M.Sc. 4th Semester Examination, 2012

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER – MTM- 401

(*Topology/Data Structure and Design and Analysis of Algorithms*)

Full Marks : 50

Time : 2 hours

The figures in the right hand margin indicate marks

GROUP – A

(*Topology*)

[*Marks* : 25]

1. Answer any *two* questions : 1 × 2

(*a*) Consider the topology on $X = \{a, b, c, d, e\}$. Find the closed subsets of X.

(*b*) Define a bicontinuous mapping.

(*c*) Define Tychonoff space.

2. Answer any *three* questions : 4×3

(*a*) Let $\mathcal{T}$ be the collection of subsets of N consisting of null set φ and all subsets of the form

$$G_m = \{m, m+1, m+2, ...\}$$

for all $m \in N$. Show that $\mathcal{T}$ is a topology.

(*b*) Prove that on the set of all real numbers the upper limit topology is finer than the usual topology.

(*c*) Let $\{\mathcal{T}_\lambda : \lambda \in \Lambda\}$, where Λ is arbitrary index set, be the arbitrary collection of topologies for X. Prove that :

$$\bigcap_{\lambda \in \Lambda} \mathcal{T}_\lambda$$

is a topology.

(*d*) Define the derived set A^d of a subset A of a topological space. If $\overline{A}$ denotes the closure of A, prove that

$$\overline{A} = A \cup A^d.$$

(*e*) Prove that a topological space X is a T_1-space if and only if every singleton subset of X is closed.

3. Answer any *one* question : 6 × 1

(*a*) Let f be a mapping from R to R defined by $f(x) = x^2$ for all $x \in R$. Find whether f is

(*i*) $\mathscr{U}$ - $\mathscr{U}$ continuous
(*ii*) $\mathscr{S}$ - $\mathscr{U}$ continuous
(*iii*) S - $\mathscr{U}$ continuous

where $\mathscr{U}, \mathscr{S}, S$ are respectively usual, indiscrete and lower limit topologies. 2 + 2 + 2

(*b*) Prove that every convergent sequence in a Hausdorff space has a unique limit. Is the converse true ? Justify your answer. 2 + 2 + 2

[*Internal Assessment* : 5 Marks]

GROUP – B

(*Data Structure and Design and Analysis of Algorithms*)

[*Marks* : 25]

Answer **Q.No. 4** and any *two* from the rest

4. Answer any *two* questions : 2 × 2

(*a*) Define the data structure "queue". How it differ from stack ?

(*b*) Describe a method to represent a graph into computer.

(*c*) Define big oh and small oh notations which are used to analysis an algorithm.

(*d*) Define heap.

5. Write merge sort algorithm to arrange a set of real numbers in ascending order. What is the time complexity of this algorithm ? 7 + 1

6. Write an algorithm for performing linear search over a linked list with one link field in each node. What is the time complexity of this algorithm ? 7 + 1

7. What do you mean by postfix expression ? Write an algorithm to evaluate a postfix expression. 1 + 7

8. (*a*) Write a recursive algorithm to traverse a binary tree in postorder, inorder and preorder.

(*b*) Write a general algorithm for divide-and-conquer method and find an expression to find time complexity of this algorithm. 4 + 4

[*Internal Assessment* : 5 Marks]