 UG/III/MATH/H/VI/18 (New)

2018

MATHEMATICS

[Honours]

PAPER – VI

Full Marks : 90

Time : 4 hours

The figures in the right hand margin indicate marks

[NEW SYLLABUS]

GROUP – A

(*Rigid Dynamics*)

[*Marks* : 30]

1. Answer any *three* questions : 8 × 3

(*a*) Let A, B, C, D, E, F be the moments and product of inertia with respect to a system of three rectangular axes passing through 0,

(*Turn Over*)

then show that the moment of inertia of the material system about any line through 0 whose direction cosines are l, m, n is $Al^2 + Bm^2 + Cn^2 - 2Dmn - 2Enl - 2Flm$. 8

(*b*) State D'Alembert's principle. A uniform rod of length 5m is freely movable on a rough inclined plane of inclination γ to the horizon and whose coefficient of friction is μ, about a smooth pin fixed through the one end; the rod is held in the horizontal position in the plane and allowed to fall from this position. If α be the angle through which its fall from rest, show that $\mu \cot \gamma = \dfrac{\sin \alpha}{\alpha}$. 2 + 6

(*c*) A hollow cylinder, of radius '*a*', is fixed with its axis horizontal and inside it moves a solid cylinder of radius '*b*' whose angular velocity in the lowest position is Ω. If the friction between the cylinders be sufficient to prevent any sliding, the least velocity of projection in order that the cylinder may roll completely is

$$\Omega = \sqrt{\frac{11g(a-b)}{3}}.$$ 8

(*d*) A rough uniform rod of length '$2a$', is placed on a rough table at right angle to its edge, if its centre of gravity be initially at a distance 'b' beyond the edge, show that the rod will begin to slide when it has turned through an angle

$$\tan^{-1}\frac{\mu a^2}{a^2+9b^2},$$

where μ is the coefficient of friction. 8

(*e*) If a rigid body moves under the action of a system of conservative forces, then show that the sum of its kinetic and potential energies remain constant throughout its motion. 8

2. Answer any *two* questions : 3 × 2

(*a*) Calculate the moment of inertia of a solid sphere about a diameter. 3

(*b*) State and prove the principle of conservation of angular momentum for finite forces. 1 + 2

(*c*) An elliptic lamina can rotate about a horizontal axis passing through a focus and perpendicular to its plane. If the eccentricity of the ellipse be $\sqrt{\frac{2}{5}}$, prove that the centre of oscillation will be at other focus. 3

GROUP – B

(*Hydrostatics*)

[*Marks* : 25]

3. Answer any *two* questions : 8 × 2

(*a*) A hallow cone, without weight, closed and filled with some liquid, is suspended from a point in the rim of its base; if θ be the angle at which the direction of the resultant pressure makes with the vertical then show that

$$\cot\theta = \frac{28\cot\alpha + \cot^3\alpha}{48}$$

α being the semi-vertical angle of the cone. 8

(*b*) A vessel contains n different liquids resting in a horizontal layers and densities $\rho_1, \rho_2, \dots, \rho_n$ respectively, starting from the highest liquid. A triangle is held with its base in the upper surface of the highest liquid, and with its vertex in the nth liquid. Prove that, if Δ be the area of the triangle and $h_1, h_2, \dots, h_n$ be the depths of the vertex below the upper surface of the 1st, 2nd, ..., nth liquids respectively, the thrust on the triangle is

$$\frac{1}{3}\frac{g\Delta}{h_1^2}\left[\rho_1\left(h_1^3 - h_2^3\right) + \rho_2\left(h_2^3 - h_3^3\right) + \cdots + \rho_n h_n^3\right]$$ 8

(*c*) Deduce the necessary and sufficient conditions for equilibrium of a fluid under the action of external forces of components (X, Y, Z) per unit mass acting at the point (x, y, z) in the fluid. Is this condition true for irrotational field of force ? Justify. 6 + 2

4. Answer any *three* questions : 3 × 3

(*a*) What happens to the position of the centre of pressure if the plane area is lowered infinitely ? 3

(*b*) Show that in a conservative field of force, the surface of equipressure and equipotential energy coincides. 3

(*c*) Equal weights of gold, silver and an alloy of gold and silver are dipped successively in a cylinder of water and cause water to rise a, b, c inches respectively. Prove that the alloy contains gold and silver in the proportion $(b-c):(c-a)$ by weight. 3

(*d*) Find the thrust of heavy homogeneous liquid on plane surface. 3

(*e*) A circular cylinder of radius 'a' and length $\frac{a}{n}$ is floating in water. If ρ be the specific gravity of the cylinder then find the distance between the centre of buoyancy and centre of gravity. 3

GROUP – C

(*Discrete Mathematics*)

[*Marks* : 20]

5. Answer any *one* question : 15 × 1

(*a*) (*i*) Prove that a connected planner graph with n vertices and e edges has $e-n+2$ regions. 6

(*ii*) What is Hasse diagram ? Draw the Hasse diagram for ≤ relation on {0, 2, 5, 10, 11, 15}. 4

(*iii*) Using generating functions solve the recurrence relation :

$$a_n = 5a_{n-1} - 6a_{n-2} \text{ for } n \geq 2$$

with initial conditions $a_0 = 6$ and $a_1 = 30$. 5

(*b*) (*i*) Prove that a nonempty connected graph G is Eulerian iff its vertices are all of even degree. 5

(*ii*) Define tree and spanning tree. Prove that an undirected graph is a tree, if and only if, there is a unique simple path between every pair of vertices. 5

(*iii*) If $(A, \leq)$ and $(B, \leq)$ be two partially order sets then prove that $(A \times B, \leq)$ is partially order set with partial order $\leq$ defined by $(a, b) \leq (a', b')$ if $a < a'$ in A and $b < b'$ in B. 5

6. Answer any *one* question : 3 × 1

(*a*) Show that the relation '$\subseteq$' defined on the power set $P(A)$ is a partial order relation. 3

(*b*) Prove that the set D of all factors of 12 under divisibility forms a lattice. 3

7. Answer any *one* question : 2 × 1

(*a*) Let $(L, \wedge, \vee)$ be a lattice and $x, y, z \in L$. Then show that L satisfies the following distributive in equalities : 2

(*i*) $x \wedge (y \vee z) \geq (x \wedge y) \vee (x \wedge z)$

(*ii*) $x \vee (y \wedge z) \leq (x \vee y) \wedge (x \vee z)$

(*b*) Show that the maximum number of edges in a simple graph with n vertices is

$$\frac{n(n-1)}{2}.$$ 2

GROUP – D

(*Mathematical Modelling*)

[*Marks* : 15]

8. Answer any *one* question : 15 × 1

(*a*) (*i*) Write the equation $\ddot{x}+2b\dot{x}+w^2x=0$ $(b>0)$ into the form of a linear plane autonomous system $\dot{x}=Ax$. Determine the critical point(s) and its nature. Discuss the stability of the critical point for $b>w$ and draw the corresponding phase portrait of the system. 1 + 2 + 2 + 2 (*ii*) Show that the two-species model represented by

$$\frac{dx}{dt} = x(4 - x - y), \ \frac{dy}{dt} = y(15 - 5x - 3y), \ x, y \geq 0$$

has a position of equilibrium, this position is stable and two species can coexist. 8

(*b*) (*i*) A prey-predator model satisfies differential equation :

$$\frac{dx}{dt} = x(a - by)$$

$$\frac{dy}{dt} = -y(p - qx)$$

with $x(0) = x_0, y(0) = y_0$, where a, b, p, q are positive constants and $x(t), y(t)$ are the population of prey-predator at time t.

Find the equilibrium position of these equations. 5

(*ii*) Find the solution and give the interpretation of the logistic growth model equation

$$\frac{dx}{dt} = \alpha x\left(1 - \frac{x}{k}\right), \alpha > 0, k > 0.$$ 5

(*iii*) What do you mean by mathematical modelling ? Formulate the simple epidemic (SI) model and show that ultimately susceptibility vanishes. 5