2015

MCA

1st Semester Examination

DISCRETE MATHEMATICS

PAPER—MCA-102

Full Marks : 100

Time : 3 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Answer any *five* questions : 5×14

1. (a) Show that the mapping f : N → N (the set of all natural numbers) defined by $f(x) = 5x - 7$ for all $x \in N$ is one-one but not onto. 7

   (b) Prove that for any three non-empty sets A, B, C,

   $$(A - B) \times C = (A \times C) - (B \times C).$$ 7

2. (a) If n be a positive integer then show by using the principle of mathematical induction that

   $$2.7^n + 3.5^n - 5 \text{ is divisible by } 24.$$ 7

(Turn Over)

(b) Construct a truth table for

$$[p \to ((q \wedge (\neg r)) \vee s)] \ \wedge \ [(\neg t) \leftrightarrow (s \wedge r)].$$ 7

3. (a) Express the given permutation as a product of disjoint cycles and examine whether it is a even or odd permutation :

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 4 & 6 & 1 & 8 & 3 & 5 & 7 \end{pmatrix}$$

Find the order of the permutation. 7

(b) Using Boolean algebra prove that

(i) $(x + y)' = x' \cdot y'$

and (ii) $(x \cdot y)' = x' + y'$. 7

4. (a) Show that

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} = (a+b+c)(b-c)(c-a)(a-b).$$ 7

(b) If $A = \begin{pmatrix} 1 & 0 & 0 \\ -1 & -2 & -1 \\ 2 & 3 & 2 \end{pmatrix}$, show that

$A^3 - A = A^2 - I$. Hence find A^{-1}. 7

5. (a) Prove that in a simple graph with $n(\geq 2)$ vertices must have at least two vertices of equal degree. 7

(b) Using the postulates and theorems of Boolean algebra reduce the function :

$$(x + y)(x + y')(x' + z')$$ 7

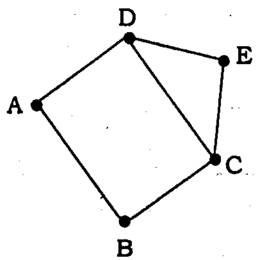
6. (a) Define Euler path and Euler circuit. Draw a graph which has Euler path but no Euler circuit. 7

(b) Let G be a connected graph with $n \geq 2$ vertices and m edges such that $m < n$. Prove that G has at least one pendant vertex. 7

7. (a) Express the following expression as a function in conjunction normal form :

$$xyz + xy'z' + x'yz' + x'y'z'$$ 7

(b) Define a spanning tree. Draw two spanning trees from the graph : 7



8. (a) Prove that a tree with n vertices contains exactly $n-1$ edges. 7

(b) Let f, g, h are three mappings from R to R defined as $f(x) = 2x$, $g(x) = x^2$, $h(x) = x + 1$ then find $h_o(g_o f)$ and $(h_o g)_o f$ and show that they are identical. 7

Internal Assessment : 30
