M.Sc. 1st Semester Examination, 2019

PHYSICS

PAPER –PHS-101

Full Marks : 40

Time : 2 hours

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

Write the answers to questions of each Paper in separate books

PHS–101.1

(*Methods of Mathematical Physics*)

[*Marks* : 20]

1. Answer any *two* questions : 2×2

(*a*) If $A = \begin{pmatrix} a \\ b \end{pmatrix} \begin{pmatrix} a-2 & b \end{pmatrix}$

Find the symmetric part of A.

(*b*) Show that

$$\int_{-1}^{+1} x^2 P_{n+1}(x) P_{n-1}(x)\, dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}.$$

(*c*) Find the Laurent series of

$$f(z) = \frac{1}{(z-1)(z-2)}$$

in the region $1 \le |z| \le 2$ and around $z = 1$.

(*d*) Evaluate :

$$\int_0^\infty x^2 e^{-2x^2} dx.$$

2. Answer any *two* questions : 4×2

(*a*) A vector $\vec{p} = 2\hat{i} + 3\hat{j} + 2\hat{k}$. The axes are rotated anti-clockwise about the Y axis by an angle of 60°. Find the vector $\vec{p}'$ in the rotated co-ordinate system.

(*b*) Evaluate

$$\int_0^{\infty} \frac{dx}{4+x^4}$$

by the method of residue.

(*c*) Evaluate $\underset{2}{\overset{5}{L}}(x)$ where $\underset{m}{\overset{n}{L}}(x)$ stands for associated Laguerre polynomial.

(*d*) If A and P be square matrices of the same type and if P be invertible, show that the matrices A and $P^{-1}AP$ have the same characteristic roots.

3. Answer any *one* question : 8 × 1

(*a*) (*i*) Evaluate

$$\int_0^{\infty} \frac{\cos mx}{\left(x^2+a^2\right)^2} dx = \frac{\pi}{4a^3}\left(1+ma\right)e^{-ma}$$

by residue theorem ($a > 0$, $m > 0$).

(*ii*) If $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

show that $e^A = I + \frac{A}{3}\left(e^3 - 1\right)$. 4 + 4

(*b*) (*i*) Prove that $H_n'(x) = 2nH_{n-1}(x)$.

(*ii*) Evaluate

$$\int_0^{2\pi} e^{\cos\theta} \cos\left(\sin\theta - n\theta\right) d\theta$$

by residue theorem. 4 + 4

PHS-101.2

(*Classical Mechanics*)

[*Marks* : 20]

1. Answer any *two* questions : 2 × 2

(*a*) Derive the Hamiltonian of a system whose Lagrangian is defined as $L = \dot{q}^2 - q\dot{q}$.

(*b*) A system is governed by Hamiltonian

$$H = \frac{1}{2}\left(p_x - ay\right)^2 + \frac{1}{2}\left(p_y - bx\right)^2,$$

a and b are constants and p_x and p_y are momenta conjugate to x and y. For what values of a and b quantities $(p_x + 5y)$ and $(p_y - 4x)$ be conserved ?

(*c*) Derive Canonical equations in terms of Poisson's bracket.

(*d*) Evaluate $[L_x, p_x]$.

2. Answer any *two* questions : 4 × 2

(*a*) A particle moves in a plane under the influence of a force, whose magnitude is

$$F = \frac{1}{r^2}\left(1 - \frac{\dot{r}^2 - 2\ddot{r}r}{c^2}\right)$$

where r is the distance of the particle to the centre of the force. Find the potential that will result in such a force and from that the Lagrangian for the motion in a plane.

(*b*) Show that the transformation $Q = p + iaq$ and

$$P = \frac{p - iaq}{2ia}$$

is Canonical transformation and find out the generating function.

(*c*) What do you mean by stable, unstable and natural equilibrium ?

(*d*) For a system consisting of single particle show that the principle of least action becomes

$$\Delta \int_{t_1}^{t_2} \sqrt{H - V} \cdot ds = 0.$$

3. Answer any *one* question : 8 × 1

(*a*) If $f = f(q_j, \dot{q}_j, t)$ then show that

$$\Delta f = \delta f + \Delta t \cdot \frac{df}{dt}$$

outline the Hamilton-Jacobi equation what is the physical significance of Hamilton's principle function. 5 + 2 + 1 (*b*) Find out the Lagrangian of a particle of charge q, mass m and linear momentum p, enters an electromagnetic field of vector potential A and scalar potential φ and obtain the Hamiltonian of the particle. 4 + 4
