M.Sc 1st Semester Examination, 2011

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-102

(*Complex Analysis*)

Full Marks : 50

Time : 2 hours

Answer **Q. No. 4** and any **two** from the rest

The figures in the right-hand margin indicate marks

1. (*a*) If $z = re^{i\theta}$ and $f(z) = u(r, \theta) + iv(r, \theta)$, obtain the Cauchy-Riemann relation in terms of r and θ. 4

(*b*) Find the harmonic conjugate function of $u(x, y) = 4xy + x + 1$ and thus construct the corresponding analytic function

$$f(z) = u(x, y) + iv(x, y)$$

for which $f(1) = 2 + i$. 4

(*Turn Over*)

(*c*) Establish the following : 4

(*i*) The poles of an analytic function are isolated.

(*ii*) A function analytic everywhere including the point at infinite is constant.

(*d*) Show that, under suitable conditions, to be stated by you

$$f(a) = \frac{1}{2\pi i} \int_C \frac{f(z)dz}{z-a}$$

where C is a closed contour surrounding the point $z = a$. 4

2. (*a*) Evaluate :

$$\oint_C \frac{e^z}{z^2 + \pi^2} dz$$

where C is the circle $|z| = 4$. 4

(*b*) Show that : 4

$$\int_{-\infty}^{\infty} \frac{x \cos x}{x^2 + 1} dx = 0.$$

(*c*) Show that :

$$w = \frac{5-4z}{4z-2}$$

transform $|z| = 1$ into a circle in the w-plane, find the centre and radius of this circle. 4

(*d*) Using Rouche's theorem prove that the seven zeros of

$$3z^7 + 5z - 1 = 0$$

lie in the interior of the circle $|z| = 2$. 4

3. (*a*) If

$$f(z) = \frac{1}{z(z-1)^2},$$

expand $f(z)$ in Laurent series at $z = 1$. Name the singularity. 4

(*b*) Define a singularity point of a complex function. Find the singular points and residues there at for the function 4

$$f(z) = \frac{e^{1/z}}{(z+1)^2}.$$

(*c*) Evaluate the following by the method of contour integration (any *two*) : 4 × 2

(*i*) $\int_{-\infty}^{\infty} \frac{dx}{x^4+1}$

(*ii*) $\int_{-\infty}^{\infty} \frac{\cos x}{x^2+1} dx$

(*iii*) $\int_{0}^{2\pi} \frac{d\theta}{(5+4\cos\theta)^2}$.

4. Answer the following : 2 × 4

(*a*) If

$$f(z) = \frac{x^2 y^5 (x+iy)}{x^4 + y^{10}}, \quad z \neq 0$$

$$= 0 \quad , \quad z = 0$$

verify whether Cauchy-Riemann relation are satisfied at the origin or not.

(*b*) Evaluate :

$$\int_C \frac{e^{2z}}{z^2}\,dz,$$

where C is the positively oriented contour $|z| = 1$.

(*c*) Prove that

$$f(z) = \text{Real}\,(z)$$

is nowhere differentiable.

(*d*) Construct the function

$$f(z) = u + iv$$

where $u = \tan^{-1}(y/x)$ and $f(1) = 0$.

[*Internal Assessment* : 10 Marks]
