PG/IIIS/MTM-301/11

M.Sc 3rd Semester Examination, 2011

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-301

(*Partial Differential Equations*)

Full Marks : 50

Time : 2 hours

Answer **Q. No. 1** and any **two** from the rest

The figures in the right-hand margin indicate marks

1. Answer any *two* questions : 4 × 2

(*a*) Explain the Charpit's method for solving the partial differential equation :

$$F(x, y, z, p, q) = 0.$$

(*b*) Show the uniqueness property of the solution of one dimensional wave equation.

(*Turn Over*)

(*c*) Find the general integral of the equation

$$(x-y)p+(y-z-x)q=z$$

and the equation of the integral surface of the differential equation which passes through the circle $z=1,\ x^2+y^2=1$.

2. (*a*) Classify the following PDE and reduce the equation to its canonical form and hence solve it : 8

$$y^2 z_{xx} - 2xy\, z_{xy} + x^2 z_{yy} = \frac{y^2}{x} z_x + \frac{x^2}{y} z_y$$

(*b*) Solve the Cauchy problem

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = F(x,t)$$

subject to the initial conditions

$$u(x,0)=\eta(x),\ \frac{\partial u(x,0)}{\partial t} = v(x)$$

by Riemann-Volterra method. 8

3. (*a*) Show that every solution $u(x,t)$ of the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2};\ 0 \le x \le l,\ 0 \le t \le T$$

defined and continuous in the rectangle $Q: 0 \le x \le l,\ 0 \le t \le T$ takes as its maximum and minimum values on the lower base ($t = 0$) and the vertical sides $x = 0$ and $x = l$. 8

(*b*) Find, for $t \ge 0$, a continuous and bounded function $u(x, t)$ which satisfies the heat conduction equation

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$$

for $t > 0$ and for $t = 0$ it becomes equal to a given continuous and bounded function 8

$$\phi(x)\ \ \forall x\ (-\infty < x < \infty).$$

4. (*a*) Find the equation of the vibration of an infinite string. 8

(*b*) Obtain the D'Alembert solution of the Cauchy problem

$$u_{tt} = c^2 u_{xx},\ -\infty < x < \infty,\ t > 0$$

$$u(x, 0) = f(x),\ \ u_t(x, 0) = g(x).$$ 8

[*Internal Assessment* : 10 Marks]