M.Sc. 3rd Semester Examination — 2012

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Partial Differential Equations*)

PAPER—MTM-301

Full Marks : 50

Time : 2 hours

Answer **Q. No. 1** and any **two** from the rest

The figures in the right-hand margin indicate marks

1. Answer any *two* questions : 4 × 2

(*a*) Let $u(x, t)$ be a solution of the wave equation $u_{tt} - c^2 u_{xx} = 0$, which is defined in the whole plane. Assume that u is constant along the line $x = 2 + ct$. Prove that $u_t + cu_x = 0$.

(*b*) Find the characteristics of the equation $pq = z$ and determine the integral surface which passes through the parabola $x = 0, y^2 = z$.

(c) Solve the equation $(p^2 + q^2)y = qz$ by Charpit's method.

2. (a) Prove that the equation $x^2u_{xx} - 2xyu_{xy} + y^2u_{yy} + xu_x + yu_y = 0$ is parabolic and find its canonical form. Find the general solution on the half-plane $x > 0$. 8

(b) (i) Let $u(x, t)$ be the solution of the cauchy problem

$$u_{tt} - 9u_{xx} = 0, -\infty < x < \infty, t > 0,$$

$$u(x, 0) = \begin{cases} 1, & |x| \le 2 \\ 0, & |x| > 2, \end{cases}$$

$$u_t(x, 0) = \begin{cases} 1, & |x| \le 2 \\ 0, & |x| > 2. \end{cases}$$

Find $u\left(0, \frac{1}{6}\right)$. Discuss the large time behaviour of the solution. 4

(ii) Consider the cauchy problem

$$u_{tt} - c^2u_{xx} = F(x, t), -\infty < x < \infty, t > 0$$
$$u(x, 0) = f(x), u_t(x, 0) = g(x), -\infty < x < \infty$$

Suppose that f, g are odd functions and for every $t \geq 0$ the function $F(., t)$ is odd too. Then prove that for every $t \geq 0$, the solution u(., t) of the cauchy problem is also odd. 4

3. (*a*) Using the method of seperation of variables solve the following problem : 8

$$u_{tt} - u_{xx} = \cos(2\pi x)\cos(2\pi t),\ 0 < x < 1,\ t > 0$$
$$u_x(0, t) = u_x(1, t) = 0,\ t \geq 0$$
$$u(x, 0) = f(x) = \cos^2(\pi x),\ 0 \leq x \leq 1$$
$$u_t(x, 0) = g(x) = 2\cos(2\pi x),\ 0 \leq x \leq 1.$$

(*b*) (*i*) Consider the heat equation $u_t - u_{xx} = 0$, $x \in R$, $t \geq 0$.

Set $u(x, t) = \phi(\lambda(x, t))$, where $\lambda(x,t) = \dfrac{x}{2\sqrt{t}}$.

Show that u is a solution of the heat equation iff $\phi(\lambda)$ is a solution of the *ODE* $\phi'' + 2\lambda\phi' = 0$, where $' = \dfrac{d}{d\lambda}$. 4

(*ii*) Give an example to show that the cauchy problem for the laplace equation is not well-posed. 4

4. (*a*) Establish poisson's formula for the dirichlet problem of the laplace equation in a disk of radius a. 8

(*b*) (*i*) State and prove strong maximum principle. 6

(*ii*) Show that the laplace operator is a self-adjoint operator. 2

[*Internal Assessment* : 10 Marks]