

# Acknowledgement

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Date:

(Tripti Bej)

# Abstract

Fuzzy set theories provide strict mathematical frameworks using vague phenomena and can be precisely and rigorously studied. The core theories in mathematics such as algebra, topology, lattices and analysis are subject to fuzzification. Many researchers have dealt with subalgebras, ideals, subimplicative ideals, P-ideals, H-ideals, implicative ideals, hyper structure etc. and their fuzzification.

This study attempts to introduce a new class of algebraic structures in intuitionistic fuzzy set and interval-valued fuzzy set. The concepts of doubt intuitionistic fuzzy subalgebra/ideals, doubt intuitionistic fuzzy H-ideals, doubt intuitionistic fuzzy subimplicative ideals, doubt intuitionistic fuzzy P-ideals and doubt intuitionistic fuzzy translations of doubt intuitionistic fuzzy sub-implicative ideals are introduced for BCK/BCI-algebra and their properties are duly characterized.

The notion of direct product of two doubt intuitionistic fuzzy subalgebras and two doubt intuitionistic fuzzy H-ideals of two *BCK/BCI*-algebras has also been discussed and some related properties have investigated. We have also found that the direct product of two intuitionistic fuzzy sets becomes doubt intuitionistic fuzzy H-ideals and doubt intuitionistic fuzzy subalgebras if and only if for any  $t, s \in [0; 1]$ , upper and lower level sets are *H*-ideals or subalgebras of *BCK/BCI* algebra  $X \times Y$ .

Results of doubt interval-valued fuzzy subalgebras and doubt interval-valued fuzzy ideals in BCK-algebras and BF-algebras are also provided. Cartesian product of two interval-valued fuzzy sets is defined and its properties are established. The relation between doubt intuitionistic fuzzy ideals and an upper level and lower level cuts of *BCK/BCI*-algebra have also been explored.

Besides doubt intuitionistic fuzzy hyper filters in hyper BE-algebras have also been established in the present work.

## List of Symbols and abbreviations

### Symbols

$\cup$	Union
$\cap$	Intersection
$\wedge$	Minimum
$\vee$	Maximum
$\leq$	Less than or equal to/ partial order
$\geq$	Greater than or equal to
$\alpha$	Membership function
$\zeta$	Non-Membership function
$X$	BCK/BCI/BF/BE-algebra
$\in$	Belonging
$\notin$	Not an element of
$*$	Binary operation
$\times$	Cartesian product/ Direct product
$\subseteq$	Contained in
$\phi$	Empty set
$\ll$	Hyper order
$\circ$	Hyper operation
$\bar{\nu}_A$	interval-valued fuzzy set
$Y$	hyper $BE$ -algebra
$\not\leq$	Not less than or equal
$\not\geq$	Not greater than or equal

## Abbreviations

SS	Subset
SA	Subalgebra
FR	Fuzzy relation
FSG	Fuzzy subgroup
FSs	Fuzzy sets
FSS	Fuzzy subsets
FSA	Fuzzy subalgebra
F-ideals	Fuzzy ideals
SI-ideal	sub-implicative ideal
IVFS	Interval-valued fuzzy set
IFSs	Intuitionistic fuzzy sets
IFSS	Intuitionistic fuzzy subsets
IF	Intuitionistic fuzzy
FSI-ideal	Fuzzy sub-implicative ideal
IFSA	intuitionistic fuzzy subalgebra
IFIs	intuitionistic fuzzy ideals
IF SI-ideals	Intuitionistic fuzzy sub-implicative ideals
IF SC-ideals	Intuitionistic fuzzy sub-commutative ideals
IF H-ideals	Intuitionistic fuzzy H-ideals
IVFSG	Interval-valued fuzzy subgroup
IVFSA	Interval-valued fuzzy subalgebra
IVF-ideal	Interval-valued fuzzy ideal
DIFSs	Doubt intuitionistic fuzzy sets
max	Maximum
min	Minimum
sup	Supremum
inf	Infimum
MSV	Membership value
MSF	Membership function
DMS	Degree of membership
DNMS	Degree of non- membership

## Abbreviations

CP	Cartesian product
DP	Direct product
UC	Upper cut
LC	Lower cut
DFSA	Doubt fuzzy subalgebra
DF-ideal	Doubt fuzzy ideal
DF H-ideal	Doubt fuzzy H-ideals
DF SI-ideal	Doubt fuzzy sub-implicative ideal
DIFSA	Doubt intuitionistic fuzzy subalgebra
DIF	Doubt intuitionistic fuzzy
DIF-ideal	Doubt intuitionistic fuzzy ideal
DIF H-ideal	Doubt intuitionistic fuzzy H-ideal
DIF SI-ideal	Doubt intuitionistic fuzzy sub-implicative ideal
DIF P-ideal	Doubt intuitionistic fuzzy P-ideal
Hyp-filter	Hyper filter
DIF Hyp-filter	Doubt intuitionistic fuzzy hyper filter
DIFI Hyp-filter	Doubt intuitionistic fuzzy implicative hyper filter
DIVFSA	Doubt interval valued fuzzy subalgebra
DIVF-ideal	Doubt interval valued fuzzy ideal
PI-ideals	Positive implicative ideals
PI Hyp-filters	Positive implicative hyper filters

## List of research papers

1. T. Bej and M. Pal, Doubt Atanassov's intuitionistic fuzzy Sub-implicative ideals in  $BCI$ -algebras, *International Journal of Computational Intelligence Systems*, **8**(2) (2015), 240-249. (**Atlantis Press and Taylor and Francis**) (**SCIE**)(I.F- 2.153)
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3. T. Bej and M. Pal, Interval-valued doubt fuzzy ideals in  $BCK$ -algebras, *International Journal of Fuzzy System Applications*, **8**(4) (2018), 101-121. (**IGI Global**) (**Scopus**)
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5. T. Bej and M. Pal, Doubt intuitionistic fuzzy ideals in  $BCK/BCI$ -algebras, *International Journal of Fuzzy Logic Systems*, **5**(1) (2015), 1-13.[UGC Approved Journal]
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7. T. Bej and M. Pal, Direct product of doubt intuitionistic fuzzy  $H$ -ideals in  $BCK/BCI$ -algebras, *International Journal of Pure and Applied Researches*, **2**(1) (2016), 11-21. [UGC Approved Journal]
8. T. Bej and M. Pal, B. Davvaz, Doubt intuitionistic fuzzy translation of doubt intuitionistic fuzzy sub-implicative ideals in  $BCI$ -algebras, (Communicated)