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2018

MATHEMATICS

[Honours]

PAPER – IV

Full Marks : 90

Time : 4 hours

The figures in the right hand margin indicate marks

[NEW SYLLABUS]

GROUP – A

(*Analytical Dynamics of Particles*)

[*Marks* : 40]

1. Answer any *one* question : 15×1

(*a*) (*i*) A heavy particle is attached to the lower end of an elastic string, the upper end of which is fixed. The modulus of elasticity of the string is equal to the weight of the

(*Turn Over*)

particle. The string is drawn vertically down length till it is four times its natural length and is then let go. Show that, the particle will return to this point in time.

$$\sqrt{\frac{a}{g}}\left(2\sqrt{3}+\frac{4\pi}{3}\right),$$

where a is the unstreched length of the string. 8

(*ii*) A particle of mass m moves in a straight line under a force mn^2 (distance) towards a fixed point in the straight line and under a small resistance to its motion equal to $m\mu$ (velocity) ; find the period of oscillation. What happen when $\mu > 2n$? 5 + 2

(*b*) (*i*) For a particle moving in a central orbit under the inverse square law (μ/r^2), prove that

(*I*) The angular momentum about the centre of force is constant.

(*II*) The velocity at any distance r is given by $v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right)$.

(*III*) Find also the period of time in the elliptic orbit. 3+3+2

(*ii*) If a particle moves in a plane with central acceleration P and an acceleration T perpendicular to P, find the differential equation of its path. 7

2. Answer any *two* questions : 8×2

(*a*) A particle of mass m is projected vertically upwards under gravity, the resistance of the air being mk times the velocity. Show that the greatest height attained by the particle is

$$\frac{V^2}{g}[\lambda - \log(1+\lambda)],$$

where V is the terminal velocity of the particle and λV is the initial velocity. 8

(*b*) A spherical raindrop of radius a cms falls from rest through a vertical height h, receiving through-out the motion an accumulation of condensed vapour at the rate of K

grammes per square cm. per second, no vertical force but gravity acting; prove that when it reaches the ground its radius will be

$$K\sqrt{\frac{2h}{g}}\left[1+\sqrt{1+\frac{ga^2}{2hK^2}}\right]$$ 8

(c) The base of a rough cycloidal arc is horizontal and its vertex downwards; a bead slides along it starting from rest at the cusp and coming to rest at the vertex. Show that $\mu^2 e^{\mu\pi}=1$, μ is the coefficient of friction. 8

3. Answer any *three* questions : 3×3

(a) If the angular velocity about origin be a constant ω, deduce the cross-radial component of rate of change of acceleration of the particle and show that if this rate of change of acceleration is zero, then

$$\frac{d^2r}{dt^2}=\frac{1}{3}\omega^2 r$$

(b) If v_1 and v_2 are the linear velocities of a planet when it is respectively nearest and farthest from the sun, prove that

$$(1-e)v_1 = (1+e)v_2$$

(*c*) Give the geometrical interpretation of the quantity *h* in case of central orbit.

(*d*) A particle falls under gravity in a resisting medium whose resistance is varies as the square of the velocity; show that the particle would not actually acquire the terminal velocity.

(*e*) A curve is described by a particle having a constant acceleration in a direction inclined at a constant angle to the tangent; show that the curve is an equiangular spiral.

GROUP – B

(*Analytical Statics*)

[*Marks* : 30]

4. Answer any *three* questions : 8×3

(*a*) Define Wrench and Pitch. Show that any system of forces acting on a rigid body can be reduced to a Wrench. 2+6

(*b*) Define angle of friction and cone of friction. A solid homogeneous hemisphere rest on a rough horizontal plane and against a smooth vertical wall, show that if coefficient of friction is greater than 3/8, the hemisphere can rest in any position and, if it is less, the least angle that the base of the hemisphere can make with the vertical is $\cos^{-1}\left(\frac{8}{3}\mu\right)$. If the wall is rough (coefficient of friction μ') show that the angle is

$$\cos^{-1}\left(\frac{8}{3}\mu\cdot\frac{1+\mu'}{1+\mu\mu'}\right).$$ 2 + 6

(*c*) State the principle of virtual work of a rigid body. A heavy uniform rod of length $2a$, rests with its ends in contact with two smooth inclined planes of inclination α and β to the horizon. If θ be the inclination of the rod to the horizon, prove by the principle of virtual work, that

$$\theta = \tan^{-1}\left[\frac{1}{2}(\cot\alpha - \cot\beta)\right].$$ 2+6

(*d*) Define central axis. Two equal forces act along each of the straight lines

$$\frac{x \mp a\cos\theta}{a\sin\theta} = \frac{y - b\sin\theta}{\mp b\cos\theta} = \frac{z}{c};$$

show that their central axis must, for all values of θ, lie on the surface

$$y\left(\frac{x}{z} + \frac{z}{x}\right) = b\left(\frac{a}{c} + \frac{c}{a}\right). \qquad 1+7$$

(*e*) (*i*) Find the co-ordinates of the centre of gravity of a lamina in the shape of a quadrant of the curve $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$, density being given by $\rho = kxy$. 5

(*ii*) Determine the equation of the line of action of resultant for a system of coplanar forces. 3

5. Answer any *two* questions : 3×2

(*a*) When is a system of coplanar forces acting on a rigid body said to be in astatic equilibrium ? Find the condition for a given system of coplanar forces to be astatic equilibrium. 3

(*b*) Find the condition of equilibrium of a particle constrained to rest on a rough curve $y = f(x)$ under any given forces. 3

(*c*) Find the centre of gravity of the arc of the cardioid $r = a(1 + \cos\theta)$ lying above the initial line. 3

GROUP – C

(*Differential Equation*-II)

[*Marks* : 20]

6. Answer any *one* question : 15 × 1

(*a*) (*i*) Find the power series solution of the equation

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} + 2y = 0$$

about $x = 1$ with $y(1) = 1, y'(1) = 2$. 8

(*ii*) Find Laplace transform of e^{at}. 2

(*iii*) Solve the following :

$$x^2 p + y^2 q = (x + y)z,$$

where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$. 5

(*b*) (*i*) Find the general solution of the following system :

$$\frac{dx}{dt} - 4x - y = 0$$

$$\frac{dy}{dt} + 8x - 8y = 0$$ 5

(*ii*) Using the Laplace transform of integration to find $f(t)$ where Laplace transform is

$$\mathscr{L}\{f(t)\} = F(p) = \frac{1}{p^2}\left(\frac{p-2}{p^2+4}\right)$$ 5

(*iii*) Using convolution theorem for inverse Laplace transform, deduce the formula

$$\int_0^1 t^{a-1}(1-t)^{b-1}\, dt = \frac{\Gamma(a)\,\Gamma(b)}{\Gamma(a+b)};$$

$a > 0, b > 0$ 5

7. Answer any *one* question : 3×1

(*a*) Show that the point (x, y) determined from

$$\frac{dx}{dt} = -ay, \text{ and } \frac{dx}{dt} = ax$$

lies on a circle. 3

(*b*) What are the basic differences between ODE and PDE ? Give examples of each case. 3

8. Answer any *one* question : 2×1

(*a*) Find Laplace transform of the function

$$f(t-a) = \begin{cases} 1, \text{ if } t > a \\ 0, \text{ if } t < a \end{cases}$$ 2

(*b*) Show that the condition for exactness of the ordinary differential equation.

$$\mu(x, y)\, M(x, y)dx + \mu(x, y)\, N(x, y)dy = 0$$

is a linear partial differential equation of order one. 2

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