2017

MATHEMATICS

[Honours]

PAPER – VIII

Full Marks : 60

Time : 3 hours

The figures in the right hand margin indicate marks

[NEW SYLLABUS]

GROUP – A

(*Numerical Analysis*)

[*Marks* : 25]

1. Answer any *two* questions : 8 × 2

(*a*) (*i*) Prove that the remainder in approximating $f(x)$ by the interpolation, polynomial

(Turn Over)

using interpolating points $x_0, x_1, ..., x_n$ is of the form

$$\frac{w(x) f^{n+1}(\xi)}{(n+1)!}$$

where $w(x) = (x - x_0)(x - x_1) \ldots (x - x_n)$ and ξ lies between the smallest and the largest of the numbers $x, x_0, x_1, ..., x_n$. 4

(*ii*) Show that the maximum error in linear interpolation is given by $\frac{h^2 M_2}{8}$ where

$$M_2 = \underset{0 \le x \le 1}{\text{Max}} |f''(x)|$$

4

(*b*) (*i*) Derive the error in Simpson's $\frac{1}{3}$ rd rule from Newton-Cotes' quadrature formula. 4

(*ii*) Describe Gauss-Seidel method for solving a system of n linear equations assuming that the system has unique solution. 4

(c) (i) Explain the Newton-Raphson method to determine approximately one simple real root of an equation $f(x) = 0$ and discuss its convergence. 4

(ii) Prove that

$$f(x_k, x_{k-1}, \ldots, x_{k-n}) \frac{\nabla^n f(x_k)}{n! h^n}$$

where the arguments are equispaced and ∇ being a backword difference operator. Hence show that

$$f(x_n, x_{n-1}, \ldots, x_0) = \frac{\nabla^n f(x_n)}{n! h^n}$$ 4

2. Answer any *three* questions : 3 × 3

(a) Using Lagrange's interpolation formula express

$$\frac{x^3 - 10x + 13}{x^3 - 6x^2 + 11x - 6}$$

as a sum of partial fractions. 3

(*b*) Using Euler's method evalute $y(1)$ correct upto three significant figures from the differential equation

$$\frac{dy}{dx} = xy,$$

given that $y(0) = 1$ and take $h = 0{\cdot}2$. 3

(*c*) Prove that sum of Lagrangian function is 1. 3

(*d*) Use Euler-Maclaurin formula to find the value of the series

$$\frac{1}{11^2} + \frac{1}{12^2} + \frac{1}{13^2} + \cdots + \frac{1}{99^2}.$$ 3

(*e*) Establish

$$\mu^2 \equiv 1 + \frac{1}{4}\delta^2$$

where μ, δ represent average and central difference operators respestively. 3

GROUP – B

(*Real Analysis* - III)

[*Marks* : 25]

3. Answer any *one* questions : 15 × 1

(*a*) (*i*) Let D be a subset of $\mathbb{R}$ and a series of function Σf_n be uniformly convergent on D to a function f. Let $x_0 \in D'$ (the derived set of D) and $\lim_{x \to x_0} f_n(x) = a_n$.

Then show that

(*I*) the series Σa_n is convergent, and

(*II*) $\lim_{x \to x_0} f(x)$ exists and equals Σa_n. 5

(*ii*) Let $\{f_n\}$ be a sequence of functions on $[a, b]$ such that for each $n \in \mathbb{N}$, $f_n'(x)$ exists for all $x \in [a, b]$. If the sequence of derivatives $\{f_n'\}$ converges uniformly on $[a, b]$ to a function g and the sequence $\{f_n\}$ converges at least at one point $x_0 \in [a, b]$, then show that the sequence $\{f_n\}$ is uniformly convergent on $[a, b]$ and if the limit function be f then show that $f'(x) = g(x)$ for all $x \in [a, b]$. 10

(*b*) (*i*) Let $\sum_{n=0}^{\infty} a_n x^n$ be a power series with radius of convergence 1. If $\sum_{n=0}^{\infty} a_n$ be convergent then show that series $\sum_{n=0}^{\infty} a_n x^n$ is uniformly convergent on [0, 1]. 5

(*ii*) If $f : [-\pi, \pi] \to \mathbb{R}$ be bounded and integrable and $\{a_n, b_n\}$ are its fourier coefficients, then show that

$\sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right)$ convergence and

$$\sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right) \leq \frac{1}{\pi}\int_{-\pi}^{\pi}[f(x)]^2 dx. \quad 5$$

(*iii*) Let $D \subset \mathbb{R}$ and for each $n \in \mathbb{N}$, $f_n : D \to \mathbb{R}$ is bounded on D. If the sequence $\{f_n\}$ be uniformly convergent on D, then show that the limit function f is bounded on D. Is the converse true ? Justify your answer. 5

4. Answer any *one* question : 8 × 1

(*a*) (*i*) Examine whether the sequence

$$\left\{\frac{nx}{1-n^2x^2}\right\}$$

is uniformly convergent on $[0, \infty]$. 5

(*ii*) Prove that a power series can be integrated term-by-term on any closed and bounded interval contained within the interval of convergence. 3

(*b*) (*i*) Find the Fourier series expansion of

$$f(x)=\begin{cases}\frac{1}{4}\pi x & \text{if } 0\le x\le \frac{\pi}{2}\\ \frac{1}{4}\pi(\pi-x) & \text{if } \frac{\pi}{2}<x\le\pi\end{cases}$$

defined on $[0, \pi]$. 3

(*ii*) Show that the series

$$\sum_{1}^{\infty}\frac{1}{n^2+[f(x)]^2}$$

is uniformly convergent on any set $D \subset \mathbb{R}$ on which f is defined. 5

5. Answer any *one* question : 2 × 1

(*i*) Determine the radius of convergence of the power series

$$\sum_{n=2}^{\infty} \frac{(x+2)^n}{\log n}.$$ 2

(*ii*) Show that the series

$$1 - \frac{e^{-2x}}{2^2-1} + \frac{e^{-4x}}{4^2-1} - \frac{e^{-6x}}{6^2-1} + \cdots\cdots$$

converges uniformly for all $x \geq 0$. 2

GROUP – C

(*Linear Algebra* - II)

[*Marks* : 10]

6. Answer any *one* question : 8 × 1

(*a*) (*i*) Let V and W be vector spaces over a field F. Let $\{\alpha_1, \alpha_2, ..., \alpha_n\}$ be a basis of V and $\beta_1, \beta_2, ..., \beta_n$ be arbitrarily chosen elements (not necessarily distinct) in W. Then show that there exists one and only one linear mapping $T : V \to W$ such that $T(\alpha_i) = \beta_i$ for $i = 1, 2, ..., n$. 5

(*ii*) Find a linear mapping $T : \mathbb{R}^3 \to \mathbb{R}^3$ such that ImT is the subspace

$U = \{(z, y, z) \in \mathbb{R}^3 : x + y + z = 0\}$ of R^3.

Where ImT represents the image of T. 3

(*b*) (*i*) Let V and W be finite dimensional vector spaces of same dimension over a field F and $T : V \to W$ be a linear mapping. Then show that T is an isomorphism if and only if T maps a basis of V to a basis of W. 5

(*ii*) Determine the linear mapping $T : R^3 \to R^3$ that maps the basis vectors $(2, 1, 1), (1, 2, 1), (1, 1, 2)$ of R^3 to the vectors (1, 1, 1,), (1, 1, 1), (1, 1,1) respectively. Find Ker T, Im T. Verify that

$$\dim \operatorname{Ker} T + \dim \operatorname{Im} T = 3.$$

Where Ker T and ImT represent the Kernel of T and image of T. 3

7. Answer any *one* question : 2 × 1

(*a*) Let V and W be vector spaces over a field F. Let $T : V \to W$ be a linear mapping. Then show that T is injective if and only if $\operatorname{Ker}(T) = \{\theta\}$. 2

(*b*) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear mapping defined by $T(x, y, z) = (x + y + z,\ 2x + y + 2z,\ x + 2y + z)$, $(x, y, z) \in \mathbb{R}^3$. Find the kernel of T. 2

NEW

Part–III 3-Tier

2017

MATHEMATICS

(Honours)

PAPER—VIII

(PRACTICAL)

Full Marks : 30

(PROBLEM – 24 + PNB & VIVA – 6)

TIME — 2 HOURS

Group—D

Answer *two* questions : 2×12

The questions must be allotted by Lottery.

Program must be written either in FORTRAN-*l*anguage or in C-*l*anguage.

Set—I

1. Write a program to find the area and circumference of a circle whose diameter is given. Demonstrate your program for the diameters 1234.78 cm and 23445.44 cm.

2. Write a program to find the roots of a quadratic equation $ax^2 + bx + c = 0$. Demonstrate your program for the equation $32.12456x^2 - 120.2256x - 332234.913 = 0$.

3. Write a program to find G.C.D between two integers. Demonstrate your program for the numbers 310298 and 23972.

4. Write a program to subtract the matrix A from the matrix 12A.

5. Write a program to subtract a matrix B from the matrix A.

6. Write a program which will convert uppercase characters of string to lowercase characters. Demonstrate your program for the string '1901. Rabindra Nath Sarkar'.

7. Write a program to sort a group names in descending order. Demonstrate your program for the set of strings Gopal, Krishna, Kanai, Surya, Barun, Pati.

8. Write a program to rewrite name of person in short form (i.g. Janaki Ranjan Sarkar in the form J. R. Sarkar).

9. Write a program in to find the mean and standard deviation of a set of 10 numbers. Demonstrate your program for the numbers 31.214, 11.82 19.08, 122.336, 22.323, 4532. 1.230, 423.21, 10323.0.

10. Write a program find a root of the equation $(x - 1.5)(x - 2.5)(x - 3.5)(x - 4.5) = 0$ by bisection method, correct up to 5 decimal places starting from x = 1.0.

11. Write a program to find a real root near x = 1 of the equation $x^{50} - 1 = 0$ using Regula-falsi method correct up to 4 decimal places.

12. Write a program to evaluate $\int_{1.6}^{2.4} \left(2\log 2x + x^{13}\right) dx$ by Simpson $\frac{1}{3}$rd rule taking 100 subintervals.

13. Write a program to find the value of y(0.2) from the differential equation $\frac{dy}{dx} = x^2 + y + 1.03$, x(0.05) = 1 by fourth order Runge-Kutta methods.

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Set—III

1. Write a program to to test whether a matrix of order n × n is unit or not.
2. Write a program to determine whether a matrix of order 5 × 5 is singular or not.
3. Write a program to check a string for palindrome without using liberary function.
4. Write a program which will converts lowercase characters of a string to uppercase characters.
5. Write a program to find the the third and fourth central moments for the sample 23.98, 34.2, 562.9, 231.4, 908.23, 342.33.
6. Write a program to compute the value of sine series up to 15 and 20 terms and compare the result when x = 0.75. (Write only one program).
7. Write a program to find the value of n! for n = 5, 15, 70 and 100.

8. Write a program to find the values of ${}^{n}C_{r}$ for given values of n and r. Demonstrate your program for $n = 19$, $r = 9$.

9. Write a program to test whether a positive integer is prime number or not. Demonstrate your program for the integers 290323, 12, 153, 34577.

10. Write a program to evaluate $\int_0^1 \left(23x + e^{\cos x}\right) dx$ by Simpson 1/3rd rule taking 500 subintervals.

11. Write a program to find a real root near $x = 1$ of the equation $x^{60} - 1 = 0$ using Regula-falsi method correct up to 4 decimal places.

12. Write a program to find a root of $x = \cos x$ by bisection method, correct up to 5 decimal places.

13. Write a program to find a real root of the equation $3x^5 - 10x^4 - 4x^2 + 2x + 8 = 0$ by Newton-Raphson method correct up to 5 decimal places.

14. Write a program to find the value of $y(0.1)$ from the differential equation

$\frac{dy}{dx} = x + y + 100$, $x(0) = 1.2$ by second order Runge-Kutta methods.

15. Write a program to find the sum of the series :

$$1 + \frac{1}{(2\times 5)^2} + \frac{1}{(2\times 5)^4} + \frac{1}{(2\times 5)^6} \text{ ..... correct up to 5 decimal places.}$$

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$1 + \frac{1}{(2\times5)^2} + \frac{1}{(2\times5)^4} + \frac{1}{(2\times5)^6}$ ..... correct up to 5 decimal places.
