M.Sc. 3rd Semester Examination, 2011

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER – MTM- 303

(*Operations Research/Dynamical Oceanology and Meteorology*)

Full Marks : 50

Time : 2 hours

The figures in the right hand margin indicate marks

(*Operations Research*)

Answer **Q.No.1** and any *two* from the rest

1. Answer any *four* questions : 2 x 4

(*a*) If in the Economic lot size system with uniform demand model, the set up cost instead of being fixed is equal to $C_3 + B_q$, where B is the set-up cost per unit item produced, then show that there is no change in the optimum order quantity produced due to this change in the set-up cost.

(*b*) What are the importance of using post optimality analysis ?

(*c*) State the 'principle of optimality' in dynamic programming.

(*d*) Write the Kuhn-Tucker conditions for the following problem

$$\text{Max. } z = 2x_1^2 + 12x_1x_2 - 7x_2^2$$

$$\text{s.t.} \quad 2x_1 + 5x_2 \leq 98$$

$$x_1, x_2 \geq 0$$

(*e*) Find the equation of the trajectory for which the functional

$$I = \int_{x_0}^{x_1} \left(y'^2 + 5\right) dx$$

will be stationary.

(*f*) Explain the concept of optimal control.

2. (*a*) Describe briefly the Beale's method for solving quadratic programming problem. 8

(*b*) Use Beale's method for solving the quadratic programming problem. 8

$$\text{Max } z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$$

$$\text{s.t.} \quad x_1 + 2x_2 \leq 2$$

$$x_1, x_2 \geq 0.$$

3. (*a*) Use dynamic programming, solve 8

$$\text{Min } \quad z = x_1^2 + x_2^2 + x_3^2$$

$$\text{s.t.} \quad x_1 + x_2 + x_3 \geq 15$$

$$x_1, x_2, x_3 \geq 0.$$

(*b*) Consider the LPP,

$$\text{Maximize } z = 3x_1 + 5x_2$$

$$\text{Subject to} \quad x_1 + x_2 \leq 1$$

$$2x_1 + 3x_2 \leq 1, \quad x_1, x_2 \geq 0.$$

Find the optimum solution. Find the variations of $C_j \, (j = 1, 2)$ which are permitted without changing the optimal solution. 8

4. (*a*) Derive an expression for economic production quantity with uniform rate of repleshment with no shortages. 8

(*b*) The demand of an item is uniform at a rate of 25 units per month. The fixed cost is Rs. 15 each time a production run in made. The production cost is Re. 1 per item, and the inventory carrying cost is Rs. 0.30 per item per month. If the shortage cost is Rs. 1.50 per item per month, determine how often to make a production run and of what size it should be ? 8

5. (*a*) Show that the functional

$$I = \int_{x_0}^{x_1} F(y, y', x)\, dx \;\; ; y' = \frac{dy}{dx}$$

will be stationary only if

$$\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0.$$ 8

(*b*) Use the revised simplex method to solve : 8

Maximize $z = 3x_1 + 5x_2$

Subject to $x_1 \le 4$

$x_2 \le 6$

$3x_1 + 2x_2 \le 18, \quad x_1, x_2 \ge 0.$

[*Internal Assessment* – 10 Marks]

(*Dynamical Oceanology and Meteorology*)

Answer any *five* questions

1. Define salinity. Deduce the following relations for seawater : 2 + 3 + 3

(*i*) $C_v = C_p + T\left(\frac{\partial \tau}{\partial T}\right)^2 \Big/ \left(\frac{\partial \tau}{\partial p}\right),$

(*ii*) $K_\eta = K_T - \Gamma\alpha,$

(symbols have their usual meaning).

2. Deduce the equations of conservation of mass of moving sea water. 8

3. Deduce the equation of motion of sea water as 8

$$\frac{D\vec{q}}{Dt} = \vec{F} + 2\vec{q} \times \vec{\Omega} - \frac{1}{\rho}\vec{\nabla}p* + \gamma\left[\frac{1}{3}\vec{\nabla}\Theta + \nabla^2\vec{q}\right]$$

(with usual notations).

4. Deduce the condition of stable, mechanical equilibrium of a stratified fluid when slightly displaced from its equilibrium in terms of Brunt-Väisälä frequency N and it express in terms of speed of sound in sea water. 6 + 2

5. Define potential temperature of dry air. State and prove hypsometric formula. $2\frac{1}{2} + 5\frac{1}{2}$

6. Define Mixing ratio, Specific humidity, Absolute humidity and Relative humidity. 2 + 2 + 2 +2

7. Mention the fundamental atmospheric forces. Deduce the equations of motion of air parcel in the following forms : 2 + 6

$$\frac{du}{dt} = -(2\Omega w\cos\phi - 2\Omega v\sin\phi) - \frac{1}{\rho}\frac{\partial p}{\partial x} + F_x$$

$$\frac{dv}{dt} = -2\Omega\, u\sin\phi - \frac{1}{\rho}\frac{\partial p}{\partial y} + F_y$$

$$\frac{dw}{dt} = 2\Omega\, u\cos\phi - \frac{1}{\rho}\frac{\partial p}{\partial z} - g + F_z$$

(with usual symbols).

[*Internal Assessment* – 10 Marks]