NEW

2018

Part I

STATISTICS

PAPER—I

(General)

Full Marks : 90

Time : 3 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Group—A

[Marks : 25]

1. Answer any *one* question : 1×10

(a) State and prove Baye's theorem. 4

What is mutual independence and what is pairwise independence ? 2

Two players agree to play under the following conditions. Taking turns, they draw the balls out of an urn containing '*a*' white and '*b*' black balls, one at a time. He who extracts the first white ball wins the game. What is the probability that the player who starts the game will be the winner ? 4

(b) Define cumulative distribution function and state its properties. 4

For a $N(\mu, \sigma^2)$ distribution, prove that

$\mu_{2n} = (2n-1)(2n-3).....3.1.\sigma^{2n}$ 4

State Chebyshev's inequality. 2

2. Answer any *three* questions : 3×5

(a) Prove the following recurrence relation for the central moment μr of a Bin (n, p) distribution—

$$\mu_{r+1} = pq\left(nr\mu_{r-1} + \frac{d\mu_r}{dp}\right)$$

(b) Three dice are rolled. If no two show the same face, what is the probability that at least one is an ace ?

(c) The two random variables x and y have the joint *pdf*

$$f(x,y)=\frac{1}{2x^2y} \text{ if } 1<x<\infty,\ \frac{1}{x}<y<x$$

$$=0, \text{ otherwise}$$

(i) Derive the marginal distributions of x and y. 3

(ii) further obtain the conditional distribution of y given $x = y$. 2

(d) If for a discrete random variable x,

$$\frac{f(x)}{f(x-1)}=\frac{n-x+1}{x}\cdot\frac{p}{q},\quad x=1,2,\ldots,n, 0<p<1, q=1-p$$

find the *pmf* of x.

(e) In a doll factory, machine M, M_2 and M_3 manufacture, respectively, 45, 25 and 30 percent of the total output. Of their output, 6, 8 and 3 percent respectively, are defective.

A doll is chosen at random from the produce and found to be defective. What is the probability that it was manufactured by M_1 ?

(f) Find the mean and standard deviation of a Poisson (λ) distribution.

Group—B

[Marks : 45]

3. Answer any *two* questions : 2×10

(a) Explain, with suitable examples, the distinction.

(i) between an attribute and a variable and

(ii) between a discrete variable and a continuous variable. 2+2

(iii) What is ratio chart ? Discuss its uses and point out its advantages over a simple line diagram. 2+4

(b) (i) Let s and R be, respectively, the standard deviation and range of a set of n values of x. Show that

$$\frac{R^2}{2n} \leq s^2 \leq \frac{R^2}{4}$$ 5

(ii) Define mean deviation. Prove that mean deviation about mean cannot exceed the standard deviation. 5

(c) (i) Define correlation coefficient. Prove that $-1 \leq r \leq 1$, where r is the correlation coefficient between x and y.

(ii) Derive the effect of change of origin and scale on the correlation coefficient. 2+4+4

(d) (i) Define multiple correlation and partial correlation, and indicate how they differ from simple (or total) correlation. 4

(ii) Define Spearman's rank correlation coefficient clearly stating the assumptions, derive the values for two extreme situations. 6

4. Answer any *five* questions : 5×5

(a) For a frequency distribution the upper class boundary bears a constant ratio r to the lower class boundary. If x_i and fi be respectively the classmark and the frequency of the i-th class and G be the geometric mean of the distribution, show that

$$\log G = \log x_1 + \frac{\log r}{n}\sum_{i=1}^{k}(i-1)fi, n = \sum_{i} fi$$

(b) Let x be a variable assuming positive values only. Show that the arithmetic mean of the square-root of x cannot be greater than the square-root of its arithmetic mean.

(c) Obtain the mean and standard deviation of the first n natural numbers.

(d) State Bowley's measure of skewness. Prove that it lies between -1 and $+1$. 2+3

(e) What do you mean by kurtosis of a frequency distribution. Describe different types of kurtosis. State a measure of kurtosis. 2+2+1

(f) Explain with suitable examples;

frequency, relative frequency, cumulative frequency, frequency density.

(g) What is correlation ration? Show that $0 \leq r^2 \leq e^2yx \leq 1$.

2+3

(h) Suppose x and y are related by the following function—

$$3x + 4y = 10.$$

There are n pairs of values are given. What will be the value of the correlation coefficient? 2

(i) Let, there are three variable x_1, x_2 and x_3. The correlation coefficient between x_1 and x_2, $r_{12} = 0{\cdot}768$; between x_1 and x_3, $r_{13} = 0{\cdot}719$; between x_1 and x_3, $r_{23} = 0{\cdot}520$.

Find the partial correlation coefficients of x_1 and x_2 eliminating the effect of x_3 and correlation coefficient of x_1 and x_3 eliminating the effect of x_2. 2½+2½

(j) Describe the situations of a 2×2 contingency table—

Positive association, negative association and independence.

Group—C

[Marks : 20]

5. Answer any *one* questions. 1×10

(a) (i) What are the different tests of consistency that an index number formula should satisfy? 5

(ii) Why is Fisher's index number known as an ideal index number? 5

(b) (i) Describe different components of a time series.

(ii) Describe the method of fitting a quadratic trend equation $T_t = a + bt + ct^2$ by the method of least squares.

(iii) How can this trend equation be used in forecasting? 4+4+2

6. Answer any *two* questions. 2×5

(a) Define Cost of Living Index number or Consumer Price index number. Discuss its different uses.

(b) Write down the different uses of statistical techniques relating to agriculture. Name two principal publications of statistical offices in the states.

(c) Describe the method of ratio to trend method for determining the seasonal indices of a time series.

(d) Write a short note on moving average method.

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