M.Sc. 3rd Semester Examination, 2011

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-306

(*Dynamical Meteorology* -I/*Operational Research Modelling* -I)

Full Marks : 50

Time : 2 hours

The figures in the right-hand margin indicate marks

(*Dynamical Meteorology* -I)

Answer **Q. No. 1** and any **four** from the rest

1. Answer any *two* questions : 2 × 2

(*a*) What is the first law of thermodynamics ?

(*b*) Define the virtual temperature.

(*c*) Write short note on geo-dynamical paradox.

2. (*a*) What is the thermal wind in the atmosphere ? Derive the thermal wind components in the atmosphere. 7

   (*b*) Derive the hydrostatic equation in the atmosphere. 2

3. Derive the thermodynamic energy equation in the atmosphere and what is represented by this equation ? 9

4. Derive the relation from which saturation temperature can be calculated when saturation of air will be done by adiabatic ascent and in this case estimate the height at which saturation will be attend. 9

5. (*a*) Obtain the momentum equation of an air parcel in the atmosphere in Cartesian co-ordinate system. 7

   (*b*) What is Coriolis force ? 2

6. (*a*) Find the vorticity equation in the atmosphere. 7

   (*b*) Determine the mixing ratio from the relative humidity and the saturation mixing ratio approximately. 2

7. (*a*) What is meant by global circulation in the atmosphere ? Derive the meridional temperature gradient in circulation. 2 + 3

(*b*) Derive the hypsometric equation. 4

8. (*a*) Deduce the equation of state for moist air in the following form :

$$p\alpha = \frac{R^*}{m_d}\left(\frac{1+\frac{w}{\epsilon}}{1+w}\right)T$$

and show that the virtual temperature is always higher than the actual temperature. 7

(*b*) Show that given volume of moist air is lighter than an equal volume of dry air at the same pressure and temperature. 2

[*Internal Assessment* : 10 Marks]

(*Operational Research Modelling* -I)

Answer **Q. No. 1** and any **four** from the rest

1. Answer any *four* questions : 2 × 4

(*a*) Write a brief note about the service mechanism of a queue.

(*b*) What are the advantages to use the simulation to solve a problem ?

(*c*) In project management, what do you mean by time-cost-trade off ? Define cost slope of an activity.

(*d*) When the group replacement policy is used to replace an item in a company ?

(*e*) What do you mean by lead time and replenishment rate in inventory control ?

(*f*) Under what conditions a real valued function

$$\psi_N(f_N, f_{N-1}, ..., f_2, f_1)$$

is decomposable ? [Assume that f_j is a function of state variable s_j and decision variable $d_j, j = 1, 2, ..., n$].

2. Using dynamic programming, show that

$$z = \sum_{i=1}^{n} p_i \log p_i$$

subject to the constraints

$$\sum_{i=1}^{n} p_i = 1 \text{ and } p_i > 0 \text{ for all } i,$$

is minimum when

$$p_1 = p_2 = \cdots = p_n = \frac{1}{n}.$$ 8

3. Stating the assumptions formulate order level lot size model with shortages to find optimal lot size and order level for which the cost is minimum. 8

4. The cost of a new machine is Rs. 5000.00. The running cost of n year is given by $R_n = 500(n - 1)$, $n = 1, 2, \ldots$. Suppose that the money is worth 5% per year. After how many years will it be economical to replace the machinery with a new one ? 8

5. Derive the differential-difference equations for (M/M/C) : (N/FCFS/∞) queueing system in transient state. 8

6. The following table gives data on normal time and cost, crashed time and cost of all activities of a project :

Activity	Normal Time(days)	Normal Cost(Rs.)	Crashed Time(days)	Crashed Cost(Rs.)
1-2	5	1000	3	2200
1-3	6	1200	3	1950
2-3	3	600	2	1000
2-4	5	800	3	1700
3-5	7	1000	4	2500
4-5	6	1200	2	2400
5-6	8	1600	6	2200

The indirect cost per day is Rs. 350·00.

Find the optimum project time and corresponding minimum total project cost by crashing appropriate activities in proper order. 8

7. The demand for a particular item has the probability distribution shown below :

Daily demand(units) :	4	5	6	7	8	9	10	11	12
Probability :	0·06	0·14	0·18	0·17	0·16	0·12	0·08	0·06	0·03

If the lead time is 5 days, using simulation study the implications of inventory policy of ordering 50 units whenever the inventory at the end of the day is 40 units. Assume the initial stock level of 75 units and run the simulation for 25 days. 8

[*Internal Assessment* : 10 Marks]