PG/IIS/MA-1204/11

M.Sc 2nd Semester Examination, 2011

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Continuum Mechanics*)

PAPER—MTM - 204

Full Marks : 50

Time : 2 hours

Answer **Q.No.1** and any **four** from the rest

The figures in the right-hand margin indicate marks

1. Answer any *two* questions : 4x2

(*a*) Show that the difference of the values of a two dimensional stream function at the two points represents the flux of a fluid across any curve joining the points ?

(*b*) Deduce the constitutive equation of a perfect fluid.

(*c*) For the displacement field $u_1 = X_1^2 X_2$, $u_2 = X_2 - X_3^2$, $u_3 = X_2^2 X_3$, determine the unit relative displacement vector at $P(1, 2, -1)$ with respect to $Q(4, 2, 3)$.

2. Show that the velocity field $u(x, y) = \frac{B(x^2 - y^2)}{(x^2 + y^2)^2}$, $v(x, y) = \frac{2Bxy}{(x^2 + y^2)^2}$ and $w = 0$, satisfies the Euler equation of motion in the absence of external forces for an inviscid incompressible flow. Also determine the pressure associated with this velocity field where B is constant. 8

3. State and prove Kelvin's Circulation theorem for a perfect fluid. Hence show that the fluid motion of once irrotational is always irrotational. 8

4. Define principal strains and principal directions of strain. Prove that all principal strains are real and principal directions of strain corresponding to the distinct principal strains are orthogonal. 8

5. (*a*) Show that in two dimensional irrotational motion, stream function satisfies Laplace's equation. 2

(*b*) Define doublet and find the complex potential for a doublet. 1 + 2

(*c*) Stress tensors at a point are given in appropriate units by

$$t_{ij} = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 4 & 0 \\ 2 & 0 & 2 \end{bmatrix}.$$

Find the stress vector at a point P on the plane through P parallel to the plane $2x_1 - 2x_2 - x_3 = 0$. 3

6. (*a*) Find the stream line and path line of a fluid motion for the velocity field: 3

$$v_1 = \frac{x_1}{1+t},\ v_2 = x_2,\ v_3 = 0.$$

(*b*) Prove that :

$$\frac{x_1^2}{a_1^2 k^2 t^4} + kt^2 \left(\frac{x_2^2}{a_2^2} + \frac{x_3^2}{a_3^2} \right) = 1$$

is a possible form of the boundary surface of the liquid. 5

7. (*a*) Find the change in volume due to strain deformation. 5

(*b*) Write down the stress invariants and why these are called invariant? 3

[*Internal Assessment* — 10 Marks]
