PG/IIIS/A.MATH/2102/10

M.Sc 3rd Semester Examination, 2010

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MA-2102

(*Integral Transform and Integral Equation*)

Full Marks : 50

Time : 2 hours

Answer **Q. No. 1** and any **three** from the rest

The figures in the right-hand margin indicate marks

1. Answer any *five* questions of the following : 2 x 5

(*a*) Define the inversion formula for Fourier sine transform of the function $f(x)$. What happens if $f(x)$ is continuous ?

(*b*) When two regular sequences are equivalence ? What do you mean by generalised function ?

(*Turn Over*)

(*c*) State the convolution theorem of Laplace transform.

(*d*) Find the exponential order of the function e^{t^n}.

(*e*) Define an integral equation. Give an example of non-linear integral equation.

(*f*) Find the Hankel transform of

$$f(x)=\begin{cases}1, & 0<x<a, \quad n=0\\ 0, & x>a, \quad n=0.\end{cases}$$

2. (*a*) Form an integral equation corresponding to the following differential equation

$$\frac{d^2y}{dx^2}+(1-x)\frac{dy}{dx}+e^{-x}y(x)=x^3-5x$$

with the given initial conditions

$$y(0)=-3, \text{ and } y'(0)=4.$$

(*b*) Derive the Laplace transform of a periodic function. Find the Laplace transform of the triangular wave function $f(t)$ which is defined as follows :

$$f(t) = \begin{cases} 1, & \text{if } 0 \le t < c \\ 2c - t, & \text{if } c \le t < 2c \end{cases}$$

where $f(t + 2c) = F(t)$. 5 + 5

3. (*a*) Show that if a function $f(x)$ defined on $(-\infty, \infty)$ and its Fourier transform $F(\xi)$ are both real, then $f(x)$ is even. Also show that if $f(x)$ is real and its Fourier transform $F(\xi)$ is purely imaginary, then $f(x)$ is odd.

(*b*) Obtain the solution of the boundary value problem

$$x^2 u_{xx} + xu_x + u_{yy} = 0, \; 0 \le x < \infty, \; 0 < y < 1.$$

$$u(x, 0) = 0, \; u(x, 1) = \begin{cases} A, & 0 \le x \le 1 \\ 0, & x > 1, \end{cases}$$

where A is a constant, using Melling transform. 4 + 6

4. (*a*) Find the zero order Hankel transform of the function x^2.

(*b*) If a and b are real constants, solve the following integral equation:

$$ax + bx^2 = \int_0^x \frac{y(t)}{(x-t)^{1/2}}\, dt.$$

(*c*) If a real valued function $f(t)$ of real variable which is piecewise continuous in any finite interval of t and is of exponential order $O(e^{\nu t})$ as $t \to \infty$, when $t \geq \underline{0}$ then prove that the integral

$$\int_0^\infty f(t)\, e^{-pt}\, dt$$

converges in the domain Real $(p) > \nu$.

$2\frac{1}{2}+5+2\frac{1}{2}$

5. (*a*) State and prove Parseval's identity on Fourier transform.

(*b*) With help of the resolvent kernel, find the solution of the integral equation

$$y(x) = 1 + x^2 + \int_0^x \left(\frac{1+x^2}{1+t^2}\right) y(t)\, dt. \qquad 5+5$$

[*Internal Assessment* – 10 Marks]