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Blow-Up Phenomena for a Class of Nonlinear Parabolic Problems Under Robin Boundary Condition

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ABSTRACT

This paper deals with the blow-up of solution to a class of nonlinear degenerate parabolic equations

$$(a(u))_t = div\left(b(u)|\nabla u|^{p-2}\nabla u\right) + f(u)$$

under Robin boundary condition. By constructing some appropriate auxiliary functions and using first-order differential inequality technique, we derive the sufficient conditions which guarantee the occurrence of the blow-up. In addition, lower bound and upper bound for blow-up time are derived when blow-up happen.

Keywords: Nonlinear parabolic equations; Blow-up; Lower bound; upper bound; Robin boundary condition

1. Introduction

In this text, we consider the following nonlinear parabolic equation

$$(a(u))_t = div\left(b(u)|\nabla u|^{p-2}\nabla u\right) + f(u) \tag{1}$$

with the following Robin boundary condition

$$\frac{\partial u}{\partial n} + \kappa u = 0$$

and the initial condition

$$u(x,0) = h(x) \geq 0.$$

Here $\vec{n}$ is the unit outer normal vector of $\partial\mathcal{O}$, and $\frac{\partial u}{\partial \vec{n}}$ is outward normal derivative of u on the boundary $\partial\mathcal{O}$ which is assumed to be sufficiently smooth.

If $p=2$, the phenomena of blow-up for parabolic problem have been extensively studied in the last 10 years for details, see Guria [7], Arunkumar, Agilan and Ramamoorthy [2], Abdol and Panchal [1], Hakim [8], Mitra, Datta, and Chanda [9], Ding and Guo [3], and Zhang [12]. Payne and Schaefer [11] have studied the following problem

$$\begin{cases} u_t = \Delta u + f(u) & \text{in } \mathcal{O}\times(0,\infty), \\ \dfrac{\partial u}{\partial n} + \kappa u = 0 & \text{on } \partial\mathcal{O}\times(0,\infty), \\ u(x,0) = h(x) \geq 0 & \text{in } \mathcal{O} \end{cases} \tag{2}$$

where $\mathcal{O}$ is a bounded domain in $\mathbb{R}^N$, Δ is the Laplace operator, ∇ is the gradient operator, n is the unit outer normal vector of $\partial\mathcal{O}$, and $\frac{\partial u}{\partial n}$ is outward normal derivative of u on the boundary $\partial\mathcal{O}$ which is assumed to be sufficiently smooth. By using a first order differential inequality technique, sufficient conditions were given to guarantee the occurrence of the blow-up. In addition, a lower bound for blow-up time was also obtained. Zhou (2009) studied the equation

$$\frac{\partial u}{\partial t} = u\mathrm{div}\left(|\nabla u|^{p-2}\nabla u\right) + \gamma|\nabla u|^p$$

with the boundary and initial value conditions. Later, Enache (2011) considered a Robin boundary value problem for quasi-linear parabolic equations of the form

$$\begin{cases} u_t = div\left(b(u)\nabla u\right) + f(u) & \text{in } \mathcal{O}\times(0,\infty), \\ \dfrac{\partial u}{\partial n} + \kappa u = 0 & \text{on } \partial\mathcal{O}\times(0,\infty), \\ u(x,0) = h(x) \geq 0 & \text{in } \mathcal{O}. \end{cases} \tag{3}$$

Under the suitable assumptions on functions b, f and h, the author established sufficient condition to guarantee the occurrence of the blow-up. Moreover, a lower bound for blow-up time was obtained. On the contrary, blow-up phenomena of general case (1) has not been studied in the literature. Our aim in this article is to fulfill this gap.

Since the initial data $u_0(x)$ in (5) is nonnegative, we have by the parabolic maximum principles (see Friedman (1958) and Nirenberg (1953)) that u is nonnegative in $\mathcal{O}\times\left(0,T^*\right)$. In section 2, we plan to present the sufficient conditions which guarantee the occurrence of the blow-up. In section 3, we will find a lower bound for the blow-up time when blow-up occurs.

2. The Blow-up solution

In this section we mainly seek the sufficient conditions which guarantee the blow-up. To this end, we define an auxiliary function of the form

$$\begin{aligned} &G(s) = 2\int_0^s yb(y)^{(p-1)p-1}a'(y)\mathrm{d}y, \quad A(t) = \int_O G\left(u(x,t)\right)\mathrm{d}x, \\ &H(s) = \int_0^s y^{p-1}b(y)^{p(p-1)}\mathrm{d}y, \quad F(s) = \int_0^s f(s)b(s)^{(p-1)p-1}\mathrm{d}s, \\ &B(t) = \int_{\mathcal{O}} F(u)\mathrm{d}x - \frac{1}{p}\int_{\mathcal{O}} b(u)^{(p-1)p}\left[(\nabla u)^2\right]^{\frac{p}{2}}\mathrm{d}x - \kappa^{p-1}\int_{\partial\mathcal{O}} H\left(u\right)\mathrm{d}x \end{aligned} \tag{4}$$

where $u(x,t)$ is the solution of problem (3).

The main result of this section is formulated in the following theorem:

Theorem 2.1. let $u(x,t)$ be the solution of problem (1). Assume that the data of problem (3) satisfy the following conditions:

$$sf(s)b(s)^{(p-1)p-1} \geq p(1+\alpha)F(s), \quad s > 0 \tag{5}$$

where α is a positive constant. We further assume

$$\lim_{y\to\infty} y^p b(y)^{p(p-1)} = 0 \quad \text{and} \quad B(0) \geq 0. \tag{6}$$

Then, we conclude that $u(x,t)$ blows up as some finite time T^* and T^* is bounded above by

$$T^* \le \frac{A(0)^{1-\frac{p}{2}(1+\alpha)}}{\left(\frac{p}{2}(1+\alpha)-1\right)2p(1+\alpha)B(0)A(0)^{-\frac{p}{2}(1+\alpha)}}.$$

Proof: We first compute

$$\begin{aligned} A'(t) &= \int_{\mathcal{O}} G'(u(x,t))u_t \mathrm{d}x = 2\int_{\mathcal{O}} ub(u)^{(p-1)p-1}(a(u))_t \,\mathrm{d}x \\ &= 2\int_{\mathcal{O}} ub(u)^{(p-1)p-1}\left[div\left(b(u)|\nabla u|^{p-2}\nabla u\right)+f(u)\right]\mathrm{d}x \\ &= 2\int_{\mathcal{O}} uf(u)b(u)^{(p-1)p-1}\mathrm{d}x - 2((p-1)p-1)\int_{\mathcal{O}} ub(u)^{(p-1)p-1}b'(u)\left[(\nabla u)^2\right]^{\frac{p}{2}}\mathrm{d}x \\ &\quad -2\int_{\mathcal{O}} b(u)^{(p-1)p}\left[(\nabla u)^2\right]^{\frac{p}{2}}\mathrm{d}x - 2\kappa^{p-1}\int_{\partial\mathcal{O}} b(u)^{(p-1)p}u^p \mathrm{d}x \\ &\ge 2\int_{\mathcal{O}} uf(u)b(u)^{(p-1)p-1}\mathrm{d}x - 2\int_{\mathcal{O}} b(u)^{(p-1)p}\left[(\nabla u)^2\right]^{\frac{p}{2}}\mathrm{d}x - 2\kappa^{p-1}\int_{\partial\mathcal{O}} b(u)^{(p-1)p}u^p \mathrm{d}x. \end{aligned} \tag{7}$$

Here, we have used the fact that $b' \le 0$. Integrating by parts and taking into account assumption (6), we have

$$\begin{aligned} H(u) &= \int_0^s y^{p-1}b(y)^{p(p-1)}\mathrm{d}y \\ &= y^p b(y)^{p(p-1)}\Big|_0^u - (p-1)\int_0^s y^{p-1}b(y)^{p(p-1)}\mathrm{d}y - p(p-1)\int_0^s y^p b(y)^{p(p-1)-1}b'(y)\mathrm{d}y \\ &\ge u^p b(u)^{p(p-1)} - (p-1)\int_0^s y^{p-1}b(y)^{p(p-1)}\mathrm{d}y = u^p b(u)^{p(p-1)} - (p-1)H(u), \end{aligned}$$

that is

$$pH(u) \ge u^p b(u)^{p(p-1)}. \tag{8}$$

Therefore, inserting (8) into (7) and using the assumption (5), we arrive at

$$\begin{aligned} A'(t) &\ge 2p(1+\alpha)\int_{\mathcal{O}} F(u)\mathrm{d}x - 2(1+\alpha)\int_{\mathcal{O}} b(u)^{(p-1)p}\left[(\nabla u)^2\right]^{\frac{p}{2}}\mathrm{d}x - 2p(1+\alpha)\kappa^{p-1}\int_{\partial\mathcal{O}} H(u)\mathrm{d}x \\ &= 2p(1+\alpha)\left[\int_{\mathcal{O}} F(u)\mathrm{d}x - \frac{1}{p}\int_{\mathcal{O}} b(u)^{(p-1)p}\left[(\nabla u)^2\right]^{\frac{p}{2}}\mathrm{d}x - \kappa^{p-1}\int_{\partial\mathcal{O}} H(u)\mathrm{d}x\right] \\ &= 2p(1+\alpha)B(t). \end{aligned} \tag{9}$$

On the other hand, we compute $B(t)$ to obtain

$$
\begin{aligned}
B'(t) &= \int_{\mathcal{O}} f(u)b(u)^{(p-1)p-1}u_t\mathrm{d}x - (p-1)\int_{\mathcal{O}} b(u)^{(p-1)p-1}b'(u)u_t\left[(\nabla u)^2\right]^{\frac{p}{2}}\mathrm{d}x \\
&\quad - \int_{\mathcal{O}} b(u)^{(p-1)p}\left[(\nabla u)^2\right]^{\frac{p}{2}-1}\nabla u\nabla u_t\mathrm{d}x - \kappa^{p-1}\int_{\partial\mathcal{O}} H'(u)u_t\mathrm{d}x \\
&= \int_{\mathcal{O}} f(u)b(u)^{(p-1)p-1}u_t\mathrm{d}x - (p-1)\int_{\mathcal{O}} b(u)^{(p-1)p-1}b'(u)u_t\left[(\nabla u)^2\right]^{\frac{p}{2}}\mathrm{d}x \\
&\quad - \int_{\mathcal{O}} b(u)^{(p-1)p}\left[(\nabla u)^2\right]^{\frac{p}{2}-1}\nabla u\nabla u_t\mathrm{d}x - \kappa^{p-1}\int_{\partial\mathcal{O}} u^{p-1}b(u)^{p(p-1)}u_t\mathrm{d}x \\
&= \int_{\mathcal{O}} b(u)^{(p-1)p-1}u_t\left[f(u) + b'(u)\left((\nabla u)^2\right)^{\frac{p}{2}} + b(u)\cdot\mathrm{div}\left[\left((\nabla u)^2\right)^{\frac{p}{2}}\right]\right]\mathrm{d}x \\
&= \int_{\mathcal{O}} b(u)^{(p-1)p-1}u_t\left(a(u)\right)_t\mathrm{d}x = \int_{\mathcal{O}} b(u)^{(p-1)p-1}a'(u)\left(u_t\right)^2\mathrm{d}x \ge 0
\end{aligned}
\tag{10}
$$

based on the fact that $a' > 0$. Thus, in view of (6) we conclude that $B(t)$ is a nondecreasing function of t and

$$B(t) \ge B(0) \ge 0\,. \tag{11}$$

Furthermore, combining (9) with (10), we use Holder inequality to get

$$
\begin{aligned}
0 \le (1+\alpha)A'(t)B(t) &\le \frac{1}{2p}\left(A'(t)\right)^2 = \frac{2}{p}\left(\int_{\mathcal{O}} ub(u)^{(p-1)p-1}a'(u)u_t\mathrm{d}x\right)^2 \\
&\le \frac{2}{p}B'(t)\left(\int_{\mathcal{O}} ub(u)^{(p-1)p-1}a'(u)u^2\mathrm{d}x\right).
\end{aligned}
\tag{12}
$$

Integrating by parts and using the assumption that $b' \le 0$, $a' > 0$ and $a'' \le 0$, it follows that

$$
\begin{aligned}
&\int_0^u sb(s)^{(p-1)p-1}a'(s)\mathrm{d}s \\
&= s^2b(s)^{(p-1)p-1}a'(s)\Big|_0^u - \int_0^u sb(s)^{(p-1)p-1}a'(s)\mathrm{d}s \\
&\quad -\left((p-1)p-1\right)\int_0^u s^2b(s)^{(p-1)p-2}b'(s)a'(s)\mathrm{d}s - \int_0^u s^2b(s)^{(p-1)p-1}a''(s)\mathrm{d}s \\
&\ge u^2b(u)^{(p-1)p-1}a'(u) - \int_0^u sb(s)^{(p-1)p-1}a'(s)\mathrm{d}s,
\end{aligned}
$$

that is

$$G(u) \ge u^2b(u)^{(p-1)p-1}a'(u)\,. \tag{13}$$

Therefore, we insert (13) into (12) to obtain

$$(1+\alpha)A'(t)B(t) \le \frac{2}{p}B'(t)\left(\int_{\mathcal{O}} G(u)\mathrm{d}x\right) = \frac{2}{p}B'(t)A(t). \tag{14}$$

This leads to

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(A^{-\frac{p}{2}(1+\alpha)}B\right) \ge 0. \tag{15}$$

An integration of (15) from 0 to t leads to

$$\frac{B(t)}{B(0)} \ge \left(\frac{A(t)}{A(0)}\right)^{\frac{p}{2}(1+\alpha)}. \tag{16}$$

Finally, it follows by combining (9) with (16) that

$$A'(t) \ge 2p(1+\alpha)B(0)A(0)^{-\frac{p}{2}(1+\alpha)}A(t)^{\frac{p}{2}(1+\alpha)}$$

or

$$\frac{A'(t)}{A(t)^{\frac{p}{2}(1+\alpha)}} \ge 2p(1+\alpha)B(0)A(0)^{-\frac{p}{2}(1+\alpha)}. \tag{17}$$

Noting that $p>2$, and integrating (17) from 0 to t, we obtain

$$A(t)^{1-\frac{p}{2}(1+\alpha)} \le A(0)^{1-\frac{p}{2}(1+\alpha)} - \left(\frac{p}{2}(1+\alpha)-1\right)2p(1+\alpha)B(0)A(0)^{-\frac{p}{2}(1+\alpha)}t. \tag{18}$$

Since inequality (18) can not hold for

$$A(0)^{1-\frac{p}{2}(1+\alpha)} - \left(\frac{p}{2}(1+\alpha)-1\right)2p(1+\alpha)B(0)A(0)^{-\frac{p}{2}(1+\alpha)}t \le 0,$$

that is , for

$$t \ge \frac{A(0)^{1-\frac{p}{2}(1+\alpha)}}{\left(\frac{p}{2}(1+\alpha)-1\right)2p(1+\alpha)B(0)A(0)^{-\frac{p}{2}(1+\alpha)}}.$$

Hence, we conclude that the solution u of problem (1) blows up at some finite time T^* and T^* is bounded above by

$$T^* \le \frac{A(0)^{1-\frac{p}{2}(1+\alpha)}}{\left(\frac{p}{2}(1+\alpha)-1\right)2p(1+\alpha)B(0)A(0)^{-\frac{p}{2}(1+\alpha)}}. \square$$

3. Lower bound for blow-up time

In this section we seek the lower bound for the blow-up time T^*. To this end, we define an auxiliary function of the form

$$v(s)=\int_0^s \frac{a'(y)}{b(y)}dy, \quad E(t)=\int_{\mathcal{O}} \left[v(u(x,t))\right]^{\mu p+2} \mathrm{d}y \quad \text{with} \quad \mu \ge 1. \tag{19}$$

Theorem 3.1. Suppose that $\mathcal{O} \subset \mathbb{R}_3$ is a bounded convex domain with smooth boundary $\partial\mathcal{O}$. Further, assume that nonlinear function a, f and g satisfy

$$0 < f(s) \le \delta b(s)\left(\int_0^s v(y)\mathrm{d}y\right)^{p-1}, \quad s>0 \tag{20}$$

where δ is a positive constant independent of $a,\ b$ and f. Then the blow-up time T^* is bounded below by

$$T^* \ge \int_{E(0)}^{+\infty} \frac{\mathrm{d}\xi}{A_0 + A_1\xi + A_2\xi^{\frac{3}{2}} + A_3\xi^3 + A_4\xi^{\frac{2(\mu p+2)-p}{2(p-2)(\mu p+2)}}}$$

where, A_1, A_2, A_3 and A_4 are positive constants to be determined later. A_0

Proof: We first compute

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}E(t)=(\mu p+2)\int_{\mathscr{O}}v^{\mu p+1}\frac{a'(u)}{b(u)}u_t\mathrm{d}x\\
&=(\mu p+2)\int_{\mathscr{O}}v^{\mu p+1}\frac{1}{b(u)}\left[div\left(b(u)|\nabla u|^{p-2}\nabla u\right)+f(u)\right]\mathrm{d}x\\
&=-\kappa^{p-1}(\mu p+2)\int_{\partial\mathscr{O}}v^{\mu p+1}|u|^{p-1}\mathrm{d}x-(\mu p+2)(\mu p+1)\int_{\mathscr{O}}v^{\mu p}\nabla v|\nabla u|^{p-2}\nabla u\mathrm{d}x\\
&+(\mu p+2)\int_{\mathscr{O}}v^{\mu p+1}\frac{b'(u)}{b(u)}|\nabla u|^{p}\mathrm{d}x+(\mu p+2)\int_{\mathscr{O}}v^{\mu p+1}\frac{f(u)}{b(u)}\mathrm{d}x.
\end{aligned}\tag{21}$$

In view of the first auxiliary function in (19), we get

$$\nabla v=\frac{a'(u)}{b(u)}\nabla u\ .\tag{22}$$

Combining (22) and $b'\le 0$, we remove the non-positive terms in (21) so that

$$\frac{\mathrm{d}}{\mathrm{d}t}E(t)\le-(\mu p+2)(\mu p+1)\int_{\mathscr{O}}\frac{b(u)}{a'(u)}v^{\mu p}|\nabla v|^{p-2}\mathrm{d}x+\delta(\mu p+2)\int_{\mathscr{O}}v^{\mu p+p}\mathrm{d}x.\tag{23}$$

Using the fact that $b(s)\ge b_m>0$ and $0<a'(s)\le a'_M$, we arrive at

$$\frac{g(u)}{a'(u)}\ge\frac{g_m}{a'_M}\ .\tag{24}$$

Therefore taking (24) in (23), we have

$$\frac{\mathrm{d}}{\mathrm{d}t}E(t)\le-(\mu p+2)(\mu p+1)(\mu+1)^{-p}\frac{b_m}{a'_M}\int_{\mathscr{O}}\left|\nabla v^{\mu+1}\right|^{p}\mathrm{d}x+\delta(\mu p+2)\int_{\mathscr{O}}v^{\mu p+p}\mathrm{d}x.\tag{25}$$

Next, we seek to bound $\delta(\mu p+2)\int_{\mathscr{O}}v^{\mu p+p}\mathrm{d}x$ in terms of $E(t)$ and $\int_{\mathscr{O}}\left|\nabla v^{\mu+1}\right|^{p}\mathrm{d}x$. By mean of Holder and Young inequalities twice, we have

$$\begin{aligned}
&\int_{\mathscr{O}}v^{\mu p+p}\mathrm{d}x\le|\mathscr{O}|^{\frac{2}{\mu p+p+1}}\left(\int_{\mathscr{O}}v^{\mu p+p+1}\mathrm{d}x\right)^{\frac{\mu p+p}{\mu p+p+2}}\\
&\le\frac{2}{\mu p+p+2}|\mathscr{O}|+\frac{\mu p+p}{\mu p+p+2}\int_{\mathscr{O}}v^{\mu p+p+2}\mathrm{d}x\\
&\le\frac{2}{\mu p+p+1}|\mathscr{O}|+\frac{\mu p+p}{\mu p+p+2}\left(\int_{\mathscr{O}}v^{\frac{3}{2}(\mu p+2)}\mathrm{d}x\right)^{\frac{2p}{\mu p+2}}\left(\int_{\mathscr{O}}v^{\mu p+2}\mathrm{d}x\right)^{\frac{\mu p+2-2p}{\mu p+2}}\\
&\le\frac{2}{\mu p+p+2}|\mathscr{O}|+\frac{\mu p+p}{\mu p+p+2}\frac{2p}{\mu p+2}\int_{\mathscr{O}}v^{\frac{3}{2}(\mu p+2)}\mathrm{d}x+\frac{\mu p+p}{\mu p+p+2}\frac{\mu p+2-2p}{\mu p+2}\int_{\mathscr{O}}v^{\mu p+2}\mathrm{d}x.
\end{aligned}\tag{26}$$

Using the integral inequality derived in Payne (2008), namely

$$\int_{\mathscr{O}}u^{\frac{3}{2}(\mu p+2)}\mathrm{d}x\le\frac{3^{\frac{3}{4}}}{2\rho_0^{\frac{3}{2}}}E(t)^{\frac{3}{2}}+\frac{\sqrt{2}}{3^{\frac{3}{4}}}\left(\frac{\rho_1}{\rho_0}+1\right)^{\frac{3}{2}}\left[\frac{E(t)^3}{4\chi^3}+\frac{3}{4}\chi\int_{\mathscr{O}}\left|\nabla u^{\frac{1}{2}(\mu p+2)}\right|^2\mathrm{d}x\right],\tag{27}$$

we obtain

$$\int_{\mathcal{O}} v^{\mu p+p}\mathrm{d}x \le \frac{2}{\mu p+p+2}|\mathcal{O}| + \frac{\mu p+p}{\mu p+p+2}\frac{2p}{\mu p+2}\frac{3^{\frac{3}{4}}}{2\rho_0^{\frac{3}{2}}}E(t)^{\frac{3}{2}}$$
$$+\frac{\mu p+p}{\mu p+p+2}\frac{2p}{\mu p+2}\frac{\sqrt{2}}{3^{\frac{3}{4}}}\left(\frac{\rho_1}{\rho_0}+1\right)^{\frac{3}{2}}\left[\frac{E(t)^3}{4\chi^3}+\frac{3}{4}\chi\int_{\mathcal{O}}\left|\nabla u^{\frac{1}{2}(\mu p+2)}\right|^2\mathrm{d}x\right] \tag{28}$$
$$+\frac{\mu p+p}{\mu p+p+2}\frac{\mu p+2-2p}{\mu p+2}\int_{\mathcal{O}} v^{\mu p+2}\mathrm{d}x.$$

For simplicity, let $w=v^{1+ns}$. Again by using Holder and Young inequalities, we obtain

$$\int_{\mathcal{O}}\left|\nabla v^{\frac{1}{2}(\mu p+2)}\right|^2\mathrm{d}x \le \frac{(\mu p+1)^2}{4(\mu+1)^2}\left(\int_{\mathcal{O}}|\nabla w|^p\,\mathrm{d}x\right)^{\frac{2}{p}}\left(\int_{\mathcal{O}} w^{\frac{p(\mu p+2)}{(p-2)(\mu+1)}-\frac{2p}{p-2}}\mathrm{d}x\right)^{\frac{p-2}{p}}$$
$$\le \frac{(\mu p+1)^2}{2p(\mu+1)^2}\int_{\mathcal{O}}|\nabla w|^p\,\mathrm{d}x+\frac{p-2}{p}\frac{(\mu p+2)^2}{4(\mu+1)^2}\int_{\mathcal{O}} w^{\frac{p(\mu p+2)}{(p-2)(\mu+1)}-\frac{2p}{p-2}}\mathrm{d}x. \tag{29}$$
$$\le \frac{(\mu p+1)^2}{2p(\mu+1)^2}\int_{\mathcal{O}}\left|\nabla v^{1+\mu}\right|^p\mathrm{d}x+\frac{p-2}{p}|\mathcal{O}|^{1-\frac{2(\mu p+2)-p}{2(p-2)(\mu p+2)}}\frac{(\mu p+1)^2}{4(\mu+1)^2}E(t)^{\frac{2(\mu p+2)-p}{2(p-2)(\mu p+2)}}.$$

Therefore, we insert (29) into (28) to arrive at

$$\delta(\mu p+2)\int_{\mathcal{O}} u^{\mu p+p}\mathrm{d}x$$
$$\le A_0+A_1E(t)+A_2E(t)^{\frac{3}{2}}+A_3E(t)^3+A_4E(t)^{\frac{2(\mu p+2)-p}{2(p-2)(\mu p+2)}}+\chi A_5\int_{\mathcal{O}}\left|\nabla v^{1+\mu}\right|^p\mathrm{d}x \tag{30}$$

where χ is a positive constant to be determined later,

$$A_0=\frac{2\delta(\mu p+2)}{\mu p+p+2}|\mathcal{O}|\ ,\quad A_1=\delta(\mu p+2)\frac{\mu p+p}{\mu p+p+2}\frac{\mu p+2-2p}{\mu p+2}\ ,$$

$$A_2=\frac{3^{\frac{3}{4}}}{2\rho_0^{\frac{3}{2}}}\delta(\mu p+2)\frac{\mu p+p}{\mu p+p+2}\frac{2p}{\mu p+2}\ ,$$

$$A_3=\frac{\delta(\mu p+2)}{4\chi_2^3}\frac{\mu p+p}{\mu p+p+2}\frac{2p}{\mu p+2}\frac{\sqrt{2}}{3^{\frac{3}{4}}}\left(\frac{\rho_1}{\rho_0}+1\right)^{\frac{3}{2}},$$

$$A_4=\frac{3}{4}\frac{\sqrt{2}}{3^{\frac{3}{4}}}\left(\frac{\rho_1}{\rho_0}+1\right)^{\frac{3}{2}}\delta(\mu p+2)\frac{\mu p+p}{\mu p+p+2}\frac{2p}{\mu p+2}\frac{p-2}{p}|\mathcal{O}|^{1-\frac{2(\mu p+2)-p}{2(p-2)(\mu p+2)}}\frac{(\mu p+1)^2}{4(\mu+1)^2}\chi\ ,$$

$$A_5=\frac{3}{4}\frac{\sqrt{2}}{3^{\frac{3}{4}}}\left(\frac{\rho_1}{\rho_0}+1\right)^{\frac{3}{2}}\delta(\mu p+2)\frac{\mu p+p}{\mu p+p+2}\frac{2p}{\mu p+2}\frac{(\mu p+1)^2}{2p(\mu+1)^2}\ .$$

Combining (30) with (25), we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}E(t)\le-(\mu p+2)(\mu p+1)(\mu+1)^{-p}\frac{b_m}{a'_M}\int_{\mathcal{O}}\left|\nabla v^{\mu+1}\right|^p\mathrm{d}y+A_0+A_1E(t)$$
$$+A_2E(t)^{\frac{3}{2}}+A_3E(t)^3+A_4E(t)^{\frac{2(\mu p+2)-p}{2(p-2)(\mu p+2)}}+\chi A_5\int_{\mathcal{O}}\left|\nabla v^{1+\mu}\right|^p\mathrm{d}x. \tag{31}$$

To make (31) useful, we must choose a suitable χ such that

$$\chi=(\mu p+2)(\mu p+1)(\mu+1)^{-p}\frac{b_m}{a'_M A_5}.$$

Thus, (31) becomes

$$\frac{\mathrm{d}}{\mathrm{d}t}E(t)\le A_0+A_1E(t)+A_2E(t)^{\frac{3}{2}}+A_3E(t)^3+A_4E(t)^{\frac{2(\mu p+2)-p}{2(p-2)(\mu p+2)}}. \tag{32}$$

Finally, an integration of the differential inequality (32) from 0 to t leads to

$$\int_{E(0)}^{E(t)}\frac{\mathrm{d}\xi}{A_0+A_1\xi+A_2\xi^{\frac{3}{2}}+A_3\xi^3+A_4\xi^{\frac{2(\mu p+2)-p}{2(p-2)(\mu p+2)}}}\le t.$$

From which we derive a lower bound for T^*, namely

$$T^*\ge\int_{E(0)}^{+\infty}\frac{\mathrm{d}\xi}{A_0+A_1\xi+A_2\xi^{\frac{3}{2}}+A_3\xi^3+A_4\xi^{\frac{2(\mu p+2)-p}{2(p-2)(\mu p+2)}}}.$$

Thus, the proof is complete. □

Remark 3.2. Theorem 3.1 remains valid if the Robin boundary condition in (1) is replaced by the following nonlinear boundary condition

$$\frac{\partial u}{\partial n}+g(u)=0.$$

Here, g is a positive function which belongs to $L^p(\mathbb{R}_+)$.

4. Conclusion

The main purpose of this paper was to present blow-up results for a nonlinear degenerate parabolic equation under Robin boundary condition, highlighting the method which has been used by Payne (2008) to some results. The techniques used to prove our results are a variety of tools such as differential inequality technique. The possible generalization is plan to present the sufficient conditions which guarantee the occurrence of the blow-up.

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