2017

MATHEMATICS

[Honours]

PAPER – III

Full Marks : 90

Time : 4 hours

The figures in the right hand margin indicate marks

[OLD SYLLABUS]

GROUP – A

(*Vector Analysis*)

[*Marks* : 27]

1. Answer any *one* question : 8 × 1

(*a*) (*i*) State and prove the Lami's theorem using vector algebra. 4

(*ii*) Let R be the region in R^2 determined by

(*Turn Over*)

the inequalities $x^2 + y^2 \leq 4$ and $y^2 \leq x^2$. Evaluate $\iint_R \sin(x^2 + y^2)dS$. 4

(*b*) (*i*) The position vector of a point on the space curve is given by

$$\vec{r} = t\vec{i} + t^2\vec{j} + \frac{2}{3}t^3\vec{k}.$$

Show that the radius of torsion = $\frac{1}{2}(1+2t^2)^2$ = radius of curvature. 4

(*ii*) Show that

$$\int_S (ax\hat{i} + by\hat{j} + cz\hat{k}) \cdot \hat{n}ds = \frac{4}{3}\Pi(a+b+c)$$

where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$. 4

2. Answer any *four* questions : 4 × 4

(*a*) A particle describes a circle $r = a\cos\theta$ with constant speed. Show that the acceleration is constant in magnitude and is directed towards the center of the circle. 4

(*b*) Write the vector equations of Osculating plane, Normal plane and Rectifying plane in terms of $\vec{r}, \dot{\vec{r}}, \ddot{\vec{r}}$. 4

(*c*) If

$$\vec{F} = (5xy - 6x^2)\vec{i} + (2y - 4x)\vec{j},$$

then evaluate $\int_C \vec{F} \cdot d\vec{r}$ along the curve C in xy plane, given by $y = x^3$ from (1, 1) to (2, 8). 4

(*d*) If $\vec{F}$ be a solenoidal vector, then show that

$$curl\ curl\ curl\ curl\ \vec{F} = \nabla^2\nabla^2\vec{F} = \nabla^4\vec{F}$$ 4

(*e*) Let C be the curve $x = \sqrt{t}$, $y = 1 + t^3$ for $0 \leq t \leq 1$. Evaluate

$$\int_C (x^3y^4dx + x^4y^3dy).$$ 4

(*f*) Show that

$$\left[\vec{\beta} \times \vec{\gamma}, \vec{\gamma} \times \vec{\alpha}, \vec{\alpha} \times \vec{\beta}\right] = \left[\vec{\alpha}\vec{\beta}\vec{\gamma}\right]$$ 4

3. Answer any *one* question : 3 × 1

(*a*) Find the directional derivative of $f(x, y, z) = x^2yz + 4xz^2$ at (1, −2, −1) along $2i - j - 2k$.

(*b*) Find the values of *a*, *b* and *c* so that

$$\vec{V} = (x + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z)k$$

is irrotational.

GROUP – B

(*Analytical Geometry*)

[*Marks* : 45]

(*Analytical Geometry of two Dimensions*)

[*Marks* : 18]

4. Answer any *two* questions : 8 × 2

(*a*) Tangents are drawn from the point (α, β) to the circle $x^2 + y^2 = a^2$, prove that area of the triangle formed by them and the straight line joining their point of contact is

$$\frac{a(\alpha^2+\beta^2-a^2)^{\frac{3}{2}}}{\alpha^2+\beta^2}$$ 8

(*b*) Show that the locus of the poles of tangents to the auxiliary circle with respect to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is given by $\frac{x^2}{a^4}+\frac{y^2}{b^4}=\frac{1}{a^2}$. 8

(*c*) Show that the distance from the orthocentre of the triangle formed by the straight lines $lx+my=1$ and $ax^2+2hxy+by^2=0$ is

$$\frac{(a+b)\sqrt{l^2+m^2}}{am^2-2hlm+bl^2}$$

Hence, show that the locus of the orthocentre of a triangle of which two sides are given in position and whose third side goes throuth a fixed point (α, β) is

$$bx^2-2hxy+ay^2=(a+b)(\alpha x+\beta y).$$ 8

5. Answer any *one* question : 2 × 1

(*a*) Define Director circle. Write the equation of the director circle of

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1(a>b).$$ 1 + 1

(*b*) Define diameter and conjugate diameter of a conic. 2

(*Analytical Geometry of three Dimensions*)

[*Marks* : 27]

6. Answer any *one* question : 15 × 1

(*a*) (*i*) A line moves so as to intersect the line $z=0$, $x=y$ and the circles $x=0$, $y^2+z^2=r^2; y=0$; $z^2+x^2=r^2$. Prove that the equations to the locus of the movingline is given by

$$(x+y)^2\{z^2+(x-y)^2\}=r^2(x-y)^3.$$ 8

(*ii*) Find the equation of the sphere which cuts orthogonally each of the four spheres $x^2+y^2+z^2+2ax=a^2$, $x^2+y^2+z^2+2by=b^2$, $x^2+y^2+z^2+2cz=c^2$ and $x^2+y^2+z^2=a^2+b^2+c^2$. 7

(*b*) (*i*) Find the equation of the enveloping cylinder of the sphere $x^2+y^2+z^2-2x+4y=1$ having its generators parallel to the line $x=y=z$. Also find its guiding curve. 7

(*ii*) The axes be rectangular and a point P moves on a fixed plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$. The Plane through P perpendicular to OP meets the axes in A, B and C where O be the origin. The Planes through A, B, C parallel to YOZ, ZOX, XOY intersect in Q. Show that the locus of Q is

$$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{ax}+\frac{1}{by}+\frac{1}{cz}.$$ 8

7. Answer any *one* question : 8 × 1

(*a*) Find the equation of the plane through the point $(-1, 0, 1)$ and the lines $4x-3y+1=0$ $=y-4z+13$; $2x-y-2=0=z-5$ and show that the equation to the line through the given point which intersects the two given lines can be written as $x=y-1=z-2$. 8

(*b*) Find the semi-vertical angle, the axis and the equation of the right circular cone with vertex at the origin and passing the straight line $\frac{x}{3}=\frac{y}{6}=\frac{z}{-2}$; $\frac{x}{2}=\frac{y}{2}=\frac{z}{-1}$ and $\frac{x}{-1}=\frac{y}{2}=\frac{z}{2}$. 8

8. Answer any *one* question : 4 × 1

(*a*) Show that the equation

$$\frac{a}{y-z}+\frac{b}{z-x}+\frac{c}{x-y}=0$$

represents a pair of planes. 4

(*b*) Find the equations of the planes bisecting the angle between the planes $x+2y+2z=19$,

$4x - 3y + 12z = -3$ and point out which bisects the acute angle. 4

GROUP – C

(*Astronomy*)

[*Marks* : 18]

9. Answer any *one* question : 15 × 1

(*a*) (*i*) At a place in north latitude ϕ, two stars *A* and *B* (declination δ and δ_1 respectively) rise at the same moment and *A* transist when *B* sets. Prove that $\tan\phi \tan\delta = 1 - 2\tan^2\phi \tan^2\delta_1$. 7

(*ii*) Explain the phenomenon of astronomical refraction. Describe Bradley's method of finding the coefficient of refraction. 8

(*b*) (*i*) State Kepler's laws of planetary motion. If the line joining two planets subtend an angle of 60° at the sum when the planets appears stationary, then show that

$$\frac{a}{b}+\frac{b}{a}=7,$$

where a and b are the distance of the planets from the sun. 7

(*ii*) Explain the phenomenon of astronomical aberration. Show that aberration varies as the sine of the earth's way. 8

10. Answer any *one* question : 3 × 1

(*a*) Show that the amount of geometric parallax varies as the sine of the apparent zenith distance. 3

(*b*) Show that the altitude of heavenly body is greatest when on the meridian. 3