M.Sc. 3rd Semester Examination, 2019

PHYSICS

PAPER –PHS-301.1 & 301.2

Full Marks : 40

Time : 2 hours

Answer **all** questions

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

PHS-301.1

(*Quantum Mechanics*-III)

[*Marks* : 20]

1. Answer any *two* : 2 × 2

(*a*) Discuss the scattering from a black disk of high energies.

(*Turn Over*)

(*b*) Given the scattering amplitude

$$f(\theta) = \frac{1}{2i\kappa} \sum (2l+1)\left[e^{2i\delta_l} - 1\right] P_l(\cos\theta).$$

show that

$$\operatorname{Im} f(0) = \kappa \sigma_t / 4\pi.$$

(*c*) A system of 2 identical bosons, each of mass 'm' is placed in a 1-D box of length 'L'. Both particles are in same spin state. The energy of the system is

$$\frac{5\pi^2\hbar^2}{2mL^2}.$$

What is the space part of the wave function ?

(*d*) Distinguish between adiabatic and sudden approximation in perturbation theory.

2. Answer any *two* : 4 × 2

(*a*) Assuming that the charge distribution in a nucleus is Gaussian.

$$\frac{\bar{e}^{\,r^2/b^2}}{\pi^{3/2} b^3}$$

then show that the form factor is also Gaussian and the mean square radius is $\frac{3b^2}{2}$.

(*b*) Distinguish between Hartree and Hartree-Fock approximation. State Koopman's theorem.

(*c*) Find the elastic and total-cross-section for a black sphere of radius 'R'.

(*d*) At what neutron lab energy will *p*-wave be important in *n*-*p* scattering ? (Impact parameter = 2 fm).

3. Answer any *one* : 8 × 1

(*a*) In the Born approximation, calculate the scattering amplitude for scattering from the square well potential $V(r) = -V_0$ for $0 < r < r_0$

and $V(r) = 0$ for $r > r_0$. If the geometrical radius of the scatterer is much less than the wavelength associated with the incident particles, show that the scattering will be isotropic. 5 + 3

(b) Consider a two-level system with $E_1 < E_2$. There is a time-dependent potential that connects the two levels as follows :

$V_{11} = 0$, $V_{22} = 0$, $V_{12} = \gamma e^{iwt}$, $V_{21} = \gamma e^{-iwt}$
(γ = real) at $t = 0$, $G(0) = 1$, $c_2(0) = 0$.

Prove that near resonance

$$\left| \begin{matrix} (i) \\ C_2(t) \end{matrix} \right|^2 = \frac{4\gamma^2}{\hbar^2 \left(w - w_0\right)^2} \sin^2 \frac{\left(w - w_0\right)}{2} t$$

where $w_0 = (E_2 - E_1)/\hbar$. 8

PHS-301.2

(*Statistical Mechanics*-I)

[*Marks* : 20]

4. Answer any *two* : 2 × 2

(*a*) N particles each of mass m confined to a box has energy eigenfunctions

$$\psi_{\kappa}(x) = \sqrt{\frac{2}{L}} \sin \kappa x.$$

Calculate the number of distinct states $\Omega(E, N, V)$ with energy E.

(*b*) A one-dimensional random walker takes steps to left or right with equal probability. Find the probability that the random walker starting from origin is back to origin after N even number of steps.

(*c*) If the Hamiltonian

$$\hat{H} = -\mu_B B \hat{\sigma}_x$$

for a spin $\frac{1}{2}$ particle, then calculate $\langle \sigma_x \rangle$.

(*d*) If the canonical partition function

$$Q = \sum_n e^{-\beta E_n}$$

Evaluate $\langle E^2 \rangle$ interms of Q.

5. Answer any *two* : 4×2

(*a*) A photon gas is at thermal equilibrium at temperature T. Calculate the mean number of photons in an energy state $\varepsilon = \hbar w$.

(*b*) If the probability of alignment of a spin $\frac{1}{2}$ particle in the upward direction is p. Find the entropy S of a system of N spins. Find the value of p at which the entropy is maximum.

(*c*) Calculate the expression of grand potential for N quantum harmonic oscillator in one dimension.

(*d*) If the density matrix in co-ordinate representation

$$\rho_{rr'} = \frac{1}{V} \exp\left[\frac{-m}{2\beta\hbar^2} \left|\vec{r} - \vec{r}'\right|^2\right]$$

for a harmonic oscillator. Prove that

$$\langle H \rangle = \frac{3}{2} \kappa_B T.$$

6. Answer any *one* : 8 × 1

(*a*) Consider a system of N particles, each of mass m, enclosed in an infinitely long cylindrical container in a uniform gravitational field. The system is in thermal equilibrium. Obtain expressions for the

(*i*) Classical partition function

(*ii*) entropy of the system

(*iii*) Internal energy

(*iv*) Specific heat of the system. 4 + 2 + 1 + 1

(*b*) (*i*) If $\hat{\rho}_1$ and $\hat{\rho}_2$ be a pair of density matrices then show that

$$\hat{\rho} = r\,\hat{\rho}_1 + (1-r)\,\hat{\rho}_2$$

is a density matrix for all real numbers r such that $0 \le r \le 1$.

(*ii*) For a spin 1 particle, single particle Hamiltonian

$$\hat{H} = -\mu_0 B \hat{\sigma} + \Delta\left(1-\sigma^2\right)$$

where $\sigma = \pm 1$ if there is spin.
$= 0$ if vacancy exists.

and Δ = vacancy formation energy.

Prove that magnetization

$$m = \frac{2\mu_0 \sin h\left(\frac{\mu_0 B}{\kappa_B T}\right)}{e^{-\frac{\Delta}{\kappa_B T}} + 2\cos h\left(\frac{\mu_0 B}{\kappa_B T}\right)}.$$

4 + 4
