 PG/IIS/MA-1202/11

M.Sc 2nd Semester Examination, 2011

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Numerical Analysis*)

PAPER—MTM-202

Full Marks : 50

Time : 2 hours

Answer **Q. No. 1** and any **two** from the rest

The figures in the right-hand margin indicate marks

1. Answer any *four* questions : 2×4

(*a*) What are the differences between single step and multistep methods to solve a differential equation ?

(*b*) What are the basic differences between Newton-Cotes method and Gaussian quadrature method ?

(*Turn Over*)

(*c*) Express the polynomial

$$5x^3 - 4x^2 + 3x + 8$$

interms of Chebyshev polynomial.

(*d*) Is LU decomposition method applicable for all system of linear equations? Explain.

(*e*) Prove that

$$\delta f(x) = 2\sin h\left(\frac{hD}{2}\right) f(x)$$

where δ and D and central difference and differential operators respectively.

(*f*) Compare direct and iteration method to solve a system of linear equations.

2. (*a*) Define cubic spline. Deduce cubic spline interpolation formula. 2 + 6

(*b*) Explain Runge-Kutta fourth order method to solve the following system of differential equations:

$$\frac{dy}{dx} = f(x, y, z), \; \frac{dz}{dx} = g(x, y, z)$$

$y(x_0) = y_0$ and $z(x_0) = z_0$. 3

(*c*) Given the following pair of differential equations :

$$\frac{dy}{dx} = \frac{x+y}{z} \text{ and } \frac{dz}{dx} = xy + z$$

with initial condition $x_0 = 0{\cdot}5$, $y_0 = 1{\cdot}5$ and $z_0 = 1$. Find y and z for $x = 0{\cdot}6$, using fourth order Runge-Kutta method. 5

3. (*a*) Explain finite difference method to solve the following equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2},\ t > 0,\ 0 < x < 1,$$

where initial conditions $u(x, 0) = f(x)$ and $\left(\frac{\partial u}{\partial t}\right)_{(x,0)} = g(x),\ 0 < x < 1$ and boundary conditions $u(0, t) = \phi(t),\ u(1, t) = \psi(t),\ t \geq 0$. Draw the suitable mesh points. 8

(*b*) Explain an efficient method to solve a tri-diagonal system of equations containing n

variables and n equations. Use this method to solve the following equations : 5 + 3

$$\begin{aligned} x_1 + 2x_2 \quad\quad &= 5 \\ -2x_1 + 3x_2 + 4x_3 &= 3 \\ 4x_2 - x_3 &= 6. \end{aligned}$$

4. (*a*) Deduce 4-point Gauss-Legendre quadrature formula. Use this method to find the value of 5 + 3

$$\int_1^2 \left(x^3 + e^x\right) dx.$$

(*b*) Describe least square method to approximate a function $y = f(x)$ with the help of orthogonal polynomials. What is the advantage to use orthogonal polynomials than other polynomials ? 6 + 2

[*Internal Assessment* : 10 Marks]