2017

MATHEMATICS

[Honours]

PAPER – IV

Full Marks : 90

Time : 4 hours

The figures in the right hand margin indicate marks

[NEW SYLLABUS]

GROUP – A

(*Analytical Dynamics of Particles*)

[*Marks* : 40]

1. Answer any *one* question : 15 × 1

(*a*) (*i*) A particle, whose mass is m, is acted upon by a force $m\mu\left(x+\dfrac{a^4}{x^3}\right)$ towards the origin; if it starts from rest at a distance a, show that it will arrive at the origin in time $\frac{\pi}{4\sqrt{\mu}}$. 8

(*ii*) Find the path described by a particle with an acceleration which is always directed towards a fixed point and varies directly as the distance from it. 7

(*b*) (*i*) A smooth straight thin tube revolves with uniform angular velocity w in a vertical plane about one extremity which is fixed; if at zero time the tube be horizontal, and a particle inside it be at a distance a from the fixed end, and be moving with velocity V along the tube, show that its distance at time t is,

$$a\cos h(wt)+\left(\frac{v}{w}-\frac{g}{2w^2}\right)\sin h(wt)+\frac{g}{2w^2}\sin wt$$ 7

(*ii*) Find the law of force towards the pole under which the curve $r^n = a^n \cos n\theta$ can be described for $n = \frac{1}{2}$ and $-\frac{1}{2}$. 8

2. Answer any *two* questions : $8 \times$?

(*a*) Find the time of description of a given arc of an elliptic orbit starting from the nearer end of the major axis.

(*b*) A comet is moving in a parabolar about the sun as focus; when at the end of its latus-rectum its velocity suddenly becomes altered in the ratio $n : 1$, where $n < 1$; show that the comet will describe an ellipse whose eccentricity is $\sqrt{1 - 2n^2 + 2n^4}$, and whose major axis is $\frac{l}{1 - n^2}$, where $2l$ was the latus -rectum of the parabolic path.

(*c*) A particle is projected with velocity V from the cusp of a smooth inverted cycloid down the arc; show that the time of reaching the vertex is

$$2\sqrt{\frac{a}{g}}\tan^{-1}\left[\frac{\sqrt{4ag}}{V}\right]$$

where the symbols have their usual meaning.

3. Answer any *three* questions : 3 × 3

(*a*) Define the following terms :

apse, apsidal distance, apsidal angle.

(*b*) What do you mean by terminal velocity ? If a particle falls under gravity (supposed constant) in a resisting medium whose resistance varies as the square of the velocity and starts from rest, show that the particle would not actually acquire the "terminal velocity" untill it had fallen an infinite distance.

(*c*) A particle of mass m moves in a central orbit of attractive force of which the intensity is $mkr^{-2}e^{-r^2}$, where K is a positive constant. Show that a circular orbit of radius r is stable, if $r^2 < \frac{1}{2}$.

(*d*) A particle describes a catenary under a force which acts parallel to its axis; show that the velocity and acceleration at any point both vary as the distance from the directrix.

(*e*) A particle describes a path which is nearly a circle about a centre of force ($=\mu u^n$) at its centre; find the condition that this may be a stable motion.

GROUP – B

(*Analytical Statics*)

[*Marks* : 30]

4. Answer any *three* questions : 8 × 3

(*a*) (*i*) State the conditions of equilibrium of a system of coplanar forces. 2

(*ii*) Show that as the forces are rotated, the value of $\frac{G}{V}$ at any assumed base O is always equal to the tangent of the angle which the straight line joining O to the centre C of the forces makes with the direction of the resultant force R, while the value of $G^2 + V^2$ is invariable and equal to $R^2 \cdot OC^2$. 6

(*b*) (*i*) State Laws of friction. 2

(*ii*) A perfectly rough plane is inclined at an angle α to the horizon; Show that the least eccentricity of the ellipse which can rest on the plane is $\sqrt{\dfrac{2\sin\alpha}{1+\sin\alpha}}$. 6

(*c*) (*i*) Mention two forces which do not appear in the equation of virtual work. 2

(*ii*) Two uniform rods AB, BC of weights W and W' are smoothy jointed at B and their middle points are joined across by a cord. The rods are tightly held in a vertical plane with their ends A, C resting on a smooth horizontal plane. Show by the principle of virtual work that the tension in the cord is $\dfrac{(W+W')\cos ACosC}{\sin B}$. 6

(*d*) (*i*) What do you mean by degrees of freedom ? How many degress of freedom a rigid body has ? Justify your answer. 3

(*ii*) A heavy uniform hemispherical shell to radius *r* has a particle attached to a point on the rim, and rests with the curved surface in contact with a rough sphere of radius *R* at the highest point. Prove that if $\frac{R}{r} > \sqrt{5} - 1$, the equilibrium is stable, whatever be the weight of the particle. 5

(*e*) (*i*) Find the co-ordinates of the centre of gravity of the arc of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ which lies in the positive quadrant. 4

(*ii*) The density of a hemisphere varies as the *n*th power of the distance from the centre; show that the centre of gravity divides the radius perpendicular to its plane surface in the ratio $n + 3 : n + 5$. 4

5. Answer any *two* questions : 3 × 2

(*a*) State the laws of statistical friction.

(*b*) Show that a given system of forces in three dimensions can have only one central axis.

(*c*) State converse of the principle of virtual work.

GROUP – C

(*Differential Equation* - II)

[*Marks* : 20]

6. Answer any *one* question : 15 × 1

(*a*) (*i*) Find the general solution of the following system

$$\frac{dx}{dt} - 7x + y = 0, \quad \frac{dy}{dt} - 2x - 5y = 0.$$ 5

(*ii*) Apply the convolution theorem for Laplace transform to show that

$$\int_0^t \sin u \cos(t-u)\,du = \frac{1}{2} t \sin t.$$ 5

(*iii*) Using Laplace transform of integration to find

$$\mathscr{L}\left(\frac{1-\cos t}{t^2}\right).$$

Hence, deduce that

$$\int_0^\infty \frac{\sin^2 t}{t^2} dt = \pi/2$$ 5

(*b*) (*i*) Apply power series method to solve the following : 9

$$2x(1-x)\frac{d^2y}{dx^2}+(5-7x)\frac{dy}{dx}-3y=0.$$

(*ii*) Solve the following :

$$x(x^2+3y^2)p-y(3x^2+y^2)q=2z(y^2-x^2)$$

where $p \equiv \frac{\partial}{\partial x},\ q \equiv \frac{\partial}{\partial y}$ 6

7. Answer any *one* question : 3 × 1

(*a*) Obtain the partial differential which characterises the set of all spheres $x^2 + y^2 + (z - c)^2 = a^2$ with centres on the z-axis.

(*b*) Find

$$L^{-1}\left\{\log\frac{p+2}{p+3}\right\}.$$

8. Answer any *one* question : 2 × 1

(*a*) What is a partial differential equation? Define the order of a partial differential equation.

(*b*) If $\mathscr{L}\{f(t)\} = F(p)$, then show that

$$\mathscr{L}\{f(at)\} = \frac{1}{a}F\left(p/a\right) \text{ where } p, a > 0.$$