M.Sc 3rd Semester Examination, 2011

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-305

(*Dynamical Oceanology* - I /*Advanced Optimization and Operations Research*)

Full Marks : 50

Time : 2 hours

The figures in the right-hand margin indicate marks

(*Dynamical Oceanology* - I)

Answer any *five* questions

1. Derive the following relations :

(*i*) $C_p = C_v + T\left\{\left(\frac{\partial\tau}{\partial T}\right)^2 / \frac{\partial\tau}{\partial p}\right\}$

(*ii*) $\Gamma = \left(\frac{T}{C_p}\right)\frac{\partial\tau}{\partial T}$

(*iii*) $K_\eta = K_T - \Gamma.\alpha = K_T(C_v/C_p)$

where symbols have their usual meanings. 8

2. Assuming that the mass exchange process across the free ocean surface $F(\vec{r}, t) = 0$ amount to a flux, b, of pure water in unit time per unit area, obtain the boundary conditions at the free ocean surface. 8

3. Explain β-plane approximation. Assuming the sea-water to be non-viscous stratified fluid, deduce the β-plane equations and examine the range of validity of these equations. 8

4. Obtain the Reynolds equations for ocean currents by averaging the Boussinesq's equation. Deduce the dynamic and kinematic condition for these equations. 8

5. Assuming the sea-water to be a viscous compressible heat conducting fluid, determine the energy equation in the form

$$\frac{\partial}{\partial t}\left(\zeta E_m\right) = -\text{div } \vec{I}_E$$

where symbols have their usual meanings. 8

6. Show that under usual notations

$$T = -\frac{1}{\lambda},\ \mu_s = -U - \frac{\lambda_s}{\lambda} + \frac{\vec{q}^{\,2}}{2}$$

$$\mu_w = -U - \frac{\lambda_w}{\lambda} + \frac{\vec{q}^{\,2}}{2},\ \vec{q} = -\frac{\vec{a}}{\lambda} - \frac{1}{\lambda}\left(\vec{b} \times \vec{r}\right)$$

are the necessary conditions of thermodynamical equilibrium of a finite volume of sea-water. Hence deduce the hydrostatic pressure equation. 8

7. Obtain the equation of motion of sea-water in the form

$$\frac{d\vec{q}}{dt} = \vec{F} + 2\vec{q} \times \vec{\Omega} - \frac{1}{\rho}\nabla p + \frac{1}{\rho}(\lambda + \mu)\nabla\Theta + \nu\nabla^2\vec{q}$$

where the symbols have their usual meanings. 8

8. Derive the equations for small amplitude wave motion in the ocean. 8

9. Show by method of separation of variables that the problem of free oscillations of the ocean reduces to the determination of the eigenvalue curves of two distinct eigenvalue problems. 8

[*Internal Assessment* : 10 Marks]

(*Advanced Optimization and Operations Research*)

Answer Q. No. 1 and any *two* from the rest

1. Answer any *one* question : 8 × 1

(*a*) Solve the following LPP by dual simplex method :

$$\text{Maximize} \quad Z = -2x_1 - 2x_2 - 4x_3$$

$$\text{subject to} \quad 2x_1 + 3x_2 + 5x_3 \leq 2$$

$$3x_1 + x_2 + 2x_3 \geq 3$$

$$x_1 + 4x_2 + 6x_3 \geq 5$$

$$\text{and} \quad x_1, x_2, x_3 \geq 0.$$

(*b*) Using cutting plane method :

$$\text{Maximize} \quad Z = 1 - 4x_1 - 2x_2$$

subject to

$$2(x_1 - 2)^2 + (x_2 - 3)^2 - 12 \leq 0$$

$$2x_1 + x_2 - 3 \leq 0$$

$$\text{and } 0 \leq x_1, x_2 \leq 5 \text{ with } \epsilon = 0{\cdot}2.$$

2. (*a*) The optimal solution of the LPD

Maximize $Z = 6x_1 - 2x_2 + 3x_3$

subject to $2x_1 - x_2 + 2x_3 \le 2$

$x_1 + 4x_3 \le 4$

and $x_1, x_2, x_3 \ge 0$

is contained in the table

		C_j	6	−2	3	0	0
C_B	Y_B	X_B	Y_1	Y_2	Y_3	Y_4	Y_5
6	Y_1	4	1	0	4	0	1
−2	Y_2	6	0	1	6	−1	2
$Z_j - C_j$		$Z = 12$	0	0	9	2	2

Find the ranges of the cost components when

(*i*) Changed one at a time

(*ii*) Changed two at a time

(*iii*) Changed all three at a time to keep the optimal solution same. 8

(*b*) Use Golden section method to

$$\text{Minimize } f(x) = \begin{cases} 5 - x, & x < 3 \\ 2x - 4, & x \ge 3 \end{cases}$$

in the interval $[0, 5]$. 8

3. (*a*) Apply Kuhn-Tucker conditions to solve the following problem :

Minimize $f(x_1, x_2) = x_1^2 + x_2^2 - 10x_1 - 2x_2 + 26$

subject to the constraints

$$x_1 - x_2^2 - 4 \leq 0$$
$$x_1 + 4x_2 - x_2^2 \leq 7$$ 8

(*b*) Maximize $Z = 7x_1 + 9x_2$

subject to $-x_1 + 3x_2 \leq 6$

$7x_1 + x_2 \leq 35$

$x_1, x_2 \geq 0$ and integers.

Use Branch and Bound method to solve this. 8

4. (*a*) If a quadratic function $Q(X) = \frac{1}{2}X^T AX + B^T X + C$ (A is +ve definite matrix) is minimized sequentially once along each direction of a set of n A-conjugate directions then prove that the global minimum of $Q(X)$ will be located at or before the nth step regardless the starting point and the order in which the directions are used. 8

(*b*) Using Fletcher and Reeves method

Minimize $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$

starting from the point $X_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. 8

[*Internal Assessment* : 10 Marks]