M.Sc. 3rd Semester Examination — 2012

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Dynamical Meteorology*-I/*Operational Research Modelling*-I)

PAPER—MTM-306

Full Marks : 50

Time : 2 hours

The figures in the right-hand margin indicate marks

(*Dynamical Meteorology*-I)

Answer **Q. No. 1** and any *four* from the rest

1. Answer any *two* questions : 2 × 2

(*a*) Define equivalent potential temperature.

(*b*) Find the relation between mixing ratio and specific humidity.

(*c*) What is the planetary vorticity ?

2. (*a*) Define potential temperature. Show that it is invariant in adiabatic motion for an air parcel in the atmosphere. 3

(*b*) Derive the saturated adiabatic lapse rate of moist air and hence show that it is less than dry adiabatic lapse rate. 6

3. (*a*) Derive the area equivalence of temphigram and discuss its important features. 7

(*b*) Explain geo-dynamical paradox. 2

4. (*a*) Derive the equation of motion of an air parcel in the atmosphere in spherical co-ordinate system. 7

(*b*) What is relative humidity ? 2

5. (*a*) Obtain the atmospheric energy equation and interpret each term. 7

(*b*) What do you mean by entropy and isentropic process ? 2

6. Define circulation and find the rate of circulation of an air parcel in the atmosphere. Interpret each term. 9

7. (*a*) Derive the effect of ascent and descent of an air parcel on lapse rate in terms of changes in pressure. 6

(*b*) Explain the convergence and divergence in the atmosphere. 3

8. (*a*) Explain the weather forecasting. Derive the formula to predict the potential temperature due to advection using finite difference. 2 + 5

(*b*) At $t = 0$, $\theta_{32} = 10\ ^\circ$C. At $t = 15$ min, $\theta_{22} = 11\ ^\circ$C, $\theta_{32} = 10 \cdot 1\ ^\circ$C, $\theta_{42} = 9\ ^\circ$C, $u_{22} = u_{32} = 5$ m/s. Numerically forecast the potential temperature θ_{32} at $t = 30$ min, assuming $\Delta x = 50$ km. 2

[*Internal Assessment* = 10 Marks]

(*Operational Research Modelling*-I)

Answer **Q. No. 1** and any *four* from the rest

1. Answer any *four* questions : 2 × 4

(*a*) What do you mean by network analysis ? What is its advantage ?

(*b*) State Belman's principle of optimality.

(*c*) Write a brief note on Monte Carlo simulation.

(*d*) State mortality theorem related to replacement of items.

(*e*) Give a real example for each of the following queueing system

(*i*) M/M/1 : ∞/FCFS/∞

(*ii*) M/M/1 : N/FCFS/∞.

(*f*) What do you mean by Economic order quantity.

2. Use dynamic programming method to solve the following problem :

$$\text{Minimize } z = \sum_{j=1}^{n} f_j(y_j)$$

Subject to the constraints

$\sum_{j=1}^{n} a_j y_j \geq b$, a_j and b are real numbers, $a_j \geq 0$, $y_j \geq 0$, $b > 0$, $j = 1, 3..., n$. 8

3. A small project is composed of seven activities whose time estimates (in weeks) are listed in the following table :

Activity	Optimistic time	Most likely time	Pessimistic time
1-2	1	1	7
1-3	1	4	7
1-4	2	2	6
2-5	1	1	2
3-5	2	5	10
4-6	2	5	8
5-6	3	6	12

(*i*) Draw the project network.

(*ii*) Find the expected duration and variance of each activity.

(*iii*) Calculate the earliest and latest occurrence time for earth event and the expected project length.

(*iv*) What is the probability that the project will be computed at least 4 weeks earlier than expected ?

(*v*) If the project due time is 19 weeks, what is the probability of meeting the due date ? 8

4. Describe a suitable method to generate a set of random numbers. Describe Monte Carlo simulation to find the value of π. 5 + 3

5. Find the optimum order level which minimizes the total expected cost under the following assumptions : 8

   (*i*) t is the constant interval between order.

   (*ii*) Q is the stock (in continuous units) at the beginning.

   (*iii*) r is the estimated random instantaneous demand of a continuous rate.

   (*iv*) Rs C_1 and Rs C_2 are the holding and shortage costs per items per t time period.

   (*v*) No set-up cost.

   (*vi*) Lead time is negligible.

6. Derive the differential-difference equations for M/M/1 : N/FCFS/∞ queueing system in steady state.

   At time zero, all the items in a system are new. Each item has a probability p of failing immediately before the end of first month of life and probability q $(= p - 1)$ of failing immediately before the end of second month.

If all items are replaced as they fail, show that the expected number of failures $f(x)$ at the end of x month is given by

$$f(x) = \frac{N}{1+q}\left[1-(-q)^{x+1}\right]$$

where N is the number of items in the system.

If the cost per item of individual replacement policy is Rs. C_1 and the cost per item of group replacement policy is Rs. C_2. Find the condition under which group replacement policy at the end of first month is most profitable over individual replacement. 8

7. Find the optimum order quantity for the following information :

Quantity	Unit price (Rs.)
$0 < Q < 200$	6·00
$200 \le Q < 1000$	5·70
$1000 \le Q$	5·40

The monthly demand is 600 units, the storage cost is 20% of unit cost and the cost of ordering is Rs. 10 per order. 8

[*Internal Assessment* = 10 Marks]