M.Sc 1st Semester Examination, 2011

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-103

(*Ordinary Differential Equations and Special Functions*)

Full Marks : 50

Time : 2 hours

Answer **Q. No. 1** and any **three** from **Q. No. 2** to **Q. No. 5**

The figures in the right-hand margin indicate marks

1. Answer any *five* questions : 2 × 5

(*a*) Write down the hypergeometric series represented by $F(a, b, c, z)$. Prove that

$$F(1,\ b,\ b,\ z) = \frac{1}{1-z}.$$

(*b*) Show that $J_n(z)$ is an odd function of z if n is odd.

(*c*) What do you mean INDICIAL equation concerning ODE ?

(*d*) Let $P_n(z)$ be the Legendre polynomial of degree n and let

$$P_{m+1}(0) = -\frac{m}{m+1} P_{m-1}(0), \quad m = 1,\ 2, \ldots$$

If $P_n(0) = -\frac{5}{16}$, then find the value of

$$\int_{-1}^{1} P_n^2(z)\, dz.$$

(*e*) What do you mean by fundamental matrix of system of linear homogeneous differential equation ?

(*f*) Show that the Green's function of a given problem is everywhere continuous.

2. (*a*) How do you solve the homogeneous vector differential equation in the form

$$\frac{dx}{dt} = Ax,$$

where $x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$ and $A = (aij)_{n \times n}$ matrix.

[Assuming that the eigenvalues of A are all real and distinct.] 5

(*b*) The Legendre polynomial $P_n(z)$ satisfies the differential equation

$$\left(1 - z^2\right) \frac{d^2 w}{dz^2} - 2z \frac{dw}{dz} + n(n+1)w = 0$$

and $P_n(1) = 1$. By changing the independent variable, show that

$$P_n(z) = F\left(n+1, -n, 1, \frac{1-z}{2}\right)$$

where $F(a, b, c, z)$ is the hypergeometric function. 5

3. (*a*) Show that

$$J_0^{\,2}(z) + 2\sum_{n=1}^{\infty} J_n^{\,2}(z) = 1$$

and prove that for real z,

$$|J_0(z)| \le 1 \text{ and } |J_n(z)| < \frac{1}{\sqrt{2}},$$

for all $n \ge 1$. 4

(*b*) Show that the solution of the differential equation

$$\frac{d^2u}{dx^2} = f(x)$$

subject to the boundary conditions $u(0) = u(a) = 0$ is given by

$$u(x) = \int_0^a G(x, \xi)\, f(\xi)\, d\xi$$

where

$$G(x, \xi) = \begin{cases} \dfrac{-x(a-\xi)}{a}, & 0 \le x \le \xi \\ \dfrac{\xi(a-x)}{a}, & \xi \le x \le a \end{cases}.$$ 6

4. (*a*) Prove that all the eigenvalues of a regular Sturm-Liouvilla system with non-negative weight function, are real. 4

(*b*) Deduce Rodrigues formula for Legendre's polynomial. 4

(*c*) Locate and classify the singular points of the following differential equation : 2

$$z^2(z^2-1)^2\,\omega''-z(1-z)\,\omega'+2\omega=0.$$

5. (*a*) Deduce the integral formula for the hypergeometric function. 4

(*b*) Find the series solution near $z=0$ of

$$(z+z^2+z^3)\,\omega''(z)+3z^2\,\omega'(z)-2\omega(z)=0.$$ 6

[*Internal Assessment* : 10 Marks]
