 

# M.Sc. 1st Semester Examination, 2019

## ELECTRONICS

( *Mathematical Methods* )

PAPER —ELC-101

*Full Marks* : 50

*Time* : 2 hours

*The figures in the right-hand margin indicate marks*

*Candidates are required to give their answers in their own words as far as practicable*

*Illustrate the answers wherever necessary*

1. Answer any *four* questions from the following : 2 × 4

(*a*) Write down Cauchy-Riemann conditions for a function $f(z)$ to be analytic in a certain region of complex plane. 2

(*b*) Define linearly dependent and linearly independent set of vectors. 1 + 1 (*c*) What do you mean by basis for a vector space and norm of a vector ? 1 + 1

(*d*) Does Laplace transform exist for all functions ? Explain with example. 2

(*e*) Round off the followings upto six significant figures : $\frac{1}{2} \times 4$

(*i*) 24.564986

(*ii*) 28.583553

(*iii*) 30.034753

(*iv*) 22.869345

(*f*) Find the truncation error in the result of the following function for $x = \frac{1}{5}$ when first three terms are used :

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!}$$ 2

(*g*) Find the mean and median of the following information : 2

| Mid Value : | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 |
|---|---|---|---|---|---|---|---|---|---|
| Frequency : | 2 | 22 | 19 | 14 | 3 | 4 | 6 | 1 | 1 |

(*h*) Calculate the standard deviation for the following data : 2

| Size of item : | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|
| Frequency : | 3 | 6 | 9 | 13 | 8 | 5 | 4 |

**2.** Answer any *four* questions from the following : 4 × 4

(*a*) Illustrate the Cayley-Hamilton theorem for the matrix $A$ where

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$ 4

(*b*) Starting from

$$I = \left\langle f - \sum_i a_i \phi_i \middle| f - \sum_j a_j \phi_j \right\rangle \geq 0$$

derive Bessel's inequality

$$\langle f_1 f \rangle \geq \sum_n |a_n|^2$$ 4

(c) Expand $\frac{1}{(1-z)}$ in a Taylor's series about $z_0 = i$ and find the radius of convergence. 3 + 1

(d) Find the Laplace transform of

$$\frac{\cos at - \cos bt}{t}.$$ 4

(e) If $u = 5xy^2/z^3$ and errors in $x, y, z$ are 0.001, compute the relative maximum error in $u$ when $x = y = z = 1$. 4

(f) Given $y = f(x)$ in the following table, find the values of $x$ for $y = 10$ and $y = 5$. 2 + 2

| | | | |
|---|---|---|---|
| $x$ | 10 | 15 | 17 |
| $y$ | 3 | 7 | 11 |

(g) Find a root of the equation

$$x^3 - 4x - 9 = 0$$

by bisection method upto six iteration. 4

(h) Show that the Legendre polynomials

$P_m(x)$ and $P_n(x)$ are orthogonal in the interval $-1 \leq x \leq 1$ if $m \neq n$. 4

3. Answer any *two* questions from the following : 8 × 2

(*a*) (*i*) In polar coordinates, show that the Cauchy-Riemann conditions become

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta} \text{ and } \frac{1}{r}\frac{\partial u}{\partial \theta} = -\frac{\partial v}{\partial r}$$

(*ii*) Evaluate using Cauchy's integral formula :

$$\int_C \frac{e^{2z}}{(z-1)(z-2)}\, dz,$$

where $C$ is the circle, $|z| = 3$. 5 + 3

(*b*) (*i*) Form a set of three orthonormal vectors by the Gram-Schmidt process using these input vectors in the order given :

$$C_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \; C_2 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \; C_3 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

(*ii*) A variate $X$ has the probability distribution

| $X$: | $-3$ | $6$ | $9$ |
|---|---|---|---|
| $P(X = x)$: | $\frac{1}{6}$ | $\frac{1}{2}$ | $\frac{1}{3}$ |

Find $E(X)$ and $E(X^2)$. 4 + (2 + 2)

(*c*) Evaluate approximately, by trapezoidal rule, the integral

$$\int_0^1 (4x - 3x^2)dx, \text{ by taking } n = 10.$$

Compute also the exact integral and find the absolute and relative error. 4 + 2 + 1 + 1

(*d*) (*i*) Find a real root of the equation

$$x^3 - 5x - 7 = 0$$

using Regula-Falsi method correct upto three decimal places.

(*ii*) Find the value of $\int_2^6 \frac{dx}{x}$ by Simpson's rule. 4 + 4

[ *Internal Assessment* : 10 Marks]

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