M.Sc 3rd Semester Examination, 2011

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-302

(*Integral Transforms and Integral Equations*)

Full Marks : 50

Time : 2 hours

Answer **Q. No. 1** and any **three** from the rest

The figures in the right-hand margin indicate marks

1. Answer any *five* questions : 2 × 5

(*a*) What do you mean by Fredholm alternative in integral equation ?

(*b*) Write the sufficient conditions for the existence of the Laplace transform of a function.

(*c*) Define Mellin transform of a function. Find the Mellin transform of $(e^x - 1)^{-1}$.

(*Turn Over*)

(*d*) Define finite Hankel transform of order n of a function $f(r)$, $0 \le r \le a$, and state its inversion formula.

(*e*) The integral of a good function is not necessarily a good function. Justify it.

(*f*) What do you mean by Fourier transform of a function ?

2. (*a*) Find the resolvent kernel of the following integral equation and then solve it : 5

$$y(x) = f(x) + \lambda \int_a^x K(x,t)\, y(t)\, dt.$$

(*b*) Prove that :

$$H_n\left\{\frac{d^2 f}{dr^2} + \frac{1}{r}\frac{df}{dr} - \frac{n^2}{r^2} f\right\} = -\alpha^2 F_n(\alpha)$$

provided both $rf'(r)$ and $rf(r)$ tend to zero as $r \to 0$ and $r \to \infty$ where H_n stands for nth order Hankel transform and $H_n\{f(r)\} = F_n(\alpha)$. 5

3. (*a*) Solve the following ODE by Laplace transform technique :

$$ty''(t) + 2y'(t) + t\,y(t) = \sin t$$

with initial condition $y(0) = 1$. 4

(*b*) State and prove convolution type theorems (both) concerning on Mellin transform. 3

(*c*) Show that the sequence $\left\{ e^{-\frac{x^2}{n}} \right\}$ is regular and defines a generalised function $I(x)$ such that : 3

$$\int_{-\infty}^{\infty} I(x)\,\gamma(x)dx = \underset{n\to\infty}{\text{Lt}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{n}}\,\gamma(x)dx,\ \forall \gamma(x) \in \hat{G}.$$

4. (*a*) If $L\{f(t)\} = F(p)$ which exists for real $(p) > \gamma$ and $H(t)$ is unit step function, then prove that for any α,

$$L\{H(t-\alpha) f(t-\alpha)\} = e^{-p\alpha} F(p)$$

which exists for real $(p) > \gamma$. 3

(*b*) Solve the following boundary value problem in the half plane $y > 0$, described by

$$\text{PDE} \quad : \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \ -\infty < x < \infty, \ y > 0$$

with boundary conditions $u(x, 0) = f(x)$, $-\infty < x < \infty$. u is bounded as $y \to \infty$; u and $\frac{\partial u}{\partial x}$ both vanish as $|x| \to \infty$. 7

5. (*a*) If the Fourier sine transform of $f(x)$ is

$$\frac{\alpha}{1+\alpha^2}$$

then find $f(x)$. 4

(*b*) Find the eigenvalues and eigen functions of the following integral equation : 6

$$y(x) = \lambda \int_0^{2\pi} \sin(x+t)\, y(t)\, dt.$$

[*Internal Assessment* : 10 Marks]