PG/IS/MTM-102/12

M.Sc. 1st Semester Examination, 2013

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Complex Analysis*)

PAPER—MTM-102

Full Marks : 50

Time : 2 hours

Answer **Q. No. 4** and any **two** from the rest

The figures in the right-hand margin indicate marks

1. (*a*) Show by an example that a function

$$f(z) = u(x, y) + iv(x, y)$$

ceases to be differentiable at the point (x_0, y_0) in the domain C even if the Cauchy-Riemann equations are satisfied at that point. 4

(*b*) Let

$$f(z) = \frac{x^3 - y^3}{x^2 + y^2} + i\,\frac{x^3 + y^3}{x^2 + y^2}, \quad z = x + iy \neq 0$$
$$= 0, \quad z = 0$$

Show that though C-R equations are satisfied at (0, 0) but $f'(0)$ does not exist. 4

(c) When is a function $f(z)$ said to have a pole of order m at z_0? If a function $f(z)$ has a pole of order m at z_0, prove that $\frac{1}{f(z)}$ has a zero of order m at z_0. 4

(d) If $f(z) = Z^5 - 3iz + 2z - 1 + i$, evaluate

$$\int_C \frac{f'(z)}{f(z)} dz$$

Where C encloses all the zeroes of $f(z)$. 4

2. (a) Expand $f(z) = \sin z$ in a Taylor series about $z = \frac{\pi}{4}$. 4

(b) Find the number of zeroes of the polynomial

$$z^4 - 5z + 1$$

in the annulus $1 < |z| < 2$. 4

(*c*) Find all the Möbius transformation which transforms the half plane $I(z) \geq 0$ onto the unit circular disc $|w| \leq 1$. 4

(*d*) Prove that the given function

$$f(z) = \frac{z^8 + z^4 + 2}{(9z^2 + 12z + 4)(z-1)^3}$$

has three singularities. 4

3. (*a*) Evaluate

$$\oint_C \frac{dz}{z-a}$$

where C is any simple closed curve and $z = a$ is outside C. 4

(*b*) State and prove Rouche's theorem. 4

(*c*) Evaluate the following by the method of contour integration (any *two*) : 4 × 2

(*i*) $\int_0^{2\pi} \frac{dx}{s + 3\sin x}$

(*ii*) $\int_0^{\infty} \frac{\sin x}{x(1+x^2)} dx$

(*iii*) $\int_{-\infty}^{\infty} \frac{dx}{x^2+x+1}$

(*iv*) $\int_0^{\infty} \frac{x^{a-1}}{1+x}\,dx,\ 0<a<1.$

4. Answer any *four* questions of the following : 2 × 4

(*a*) Find Res $f(z)$ at $z=0$ where

$$f(z)=\frac{z-3}{z^2}\sin\frac{1}{1-z}$$

(*b*) When is a function $f(z)$ said to be analytic in a given domain in the complex z-plane ?

(*c*) Locate and name the singularity of

$$f(z)=\frac{z}{(z^2+4)^2}.$$

(*d*) Construct the analytic function $w=f(z)$ if its real part is $e^x\cos y$ and if $f(0)=1$.

(*e*) Evaluate

$$\int_C \frac{\cos z}{z^3},$$

where C is a positively oriented closed curve around the origin.

(*f*) Define residue theorem.

[*Internal Assessment* : 10 Marks]