical Review of Type-2 Fuzzy Transportation Problem

The concept of a type-2 fuzzy set was first proposed in [154] as an extensions of type-1 fuzzy sets. While the membership grade of a type-1 fuzzy set is a real number in $[0,1]$, the membership grade of a type-2 fuzzy set (T2 FS) is a fuzzy number with a support bounded by the interval $[0,1]$. Human judgements are not always precise and also a word does not have the same meaning to different people. Mendel [102] explained that a sensible way to model a word is to using T2 FS, more precisely interval T2 FS.

Since then, many researchers have employed the theory in their studies. For example, Mitchell [104] used the concept of an embedded type-1 fuzzy number to give a method for ranking type-2 fuzzy numbers. Yang et al. [148,149] explained briefly the reduction methods of type-2 uncertain variables. After, a many research (cf Martnez et al. [94], Martnez-Soto et al. [93], Melin et al. [96], Hidalgo et al. [55]) studies the development of interval type-2 fuzzy logic theoretically.

1.4 Motivation and Objective of the Thesis

This thesis is made on the aim of some basic objectives and it is motivated by the following situations

Motivation

Transportation problem is an old age problem since the investigation of F.L. Hitchcock [57]. Normally, these problems are formulated in crisp environment as single or multi-objective linear programming problems and solved by linear or non-linear optimization methods. For two-dimensional single objective transportation problems, there are some special techniques such as VAM, etc. to solve the problems.

It is fact that the real world is full of uncertainties. It may be random i.e., uncertainty in stochastic sense or imprecise i.e., uncertainty in non-stochastic sense. The unit transportation costs, resources, demands, constraints, etc, in transportation problems are normally uncertain. This uncertainty may be in stochastic or fuzzy senses. This prompted us to consider some innovative general and solid transportation problems in uncertain environments though there are some transportation or solid transportation problems in stochastic and fuzzy environments.

This situation induced us to consider some transportation problems in fuzzy, bi-fuzzy environment. This environment is an emerging area and is yet to be developed.

The objectives of this dissertation are given below:

(i) To present some innovative general and solid transportation models in uncertain environments not considered by earlier researchers:

In this dissertation, ten general/ solid transportation problems- three in fuzzy, two in stochastic and five in fuzzy-stochastic environments are presented and solved by different methods. These models were not considered by earlier researchers.

(ii) To illustrate the use of entropy:

Attempt has been made to illustrate the effect of the use of 'entropy' in transportation problems. In one innovative transportation problems, 'entropy' has been included as an additional constraint in addition to the usual minimization of total transported cost and time. It has been illustrated that inclusion of 'entropy' gives balanced allotments to the cells of transportation cost matrix though it fetches more costs than the corresponding minimum costs.

(iii) To introduce the transportation cost discount against the transported amount:

Till now, no researcher has incorporated quantity discount in any transportation problem. But, this is a natural phenomenon in transportation system. We have studied two transportation problems introducing AUD. Here, discount in unit transportation cost is allowed against the transported amount.

(iv) To consider the transportation of breakable items:

Till now, in the literature, none has considered the transportation of breakable items such as the units made of glass, ceramic, mud, etc. Normally, amount of broken units linearly depends on the size of consignment. If price discount is allowed, then the retail-

ers are in fix/ dilemma i.e., if size of consignment is large, the amount of broken units is also large by which heavy loss is incurred, on the other hand, for large transported amount, discount in unit cost is high i.e., unit transportation cost is low. If the amount of consignment is low, then the above mentioned contradictions are reversed. In one transportation problem, such a conflicted situation has been considered and investigated.

(v) To introduce fuzzy logic / inference in transportation problems:

Now-a-days, fuzzy logic / inference is used to make the models in different areas more realistic. Till now, in the literature, none has considered fuzzy inference in transportation problems. Fuzzy relations connecting unit cost and transported amount have been introduced in a transportation problem and these relations are termed with imprecise verbal words 'low', 'medium', and 'high' etc.

(vi) To use soft computing methods- such as GA for solution:

There are some advantages in using the soft computing technique such as Genetic Algorithm etc., than the conventional optimization techniques. Moreover, Multi-objective Genetic Algorithm (MOGA) gives better results for multi-objective optimization problems than the other conventional non-linear optimization methods such as Global Criteria Method, Fuzzy Programming technique etc., which convert multi-objective general and solid transportation problems to single objective problems. Some two-dimensional and three dimensional multi-objective transportation problems have been developed and directly solved by MOGA. In addition to this, some single objective transportation problems have been solved by GA, specially developed for transportation problems.

1.5 Organization of the Thesis

In this proposed thesis, some real-life Transportation problems / Solid Transportation problems have been developed and solved in fuzzy and bi-fuzzy environments. The thesis has been divided into four parts and eleven chapters.

Part-I: General Introduction, Basic Concepts and Methodologies

Part-II: Transportation Problems in Fuzzy and Bi-fuzzy Environments

Part-III: Summary of the Thesis

Part-IV: Bibliography

Part-I:**(General Introduction, Basic Concepts and Methodologies)**

The Part-I is divided into two chapters such as Chapters-1 and Chapters-2.

Chapter-1**(General Introduction)**

An introduction giving an overview of the development on transportation problems in fuzzy and bi-fuzzy environments, has been discussed in this chapter.

Chapter-2**(Solution Methodologies)**

In this chapter, the methods/ techniques used in this dissertation to solve different types of single / multi objective transportation problems in different environments have been described and modified wherever it is required. These methods are Nearest Interval Approximation Method, Weighted Sum Method, Critical value method, Genetic algorithm, Multi-objective Genetic algorithm.

Part-II**(Transportation Models in Fuzzy and Bi-fuzzy Environment)**

The Part-II contains seven Chapters in which fuzzy and bi-fuzzy transportation models are derived and solved.

Chapter-3: A profit maximization solid transportation problem in fuzzy environment using genetic algorithm.

A solid transportation problem has been considered in which the transportation is accomplished in two stages. Firstly, from the origin(s) to the near by station(s) of the destination(s) and secondly, from the near by station(s) to the exact destination(s). Here, a fuzzy *AUD* (all unit discount) policy has been introduced based upon the amount of

transportation along with a fuzzy fixed charge. In addition, a budget constraint has been incorporated taking fuzzy unit transportation cost. Then the proposed model has been converted into a single objective optimization problem using interval arithmetic of the α -cut of the fuzzy profit function. To solve the proposed model, Genetic Algorithm(*GA*) has been used. Finally, a numerical example has been illustrated to study the feasibility of the model.

Chapter-4: Two staged solid transportation problem of breakable items with safety cost.

Transportation of a commodity in supply chain management system from the origin to the user customer may not be a single stage problem. In this chapter, we demonstrate a real-world two-stage solid transportation problem (*TSSTP*) where, the items are transported from origin/ source to distribution center / warehouse, then to the destinations through different conveyances. A safety cost is taken into account as an additional cost to reduce the rate of breakability of the items. In the model, unit transportation costs, safety costs, supplies, demands and capacities of the conveyances are taken to be uncertain parameters. There are restrictions on breakability of the items in the conveyances. Using the α -cut of the fuzzy number, the *TSSTP* is first converted into an equivalent deterministic *TSSTP*. The deterministic *TSSTP* is reduced into single objective programming problem. The single objective programming problems are solved using *Lingo – 11.0* optimization toolbox. Finally, a numerical example is given to illustrate the performance of the models.

Chapter-5: Solving a fuzzy solid transportation model with fuzzy ranking.

The Vogel's approximation method (*VAM*) with imprecise parameters is performed in a solid transportation problem (*STP*). Here, for the first time, fuzzy ranking tool is introduced for the different type of operations and compression presence in *VAM*. The

obtained basic feasible solution (*BFS*) has been converted into an optimal solution by the method fuzzy modi-indices. Finally, a numerical example is taken into account to support the method in imprecise environment. Moreover, sensitivity analysis also has been presented.

Chapter-6: Genetic algorithm approach of a solid transportation problem based on fuzzy logic

Traditionally, the unit transportation cost may vary depending on various parameters, like distance of transportation, amount of transportation, type of transportation item, etc. In this chapter, an imprecise solid transportation problem is considered where the unit cost depends into two parts: one for type of item and another for distance of transportation. Here, both the unit cost are logically executed from three different membership functions. More over, a fuzzy logic based genetic algorithm has been applied to optimize such model. Finally, a numerical example is presented to illustrate model.

Chapter-7: A solid transportation problem with entropy using fuzzy logic & genetic algorithm

Two items solid transportation problem has been studied, where the items are substitutable in nature. Due to this substitutability, the demand and availability of one item depends on that of the other which can be calculated from the concept of fuzzy logic. Here holding cost has been considered in both of the origin and destination. The model is optimized using genetic algorithm for two different models, model with and with out entropy. Finally, the models are illustrated through numerical examples

Chapter-8: A solid transportation problems with non-linear transportation cost and type-2 fuzzy variables

In reality, the unit transportation cost of a commodity is not fixed, but it decreases at

an increasing rate of the quantity. In this chapter, we consider a solid transportation problem with non-linear cost coefficients is considered where the unit transportation cost is depend on the amount of transportation. The unit transportation cost decreases at an inverse exponential rate if the transported amount increases. Here, all the parameters related to transportation problem are assumed as a type-2 fuzzy numbers. The model is solved following CV-based reduction method, nearest interval approximation method and Chance constrained programming based credibility measures. The deterministic non-linear solid transportation problem is optimized through *Lingo – 11.0* software. Finally, the models are illustrated through numerical examples

Chapter-9: A multi-objective solid transportation problem with discount and two-level fuzzy programming technique.

A multi-objective solid transportation problem (*MOSTP*) is optimized with two relevant criteria: cost and time for transportation. A realistic discounts policy [all unit discount (*AUD*)] has been proposed for the transportation cost depending on the amount of transportation units. In the proposed model, the shipment cost, supply amount, demand, etc., are as fuzzy in nature. A two-level fuzzy mathematical programming has been developed on the basis of fuzzy extension principle to solve the proposed model. Finally, the problem is demonstrated by taking a numerical example.

Part-III

Chapter-10: Summary and future work

At the end, a summary of thesis, its limitation and the scope of future research work have been given.

Part-IV

Chapter-11: Bibliography