M.Sc 2nd Semester Examination, 2011

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Functional Analysis*)

PAPER —MTM - 205

Full Marks : 50

Time : 2 hours

Answer **Q.No.1** and **2** and any **four** from **Q.No.3** to **Q.No.8**

The figures in the right-hand margin indicate marks

1. Answer any *two* : 2×2

(*a*) State the Banach fixed point theorem.

(*b*) State the open mapping theorem.

(*c*) Define a Hausdorff metric space.

2. Answer any *one* : 8

(*a*) Prove that $C[a, b]$ is a normed linear space with norm

$$\|x\| = \max\{x(t) : a \leq t \leq b\}.$$

(*b*) Let $f(x, y)$ and $\frac{\partial f}{\partial y}$ be continuous in the closed rectangle
$D = \{(x, y): a_1 \leq x \leq a_2,\ b_1 \leq y \leq b_2\}$ and (x_0, y_0) be an interior point of D. Prove that the differential equation $\frac{dy}{dx} = f(x, y)$ has a unique solution $y = g(x)$ which passes through (x_0, y_0).

3. (*a*) Prove that every normed linear space is a Metric space.

(*b*) Let $T: X \to Y$ be a linear operator where X and Y are normed linear spaces. If T is continuous at the particular part x_0 of X then prove that T is continuous at any point x of X.

4 + 3

4. Let $B(X, Y)$ be the set of all bounded linear transformations from nls X into nls Y. Assumeing that $B(X, Y)$ is a nls, prove that it is complete if y is complete. 7

5. Let X be a real normed linear space and M be a subspace of X. If f is a bounded linear functional defined on M then prove that there exists a bounded linear function g on E such that $f(x) = g(x)$ for all $x \in M$ and $\| g \| = \| f \|$. 7

6. State and prove that closed graph theorem. 7

7. (*a*) Prove that inner product is a continuous function.

(*b*) State and prove the Pythagorean theorem in an inner product space. 3 + 4

8. (*a*) Define an Adjoint operator of a bounded linear operator.

(*b*) Prove that :

$$(\lambda T_1 + \mu T_2)^* = \bar{\lambda} T_1^* + \bar{\mu} T_2^*.$$

(*c*) Prove that $T^* T$ is a positive operator. 1 + 3 + 3

[*Internal Assessment* : 10 Marks]