UG/III/MATH/H/VII/18(New)

2018

MATHEMATICS

[Honours]

PAPER – VII

Full Marks : 90

Time : 4 hours

The figures in the right hand margin indicate marks

[NEW SYLLABUS]

GROUP – A

(*Elements of Computer Science*)

[*Marks* : 30]

1. Answer any *two* questions : 8 × 2

(*a*) (*i*) Write down the block diagram of a computer. What is the purpose of memory in a computer ? 4

(*Turn Over*)

(*ii*) What are source program and object program ? Define a compiler and an interpreter. State the difference between them. 4

(*b*) (*i*) Let

$$B = \{1, 2, 5, 7, 10, 14, 35, 70\},$$

the set of all positive integral divisors of 70. Let

$x + y$ = least common multiple of x and y for all $x, y \in B$.

$x \cdot y$ = greatest common divisor of x and $y, \forall x, y \in B$ and $x' = 70/x \; \forall x \in B$.

Show that $\{B, +, \cdot, 1\}$ is a Boolean Algebra. 4

(*ii*) Write a flowchart to determine the minimum value from a list of numbers. 4

(*c*) (*i*) Write down Huntington postulates in Boolean algebra. 4

(*ii*) Describe NOR and NAND gates. Explain why they are called universal gate. 4

2. Answer any *two* questions : 4 × 2

(*a*) Explain IF statement with block diagram and suitable example. 4

(*b*) Give a brief description of while loop and do-while loop with example in C. 4

Or

Give a brief description of Logical-IF and Block-IF statement in FORTRAN-77.

(*c*) Distinguish between computed GOTO and assign GOTO statements with examples in FORTRAN-77. 4

Or

Explain 'while' loop and 'do-while' loop with examples in C.

3. Answer any *two* questions : 3 × 2

(*a*) Write a program to determine the prime factors of a number. 3

(*b*) Write short notes on Do-Statement in FORTRAN-77 or for loop in C. 3

(*c*) Write a short note on 'Call by value' and 'Call by reference' in C. 3

Or

What are the difference between FUNCTION subprogram and SUBROUTINE Subprogram in FORTRAN-77. 3

GROUP – B

(*Mathematical Theory of Probability*)

[*Marks* : 35]

4. Answer any *one* question : 15 × 1

(*a*) (*i*) Let the probability p_n that a family has n children be αp^n, when $n \geq 1$ and let $p_0 = 1 - \alpha p(1 + p + p^2 + ...)$ be the probability that a family has no child. Suppose that a child is as likely to be a male as to be a female. Show that for

$k \geq 1$, the probability that a family contains exactly k boys is

$$\frac{2\alpha p^k}{(2-p)^{(k+1)}}.$$

Given that a family includes at least one boy, what is the probability that there are two or more boys ? 5

(*ii*) If X is Poisson distributed with parameter μ, then prove that

$$P(X \leq n) = \frac{1}{n!}\int_{\mu}^{\infty} e^{-x} x^n dx,$$

where n is any positive integer. 5

(*iii*) An urn contains a white and b black balls. Balls are drawn one by one until only those of the same colour are left. What is the probability that they are white ? 5

(*b*) (*i*) Define characteristic function of a random variable X. Find the characteristic function of a continuous random variable X with probability density function

$$f(x) = \begin{cases} 1-|x|, & \text{if } |x|<1 \\ 0\ , & \text{elsewhere.} \end{cases}$$ 1 + 4

(*ii*) State the central limit theorem for the case of equal components and show that the theorem implies the Law of Large numbers. Explain why the central limit theorem does not hold for the sequence of independent random variable $\{X_i\}$ $i = 1, 2 \ldots\ldots$ where the random variable X_i follows the Cauchy distribution ? 1 + 2 + 2

(*iii*) A point a is fixed in the interval (0, 1). A random variable X is uniformly distributed in that interval. Find the correlation co-efficient between X and the distance Y between X and a. For what value of a, X and Y are uncorrelated ? 3 + 2 (*c*) (*i*) Let X is a continuous random variable having spectrum $(-\infty, \infty)$ and distribution function $F(x)$. Show that the expected value of X is given by

$$E(X) = \int_0^\infty \{1 - F(x) - F(-x)\} dx$$

provided

$$x\{1 - F(x) - F(-x)\} \to 0 \text{ as } x \to \infty.$$ 5

(*ii*) A Secretary writes four letters and the corresponding addresses on envelopes. He inserts the letters in the envelopes at random irrespective of the address. What is the probability that all letters are wrongly placed? Also find the probability that one letter is placed in right envelope. $2\frac{1}{2} + 2\frac{1}{2}$

(*iii*) If $X_1, X_2, \ldots, X_n$ are n mutually independent standard normal variates then prove that

$$X_1^2 + X_2^2 + \ldots + X_n^2$$

is χ^2 distributed with n degrees of *freedom.* 5

5. Answer any *two* questions : 8 × 2

(*a*) (*i*) Let X and Y be independent random variables representing the length and width of a rectangle with probability density functions :

$$f(x)=\begin{cases}\frac{1}{10}; & 100<x<110\\ 0\ ; & \text{elsewhere}\end{cases}.$$

$$f(y)=\begin{cases}\frac{1}{2}; & 10\le y\le 12\\ 0\ ; & \text{elsewhere}\end{cases}.$$

Find the probability density functions of perimeter of the rectangle. 5

(*ii*) Two numbers are selected at random from the set $\{1, 2, 3, ... n\}$. What is the probability that the absolute difference between the first and second chosen number is $\geq m$, a positive integer ? 3

(*b*) (*i*) Show that the 1st absolute moment about the mean for the normal (m, σ) distribution is

$$\sqrt{\frac{2}{\pi}}\sigma.$$ 4

(*ii*) The joint probability density function of the random variables X, Y is given by

$$f(x, y) = k(3x + y), 1 < x < 3, 0 < y < 2.$$
$$= 0, \text{ elsewhere.}$$

Find the value of the constant k and $P(X + Y < 2)$. Examine if X and Y are independent. 1 + 2 + 1

(*c*) (*i*) If $f(x, y) = xe^{-x(y+1)}$, $(x \geq 0, y \geq 0)$, find the marginal and the conditional density functions. 4

(*ii*) Let X be a $N(0, 1)$ variate, and $Y = X^2$. Show that $\rho_{XY} = 0$, though X and Y are not independent. 4

6. Answer any *one* question : 4 × 1

(*a*) Let X and Y respectively denote the number of heads and the longest run of heads in four tosses of a coin. Compute the means and variances. 4

(*b*) For any two events A and B, show that 2 + 2

$$\text{Max}[P(A), P(B)] \leq P(A \cup B) \leq \text{Min}\{P(A) + P(B), 1\}$$

and $P(A) + P(B) - 1 \leq P(A \cap B) \leq \text{Min}[P(A), P(B)]$

GROUP –C

(*Mathematical Statistics*)

[*Marks* : 25]

7. Answer any *one* question : 15 × 1

(*a*) (*i*) Define the terms : Sample and Sampling distribution of a statistic. Explain briefly, how the distribution of sample is the statistical image of the distribution of the population. 1 + 2 + 2 (*ii*) Consider a sample of size n without replacement from a finite population of size N with variance σ^2. Show that the standard error of the sample mean is

$$\frac{\sigma}{\sqrt{n}} \cdot \sqrt{\frac{N-n}{N-1}}.$$ 5

(*iii*) To test the claim of a book publisher that m, the average number of misprints per page of a book is 1. If a randomly chosen page of that book contains more than two misprints, then the null hypothesis $H_0 : m = 1$ is rejected. What are the probability of type-I and type-II errors ? Find the power of the test, when the alternative hypothesis $H_1 : m = 2$.

2 + 2 + 1

(*b*) (*i*) A population is defined by the density function

$$f(x\,;\alpha) = \frac{x^{l-1} e^{-x/\alpha}}{\Gamma(l)\,\alpha^{l}}, 0 < x < \infty,$$

l is positive constant. Show that an approximate confidence interval for $\alpha > 0$ (when the sample size n is large) with confidence coefficient $1-\in$ $(0<\in<1)$ is

$$\left(\frac{\bar{x}}{l}-\frac{u_{\in}}{\sqrt{nl}}\cdot\frac{\bar{x}}{l},\ \frac{\bar{x}}{l}+\frac{u_{\in}}{\sqrt{nl}}\frac{\bar{x}}{l}\right)$$ where $u_{\in}$

is given by $\frac{1}{\sqrt{2\pi}}\int_{u_{\in}}^{\infty}\bar{e}^{\,t^2/2}dt=\in/2.$ 5

(*ii*) Let X_1, X_2, X_3 be a random sample of size 3 (independent) from a population with mean μ and variance σ^2. T_1 T_2 and T_3 are the estimators used to estimate μ where

$$T_1=X_1-X_2+X_3,\ T_2=2X_1-3X_3+2X_2,$$
$$T_3=(\lambda_1 X_1+X_2+X_3)/3.$$

(*a*) Are T_1 and T_2 unbiased estimators ?

(*b*) Find λ such that T_3 is unbiased.

(*c*) Which is the best estimator ? 1 + 2 + 2 (*iii*) It is required to estimate the mean of a normal population using a sample sufficiently large so that the probability that the sample mean will not differ from the population mean by more than 25% of the population standard deviation will be 0·95. How large should be the sample?

(Given $\frac{1}{\sqrt{2\pi}}\int_{1.96}^{\infty} \overline{e}^{t^2/2}dt = 0\cdot025$). 5

8. Answer any *one* question : 8 × 1

(*a*) (*i*) What is Chi-square test of goodness of fit ? The values of the variable x and the corresponding frequencies f in a sample of size 200 are given below :

x :	0	1	2	3	4	5	6	7	8	9
f :	18	19	23	21	16	25	22	20	21	15

Test the hypothesis that all the values of x in the above are equally likely. [Given that $P(\chi^2 > 14\cdot68) = 0\cdot10$ for *9 degrees of* freedom). 1 + 4 (*ii*) If X_1, X_2, ... X_n are n random samples drawn from a finite population of size N with mean μ and variance σ^2, and $\bar{x}$ is the mean of samples, then show that $Var(\bar{X}) = \frac{\sigma^2}{N}$, if the sampling with replacement, where

$$\bar{X} = \frac{1}{n}(X_1 + X_2 + ... + X_n).$$ 3

(*b*) (*i*) A Computer operator while calculating the coefficient between two variates x and y for 25 pairs of observations obtained the following constants : $n = 25$, $\Sigma x = 125$, $\Sigma x^2 = 650$, $\Sigma y = 100$, $\Sigma y^2 = 460$, $\Sigma xy = 508$. It was, however, later discovered at the time of checking that he had copied down two pairs as (6, 14) and (8, 6) while the correct pairs were (8, 12) and (6, 8). Obtain the correct value of the correlation coefficient. 4

(*ii*) Find the sampling distribution of the mean for the gamma population. 4

9. Answer any *one* question : 2 × 1

(*a*) What do you mean by null hypothesis and alternative hypothesis ? 2

(*b*) If the standard deviation of 1, 2, 3, ..., k is 2. Then find k. 2