PG/2nd Sem/PHS/19

2019

PG

2nd Semester Examination

PHYSICS

Paper - PHS 201

Full Marks : 40

Time : 2 Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Use separate Scripts for Group 201.1 and 201.2.

Group - 201.1

(Quantum Mechanics - II)

1. Answer any *two* : 2×2=4

(a) Consider the operator $\hat{A} = \frac{1}{2}\{\hat{J}_x, \hat{J}_y\}$.

Calculate the expectation value of $\hat{A}$ and $\hat{A}^2$ with respect to the state $|j, m\rangle$.

[Turn Over]

(b) Find the energy level of spin $S=\frac{1}{2}$ particle whose Hamiltonian is given by

$$\hat{H}=\frac{\alpha}{\hbar^2}\left(S_x^2+S_y^2-2S_z^2\right)-\frac{\beta}{\hbar}S_z$$

where α and β are constants. What is degeneracy of the energy?

(c) Find the rotation matrix $d^{(1/2)}$ for $j=\frac{1}{2}$.

(d) Write down Klein Gordon equation in covariant form.

2. Answer any *two* : $4\times2=8$

(a) Deduce an expression of j_μ for spin - 0 particle with interaction of e.m. field.

(b) Prove that $\bar{\psi}\psi$ and $\bar{\psi}\gamma^\mu\psi$ are scalar and vector under Lorentz transformation.

(c) Final the C. G. coefficients for $j_1=j_2=\frac{1}{2}$

(d) Show that the current $j_\mu = \frac{-i}{2}\left(\phi\partial_\mu\phi^* - \phi^*\partial_\mu\phi\right)$ satisfies continuity equation.

3. Answer any *one* : 8×1=8

(a) (i) A spin zero particle of charge q and mass m is incident on a potential barrier

$$A^0 = 0 \quad \text{for } z < 0; z > a$$

$$= V_o \quad \text{for } 0 < z < a$$

where V_o is a positive constant.

Find the transmission coefficient Also, find the energy of the particle for which the transmission coefficient is equal to one. 4

(ii) Prove that $\sum_{r=1}^{2} u_r(p)\bar{u}_r(p) = \frac{\not{p}+m}{2m}$

where $u_r(p)$ is the positive energy spinor. 4

[Turn Over]

(b) (i) A rigid rotator in a plane is acted on by a perturbation represented by

$$H' = \frac{V_o}{2}\left(3\cos^2\phi - 1\right) \quad V_o = \text{constant}$$

Calculate the ground state energy upto the second order in perturbation.

(ii) Find the ground state energy up to the second order in perturbalion.

Find the ground state energy of H - atom using trial function

$$\psi(r) = A\exp(-\alpha r^2) \text{ if } \hat{H} = \frac{-\hbar^2}{2\mu}\nabla^2 - \frac{e^2}{r}.$$

4+4=8

Group - 201.2

(Methods of Math. Physics - II)

1. Answer any *two* : 2×2=4

(a) Find the Fourier transform of

$$f(x) = 1 - x^2 \text{ if } |x| \le 1$$
$$= 0 \quad \text{if } |x| > 1$$

(b) Find inverse Laplace transform of

$$\log\left(\frac{k^2-1}{k^2}\right)$$

if $\hat{L}\left[f(x)\right]=\int_0^\infty e^{-kx} f(x)\,dx$

(c) $x'=(x-vt)\gamma$

$y'=y$

$z'=z$

$t'=\left(t-\frac{vx}{c^2}\right)\gamma$

Find the generator of this Lorentz transformation.

(d) Prove that a finite group whose order is a prime number must be a cyclic group.

2. Answer any *two* : 4×2=8

(a) Prove that there is a homomorphism between $SU(2)$ and $SO(3)$.

[Turn Over]

(b) Solve

$$y'' + 9y = 9\theta(t-3),\ y(0) = y'(0) = 0$$

where $\theta(t-3)$ is the unit step function.

(c) $f(x) = 1$ for $2n \le x \le 2n+1$

$= 0$ for $2n+1 \le x \le 2n+2$

where $n = 0, 1, 2, \ldots\ldots\ldots\ldots$

Find the Laplace transform of $f(x)$.

(d) $$f(x) = \delta(x) + \sum_{n=1}^{\infty} \frac{d^n}{dx^n}\delta(x)$$

Find the F. T. of $f(x)$.

3. Answer any *one* : 4×2=8

(a) (i) Solve

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial x\partial y} - 6\frac{\partial^2\psi}{\partial y^2} = y\cos x$$

(ii) Find the Green's function of the differential equation :

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + (k^2x^2 - n^2)y = f(x)$$

$y(0)$ is finite, $y(a) = \beta$. 4+4

(b) (i) Solve $\phi(x)$ by the method of F.T.

$$\phi(x) = f(x) + \lambda \int_{-\infty}^{+\infty} K(x-t)\phi(t)\,dt$$

(ii) $\hat{T}(\phi)\, f(x, y) =$
$f(x\cos\phi + y\sin\phi,\ -x\sin\phi + y\cos\phi)$

Find the generator. 4+4