

M.Sc 1st Semester Examination, 2011

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-101

(*Real Analysis*)

Full Marks : 50

Time : 2 hours

Answer **Q. No. 1** and any **three** from **Q. No. 2** to **Q. No. 6**

The figures in the right-hand margin indicate marks

1. Answer any *two* questions : 2 × 2

(*a*) If $f(x) = 2x$ and $g(x) = 5$ find the R.S. integral

$$\int_3^7 f(x)\, dg(x).$$

(*b*) What do you mean by σ-algebra of sets.

(*c*) Write a norm for $BV([a, b])$ i.e., the vector space of all functions of bounded variation on $[a, b]$.

(*Turn Over*)

2. (*a*) Define a function of bounded variation over a closed interval $[a, b]$.

(*b*) Show that the function $f(x)$ defined on $[2, 5]$ by

$$f(x) = 3 \text{ for rational } x \text{ in } [2, 5]$$
$$= 4 \text{ for irrational } x \text{ in } [2, 5]$$

is not of bounded variation on $[2, 5]$.

(*c*) Prove that addition of two functions of bounded variation over $[a, b]$ is also so over $[a, b]$.

(*d*) Prove that a function of bounded variation on $[a, b]$ is bounded on $[a, b]$. 2 + 3 + 4 + 3

3. (*a*) Define rectifiable path in $\mathbb{R}^n$.

(*b*) Let $\vec{f} : [a, b] \to \mathbb{R}^n$ be a path in $\mathbb{R}^n$ with components $\vec{f} = (f_1, f_2, ..., f_n)$. Show that $\vec{f}$ is rectifiable if and only if each component f_K is of bounded variation on $[a, b]$.

(*c*) Let $f : [a, b] \to \mathbb{R}$ and $g : [a, b] \to \mathbb{R}$ be functions with $0 < f(x) < g(x) \forall x \in (a, b)$ and $f(a) = g(a), f(b) = g(b)$.

Let $\vec{h} : [a, 2b-a] \to \mathbb{R}^2$ be defined by

$$\vec{h}(t) = (t,\ f(t)) \text{ if } a \le t < b$$
$$= (2b-t,\ g(2b-t)) \text{ if } b \le t \le 2b-a.$$

Show that $\vec{h}$ is a rectifiable path in $\mathbb{R}^2$. 2 + 5 + 5

4. (*a*) Establish 'Cauchy condition' for Riemann-Steiltjes integrability.

(*b*) Let $f : [a, b] \to \mathbb{R}$ and $g : [a, b] \to \mathbb{R}$ be functions. Show that if f is Riemann-Steiltjes integrable with respect to g on $[a, b]$, then g is also Riemann-Steiltjes integrable with respect of f on $[a, b]$ and

$$\int_a^b f(x)dg(x) = f(b)g(b) - f(a)g(a) - \int_a^b g(x)df(x).$$

6 + 6

5. (*a*) Let $f : [a, b] \to \mathbb{R}$ be Riemann integrable. Show that the points of discontinuity of f in $[a, b]$ has measure zero.

(*b*) Let X be a nonempty set. Let $\mu : \mathcal{P}(X) \to [0, \infty)$ be an outer measure. Show that the set of all measureable sets with respect to μ is a σ-algebra.

6 + 6

6. (*a*) Prove that every bounded Riemann integrable function over $[a, b]$ is Lebesgue integrable and the two integrals are equal. Is the converse true ? Explain your answer by an example.

(*b*) Prove that every bounded measurable function on $[a, b]$ is Lebesgue integrable on $[a, b]$. 6 + 6

[*Internal Assessment* : 10 Marks]