PG/IIS/MA-1203/11

M.Sc 2nd Semester Examination, 2011

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM - 203

Full Marks : 50

Time : 2 hours

The figures in the right-hand margin indicate marks

GROUP—A

(*Abstract Algebra*)

[*Marks* : 25]

Answer **Q.No.1** and any **two** questions from the rest

1. Answer any *two* questions : 2x2

(*a*) State Cayley's theorem for finite group G. 2

(*b*) Define conjugate classes for a group G with two examples. 2

(*c*) Define prime ideal in a ring R with two examples. 2

2. (*a*) Define normal subgroup. Show that a Kernal of a group homomorphism is always a normal subgroup. 1 + 3

(*b*) State and prove first isomorphism theorem for a group G. 4

3. (*a*) Show that a group G be a direct product of subgroup H and K iff (*i*) every $x \in G$ can be uniquely expressed as $x = hk$, $h \in H$, $k \in K$. (*ii*) $hk = kh$, $h \in H$, $k \in K$. 4

(*b*) Let R be the field of real numbers. Then show that the only isomorphism of R onto R is the identity mapping I_R. 4

4. (*a*) Define simple ring with an example. Show that any field is a simple ring. 1 + 3

(*b*) Let R be a finite integral domain. Then prove that R is a field. 4

[*Internal Assessment* : 5 Marks]

GROUP—B

(*Linear Algebra*)

[*Marks* : 25]

Answer **Q.No.5** and any **two** from the rest

The symbols have their usual meanings

5. Answer any *two* questions : 2×2

(*a*) A linear transformation $T: E^n \to E^1$ is non-null i.e., $T(x) \neq \theta$, $\forall\ x \in E^n$, where E^n is the n-dimensional Euclidean space. Find the dimension of the null space of T.

(*b*) Let T be a linear mapping on a vector space V such that $T^2 = \theta$. Find the relationship between ker T and $R(T)$.

(*c*) Define complete lattice with an example. Also give an example of a lattice which is not complete.

6. (*a*) Is there a linear transformation $T: R^3 \to R^2$ such that $T(1, 0, 3) = (1, 1)$ and $T(-2, 0, -6) = (2, 1)$? Justify your answer.

(*b*) Prove that for any lattice, the distributive inequalities hold.

(*c*) Let

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}, B = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

Find the minimal polynomial of

$$\begin{bmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{bmatrix}.$$

3 + 2 + 3

7. (*a*) Give the definition of lattice with respect to poset and also give the definition of lattice with respect to algebra. Show that the two definitions are equivalente.

(*b*) Let $T: V \to W$ be a linear transformation and dim $V = n$. Then prove that the following are equivalent.

(*i*) T is injective

(*ii*) Rank of $T = n$

(*iii*) $\beta = \{ v_1, v_2, \ldots, v_n \}$ is a basis of $V \Longrightarrow T(\beta) = \{ T(v_1), T(v_2), \ldots, T(v_n)\}$ is a basis of Im T , i.e., dim V = dim T.

5 + 3

8. (*a*) Let V and W be vector spaces, and let $T: V \to W$ be linear. Then prove that T is one–to–one iff $N(T) = \{ \theta \}$.

(*b*) For the following linear transformations T, determine whether T is invertible and justify your answer.

$T: M_{2\times 2}(R) \to P_2(R)$ defined by

$$T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = a + 2bx + (c + d)x^2.$$

(*c*) Let F be a field and let V be the set of all polynomials in x of degree $(n-1)$ or less over F. Define a mapping $T: V \to V$ and $T(f) = f'$. Find the matrix of T with respect to the bases $\{ 1, x, x^2, \ldots, x^{n-1)} \}$ 3 + 2 + 3

[*Internal Assessment* : 5 Marks]
