PG/2nd Sem/PHS/19 (Old)

2019

PG

2nd Semester Examination

PHYSICS

Paper - PHS 201

[Old Syllabus]

Full Marks : 40

Time : 2 Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

PHS - 201 (A)

(Quantum Mechanics - II)

(Marks : 20)

Answer Q. No. 1 & 2 and any *one* from the rest.

4×2=8

1. Answer any *three* bits : 2×3=6

(a) If $\hat{p}_r = -i\hbar\left(\dfrac{\partial}{\partial r} + \dfrac{1}{r}\right)$

prove that

$$\frac{p^2}{2m}=\frac{p^2{}_r}{2m}+\frac{L^2}{2mr^2}$$

(b) If $\hat{H}_D = c\hat{\alpha}.\hat{p}_z + \beta mc^2 + V(z)$ be the one dimentional Dirac Hamiltonian, then prove that $\left[\hat{\sigma}, \hat{H}_D\right] = 0$.

(c) A spin $\frac{1}{2}$ particle is in the state $\frac{1}{\sqrt{6}}\begin{pmatrix}1+i\\2\end{pmatrix}$.

What is the probability of getting $\frac{\hbar}{2}$ when be measure S_x?

(d) $S_r = \frac{\vec{S}.\vec{r}}{r}$ is the component of electron spin in the direction of $\vec{r}$ commutes with each component of total angular momemtum $\vec{J} = \vec{L} + \vec{S}$, then prove that $\left[\vec{S}, S_r\right] = i\hbar\frac{\vec{r}\times\vec{S}}{r}$.

(e) A hydrogen atom in its ground state is placed under electric field $\bar{E}$. Find the change in its eigen value in the second order.

$$\left[\psi_\circ = \frac{1}{\sqrt{\pi a_\circ^3}} e^{-r/a_\circ}\right]$$

2. Answer any *one* : 4

(a) Prove that $\left[\bar{a}.\bar{L}, \bar{b}.\bar{L}\right] = i\hbar\left(\bar{a}\times\bar{b}\right).\bar{L}$, [$\bar{a}.\bar{b}$ commute with each other and with $\bar{L}$].

(b) Prove that Dirac equation in .e. field in of the form $\left[(i\partial - eA)^2 - \frac{e}{2}\sigma^{\mu\nu}F_{\mu\nu} - m^2\right]\psi = 0$ and $\sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu, \gamma^\nu\right]$.

3. (a) Prove that $\sum_r u_r(p)\bar{u}_r(p) = \frac{\not{p}+m}{2m}$.

and $\sum_r v_r(p)\bar{v}_r(p) = \frac{\not{p}-m}{2m}$

(b) Show that $\not{p}^2 = p^2$.

[Turn Over]

4. Write down Block-equation for spin and solve it a magnetic field $\bar{H} = H_o \hat{e}_z + \hat{e}_x H_1 \sin wt + \hat{e}_y H_1 \cos wt$

2+8

PHY 201 (B)

(Method of Mathematical Physics - II)

Answer Q. No. 1 & 2 and any *one* from the rest.

2×5=10

1. Answer any *five* bits :

   (a) $x' = ax + b$ find the generator.

   (b) State & prove Lagrange's theorem for group.

   (c) If a group $a * b = a + b - 1$

   Find the inverse of the elements.

   (d) Prove that $\delta(x)\delta(y) = \frac{\delta(r)}{2\pi r}$

   (e) Define the structure constant of a Lie group with examples.

   (f) $f(t) = \frac{2t}{3}, 0 \le t \le 3$

   Find the L.T. of $f(t)$.

(g) If

$$\hat{T}(p)f(x,y)=f(x\cos\phi+y\sin\phi,-x\sin\phi+y\cos\phi)$$

Find the generator of this Lie group.

(h) Find the inverse F.T. of $\delta(w)$.

2. (a) Find the Green's function for the boundary value problem.

$$\frac{d^2y}{dx^2}+k^2x=f(x)$$

$$y(0)=0$$

$$y(1)=0$$

(b) Solve

$$y(x)-\int_0^1(2x-t)y(t)\,dt=Cos\,2\pi x\,.$$ 5+5

3. (a) Solve

$$4\frac{\partial^2 z}{\partial x^2}-4\frac{\partial^2 z}{\partial x\,\partial y}+\frac{\partial^2 z}{\partial y^2}=16\log(x+2y)$$

[Turn Over]

(b) For group T_d, charater table for reducible representation is

T_r:	E	$8C_3$	$3C_2$	$6S_4$	$6\sigma_d$
	4	1	0	0	2

T_d	E	$8c_3$	$3c_2$	$6S_4$	$6\sigma_d$
A_1	1	1	1	1	1
A_2	1	1	1	−1	−1
E	2	−1	2	0	0
T_1	3	0	−1	1	−1
T_2	3	0	−1	−1	1

Find the number of irreducible representaion in T_d and show that $T_r = A_1 \oplus T_2$. 5+5