UG/II/MATH/H/IV/17(Old)

2017

MATHEMATICS

[Honours]

PAPER – IV

Full Marks : 90

Time : 4 hours

The figures in the right hand margin indicate marks

[OLD SYLLABUS]

GROUP – A

(*Analytical Dynamics*)

[*Marks* : 40]

1. Answer any *one* question : 15 × 1

(*a*) (*i*) A heavy particle slides down a rough cycloid whose base is horizontal and vertex downwards. Show that (*I*) if it starts from rest at the cusp and comes to rest at the vertex then $\mu^2 e^{\mu\pi} = 1$ (*II*) if it starts from rest at a point where the tangent makes an angle θ with the horizon and comes to rest at the vertex then

$$\mu e^{\mu\theta} = \sin\theta - \mu\cos\theta$$

where μ is the coefficient of friction. 7

(*ii*) A particle moves in a plane with an acceleration which is parallel to the axis of Y and varies as the distances from the axis of X. Write the equation of motion and show that the equation of the path may be written in the form

(*I*) $y = a\cos(Ax + B)$ when acceleration is attractive.

(*II*) $y = Ae^{\alpha x} + Be^{-\alpha x}$ when the acceleration is repulsive. The curve $x = a(\theta - \cos\theta)$, $y = a(1 - \cos\theta)$, where a, e are constants and θ is a parameter, is described under the action of force parallel to the axis of X, show that the force varies as $\frac{(e-\cos\theta)}{\sin^3\theta}$. 2 + 2 + 4

(b) (i) A particle of mass m moves under a central attractive force $m\mu\,(5u^3 + 8c^2u^5)$ and is projected from an apse at a distance C with velocity $3\sqrt{\mu}/C$; prove that the orbit is $r = C\cos^{2/3}\theta$ and that it will arrive at the origin after a time $\frac{\pi c^2}{8\sqrt{\mu}}$. 7

(ii) One end of an elastic string, modulus of elasticity X and of natural lenght l is fixed to a point on a smooth horizontal table and the other end is tied to a particle of mass 'm' lying on the table. The particle is pulled to a distance such that the length of the string is equal to twice its natural length l and is released.

Show that the time of a complete oscillation is

$$2(\pi+2)\sqrt{\frac{lm}{\lambda}}.$$ 8

2. Answer any *two* questions : 8×2

(*a*) Find the radial and cross-radial components and velocity and acceleration of a particle moving in plane in polar co-ordinates (r, θ). 8

(*b*) If the velocity of a body in an elliptic orbit, major axis $2a$, is the same at a certain point P, whether the orbit being described in a periodic time T about one focus S or in periodic time T' about the other focus S', prove that

$$SP=\frac{2aT'}{T+T'} \text{ and } S'P=\frac{2aT}{T+T'}$$ 8

(*c*) A particle describes a path which is nearly a circle about a centre of force $f(x)$ at its centre where $u = l/r$. Find the condition under which this may be a stable motion. 8

3. Answer any *three* questions : 3 × 3

(*a*) Find the escape velocity at an altitude of 900 km above the surface of the earth.

(*b*) A shell of mass m is ejected from a gun of mass M by an explosion which generates kinetic energy E. Prove that the initial velocity of the shell is

$$\sqrt{\frac{2ME}{(M+m)m}}.$$

(*c*) A spherical drop of liquid falling freely in a vapour acquires mass by condensation a constant rate K. Show that velocity after falling from rest in time t is

$$\frac{1}{2}gt\left(1+\frac{m}{M+kt}\right).$$

Where M is the initial mass of the drop.

(*d*) The velocity V of a particle moving along the x axis given by $V^2 = 16 - x^2$. Prove that the motion is simple harmonic.

GROUP – B

(Linear Programming and Game Theory)

[*Marks* : 36]

4. Answer any *one* question of the following : 15×1

(*a*) (*i*) Using simplex method to solve the following LPP : 8

$$\text{Maximize } Z = 2x_1 + 9x_2 + x_3$$
$$\text{subject to } \quad x_1 + 4x_2 + 2x_3 \geq 5$$
$$3x_1 + x_2 + 2x_3 \geq 4$$
$$x_1, x_2, x_3 \geq 0.$$

(*ii*) In rectangular game the pay-off matrix given below :

$$\begin{bmatrix} 2 & 2 & 1 & -2 & -3 \\ 4 & 3 & 4 & -2 & 0 \\ 5 & 1 & 2 & 3 & 6 \end{bmatrix}$$

use dominance to reduce the game 2 × 2 and then solve the game. 5

(*iii*) Solve the following 2×4 game geometrically

Player B

$$\text{Player } A \quad \begin{array}{c} \\ A_1 \\ A_2 \end{array} \begin{array}{c} \begin{array}{cccc} B_1 & B_2 & B_3 & B_4 \end{array} \\ \begin{bmatrix} 3 & 2 & -1 & 4 \\ 2 & 5 & 6 & -2 \end{bmatrix} \end{array}$$ 2

(*b*) (*i*) Use duality to solve the following LPP :

Maximize $Z = 3x_1 - 2x_2$

subject to

$$x_1 \leq 4$$
$$x_2 \leq 6$$
$$x_1 + x_2 \leq 5$$
$$x_2 \geq 1$$

and $x_1, x_2 \geq 0$. 8

(*ii*) Solve the following transportation problem : 7

	D_1	D_2	D_3	
O_1	8	7	3	60
O_2	3	8	9	70
O_3	11	3	5	80
	50	80	80	

5. Answer any *two* of the following questions : 8 × 2

(*a*) Use two phase simplex method to

$$\begin{aligned} \text{Minimize} \quad & Z = x_1 - 3x_2 + 2x_3 \\ \text{subject to} \quad & 3x_1 - x_2 + 2x_3 \le 7 \\ & -2x_1 + 4x_2 \le 12 \\ & -4x_1 + 3x_2 + 8x_3 \le 10 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$ 8

(*b*) Solve graphically the following LPP

$$\begin{aligned} \text{Minimize} \quad & Z = 2x_1 + 3x_2 \\ \text{subject to} \quad & -x_1 + 2x_2 \le 4 \\ & x_1 + x_2 \le 6 \\ & x_1 + 3x_2 \ge 9 \\ \text{and} \quad & x_1, x_2 \ge 0. \end{aligned}$$

Hence show that feasible region of a LPP is a convex set. 8

(*c*) A transport company has offices in five localities *A*, *B*, *C*, *D* and *E*. Some day the offices located at *A* and *B* has 8 and 10 spare trucks whereas offices at *C*, *D*, *E* required

6, 8, 4 trucks respectively. The distance in kilometer between the five localities given below :

To		C	D	E
From	A	2	5	3
	B	4	2	7

How should the trucks from *A* and *B* be sent to *C*, *D* and *E* so that the total distance covered by the trucks is minimum. Formulate the problem as L.P.P. and hence solve it graphically. 8

6. Answer any *one* of the following : 3 × 1

(*a*) Define convex set in $\mathbb{R}^n$. What do you mean by convex combination of vectors in $\mathbb{R}^n$?

(*b*) Show that although (2, 3, 2) is a feasible solution to the system of equations

$$x_1 + x_2 + 2x_3 = 9$$
$$3x_1 + 2x_2 + 5x_3 = 22$$
$$x_1, x_2, x_3 \geq 0$$

it is not a basic solution. Find all the basic feasible solution of the given system.

7. Answer any *one* question : 2 × 1

(*a*) Show that $X = \{x : |x| \leq 2\}$ is a convex set.

(*b*) Graph the convex hull of the point : (0, 0), (0, 1), (1, 2) (1, 1), (4, 0). Which of these points is an interior point of the convex hull ? Explain it as a convex combination of the extreme points.

GROUP – C

(*Tensor Calculus*)

[*Marks* : 14]

8. Answer any *one* question : 8 × 1

(*a*) (*i*) If A^i be an arbitrary contravariant vector and $C_{ij} A^i A^j$ be an invariant, then show that $(C_{ij} + C_{ji})$ is a covariant tensors of the second order. 5

(*ii*) If a_{ij} be skew-symmetric tensor and A^j be a contravariant vector, then show that $a_{ij}A^iA^j = 0$. 3

(*b*) Define Ricci Tensor. Show that Ricci tensor is symmetric. 2+6

9. Answer any *two* questions : 3×2

(*a*) Prove that the inner product of tensors A_q^p and B_l^{ij} is a tensor of rank three.

(*b*) If f be a scalar function of co-ordinates x^j, then prove that

(*i*) $\dfrac{\partial f}{\partial x^j}$ is a covariant vector.

(*ii*) d^{xj} is a contravariant vector.

(*c*) If A_i is a covarient vector, examine whether $\dfrac{\partial A_i}{\partial x^j} - \dfrac{\partial A_j}{\partial x^i}$ are the component of a tensor or not.
