A COMPREHENSIVE STUDY ON TRANSPORTATION PROBLEMS UNDER MULTI-CHOICE ENVIRONMENT

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A Comprehensive Study on Transportation Problems under Multi-Choice Environment

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by

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Dedicated to

my father

Late Dudal Chandra Maity

CERTIFICATE

This is to certify that the thesis entitled **"A Comprehensive Study on Transportation Problems under Multi-Choice Environment"**, submitted by **Gurupada Maity** for the award of the degree of Doctor of Philosophy to the Vidyasagar University is an authentic record of bonanza original research work carried out under my supervision and guidance. The thesis is worthy of consideration for awarding the degree of Doctor of Philosophy and it satisfies and fulfils the requirements in accordance with regulation of the University.

It is also certified that this original thesis has not been submitted to any other University or Institution for any degree/diploma.

Date: **Supervisor**

DECLARATION

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a. The research work presented in the thesis is original and has been done by myself under the general supervision of my supervisor.

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Abstract

Transportation problems sustain economic and social activity and are treated as central nerve system to operations research and management science. Based on day-by-day competitive market scenario several types of decision making problems are introduced using the classical sense of transportation problem to find better decisions. Although, the classical transportation problem defines the way to minimize total cost of a transportation system, but nowadays it is used for several objectives like optimizing profit, minimizing transportation time, fixing of cost of goods, etc. by using several methodologies. Considering the real-world situations, transportation problems are developed in many decision making problems under several uncertain environments. Optimization under multi-choice environment is also a kind of uncertain programming problem which mainly occurred due to the presence of multi-choice parameter in optimizing function or in the feasibility conditions or in both. Incorporating the daily-life transportation situations for selection of route under multiple choice possibilities in a transportation problem, the multi-choice TP is developed. This thesis is devoted to transportation problem under the different environments considering the multi-choice programming framework.

The single-objective TP is not adequate to handle real-life decision making problems, owing to present competitive market scenario, we consider our study in multi-objective window. To cover all real-life situations on TPs, we introduce the multi-objective function in our considered TP in this thesis.

Again, linear programming problems are not sufficient for formulating all types of decision making problem in real-life situations. As a result, non-linear programming problems have been incorporated into the multi-objective TPs. Here, in the thesis, the non-linear objective function is occurred in the TP due to some goods are left after distributing the goods from the origin to the destination points.

An optimization problem becomes a goal programming problem if the objective functions have some specific aspiration level of satisfaction which are known as the goal of the objective functions. Goal programming approach is a well known technique to solve multi-objective transportation problems. But GP is not always producing better optimal solutions. Here, we propose a new way to solve MOTP by revised multi-choice goal programming and utility function

approach. In this study, we present a better result of MOTP using utility function approach in compare to GP or RMCGP. Again, conic scalarization function is incorporated to solve the multi-choice MOTP. The CSF is presented as much better technique in compare to GP and RMCGP technique.

Transportation time is also an important factor in a TP and so it is also required to minimize transportation time along with the minimization of transportation cost. In this research contents, we introduce a new procedure to solve bi-objective transportation problem namely time and cost minimizing TP under multi-choice interval cost parameters.

Considering the real-life situations, we develop a transportation problem under fuzzy decision variable. Considering the fuzzy multi-choice goal corresponding to each of the allocations, we incorporate a new class of TP namely FTP. We extend the study into multi-objective environment for better results of FTP.

We initiate the study of cost reliability in the multi-objective transportation problem under uncertain environment. Assuming the uncertainty in real-life decision making problems, the concept of reliability is incorporated in the transportation cost and the effectiveness is justified through the proposed MOTP. Furthermore, considering the real phenomenon in the MOTP, we treat the transportation parameters, like as supply and demand as uncertain variables and obtain a better solution in compare to traditional TPs.

In this thesis, we attempt to formulate the mathematical model of Two-Stage multi-objective transportation problem where we design the feasibility space based on the selection of goal values. Considering the uncertainty in real-life situations, we incorporate the interval grey parameters for supply and demand in the Two-Stage MOTP, and a procedure is applied to reduce the interval grey numbers into real numbers.

Choosing several modes of transportation in a TP, a new method is designed for solving transportation problem by introducing the multi-modal transport systems. Here we incorporate the situation of multi-mode of transportation and analyze the way to solve TP under this situation and propose a better mode of transportation for optimal solution.

Again, we consider the study of multi-choice multi-item TP in the inventory optimization problem. Two different classes of TP such as inventory optimization and transportation optimization made under the consideration of a single mathematical model and noted as a new model namely IOIT.

The proposed mathematical models and methodologies are justified by constructing the real-life examples. Finally, conclusions are described according to the studies and the new ways are sighted for the future scope of studies.

Key Words: Transportation Problem, Multi-Choice Programming, Fuzzy Programming, Fuzzy Decision Variable, Multi-Objective Optimization, Decision Making Problem, Interval Number, Goal Programming, Revised Multi-Choice Goal Programming, Conic Scalarization Function, Utility Function, Linear Programming Problem, Multi-modal Transportation Problem, Both-Stage Transportation Problem, Uncertain Programming, Cost Reliability, Grey Number.

Abbreviations

CSF	Conic Sclarization Function
DM	Decision Maker
FM	Fuzzy Mathematics
FDV	Fuzzy Decision Variable
FMCGP	Fuzzy Multi-Choice Goal Programming
FTP	Fuzzy Transportation Problem
LINDO	Linear Interactive and Discrete Optimization
LPP	Linear Programming Problem
GP	Goal Programming
IO	Inventory Optimization
IOIT	Inventory Optimization in Integrated Transportation
MATLAB	Matrix Laboratory
MCITP	Multi-Choice Interval-Valued TP
MCDM	Multi-Criteria Decision Making
MCMTP	Multi-Choice Multi-Objective Transportation Problem
MCP	Multi-Choice Programming
MODM	Multi-Objective Decision Making
MOO	Multi-Objective Optimization
MOTP	Multi-Objective Transportation Problem
MMTP	Multi-Modal Transportation Problem
NLP	Non Linear Programming
OR	Operations Research
OP	Optimization Problem
TP	Transportation Problem
RMCGP	Revised Multi-Choice Goal Programming

List of Symbols

$\mathbb{R}$	Set of real numbers
$\mathbb{R}^n$	Euclidean space in n-dimensions
$I = [a, b]$	Closed interval on $\mathbb{R}$
z	Binary variable
Pr	Probability distribution
C_{ij}^k	Cost coefficient of t-th route from i-th origin to j-th destination
Cl	Closure of a set
Z^k	k-th objective function of MOO
a_i	Availability of i-th constraint
b_j	Demand of j-th constraint
α_i	Pre-deterministic confidence level of i-th constraint
β_j	Pre-deterministic confidence level of j-th constraint
η_k	Pre-deterministic confidence level of k-th constraint
w_i	Weight attached with i-th goal
min	Minimize
max	Maximize
$\mathbb{M}$	Measure of uncertain distribution

Contents

Chapter 1

Introduction and Literature Survey

This chapter depicts on mathematical programming, more specifically in the field of Transportation Problem (TP). Different types of TP are presented under various environments, such as multi-objective, multi-choice, fuzzy, interval, stochastic environments, etc. Several methodologies such as fuzzy programming, goal programming, revised multi-choice goal programming, conic scalarization approach, etc. are introduced according to the mathematical point of view. A brief literature survey of the proposed research work, objective, scope and organization of the thesis are included in this chapter.

1.1 Introduction

Operations Research (OR) is a discipline which encompasses a wide range of real-life problem involving solution techniques and methods applied in pursuit of improved decision-making and efficiency such as mathematical optimization, econometric methods, simulation, neural networks, data envelopment analysis, decision analysis and the analytic hierarchy process, etc. The modern ground of OR arose during World War II. In the World War II, OR was defined as a scientific method for providing executive departments with a quantitative basis for decisions regarding the operations under the control. The term "optimization" is the root of the study of OR. The optimization is used in different areas of study like Mathematical optimization, Engineering optimization, Economics and business, Information technology, etc.

A mathematical problem is an Optimization Problem (OP) where the objective function is maximized or minimized with or without some prescribed set of constraints. Requirements in real-life decision making situations enlarge the area of Mathematical optimization problems in different fields like Multi-Objective Optimization (MOO) problem, Multi-Choice Optimization Problem (MCOP), Multi-Modal Optimization Problem (MMOP), Optimization under uncertainty, Transportation Problem (TP), etc.

1.1.1 Optimization problem

Optimization is the mathematical discipline which is concerned with finding the maximum and minimum of functions with or without constraints. In the study of optimization, basically we need to optimize a real function of n variables $f(x_1, x_2, \ldots, x_n)$ with or without constraints. In an Optimization Problem (OP) for modeling a physical system, if there be only one objective function, and the task is to find the optimal solution, then it is called a single-objective optimization problem. The general form of single-objective optimization problem is as follows:

$$\begin{aligned} \text{minimize/maximize} \quad & f(x_1, x_2, \ldots, x_n) \\ \text{subject to} \quad & g(x_1, x_2, \ldots, x_n) \leq 0, \\ & h(x_1, x_2, \ldots, x_n) \geq 0, \\ & l(x_1, x_2, \ldots, x_n) = 0, \\ & \forall\ (x_1, x_2, \ldots, x_n) \in\ F \in R^n,\ \text{F is the feasible region.} \end{aligned}$$

Furthermore, single objective OP can be broadly divided into two different types of problem, namely, linear OP and non-linear OP. If the objective function or a constraint or a set of constraints or both be of non-linear type, then the OP is a non-linear OP otherwise it is a linear OP. Again, according to real-life situations, OP may be deterministic or fuzzy or interval order relation or multi-choice programming depending on parameter space.
Many real-world OPs cannot be formulated by a single objective function. When an OP is used for modeling a real-life problem which involves more than one objective function, the task of finding the optimal solution is called the MOO problem. It has been observed that the parameters which form a parameter space may be multiple types in which only one is to be selected which optimizes the objective functions. The most general mathematical model of the MOO problem is as follows:

$$\text{minimize/maximize} \quad f = f(f_1, f_2, \dots, f_n) \tag{1.1}$$

$$\text{subject to} \quad g(x_1, x_2, \dots, x_n) \leq 0, \tag{1.2}$$

$$h(x_1, x_2, \dots, x_n) \geq 0, \tag{1.3}$$

$$l(x_1, x_2, \dots, x_n) = 0, \tag{1.4}$$

$$\forall \ (x_1, x_2, \dots, x_n) \in \ F, \tag{1.5}$$

where $f_1, f_2, \dots, f_n$ are the objective functions of the decision variables $x_1, x_2, \dots, x_n$ are called decision variables. Here, MOO problem is also studied in different environments.

1.1.2 Transportation problem

The TP is a kind decision making problem which may be considered as the central nerve system to keep the balance in economical world from ancient day to till today. It can be delineated as a special case of a Linear Programming Problem (LPP). The classical sense of TP determines how many units of a commodity are to be shipped from each point of origin to various destinations, satisfying source availabilities and destination demands, while minimizing the total cost of transportation along with cutting down the costs per unit of items for the purchasers.

The mathematical model of transportation problem is as follows:

$$\begin{aligned}
\text{minimize} \quad & Z = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}x_{ij} \\
\text{subject to} \quad & \sum_{j=1}^{n} x_{ij} \leq a_i \quad (i = 1, 2, \ldots, m), \\
& \sum_{i=1}^{m} x_{ij} \geq b_j \quad (j = 1, 2, \ldots, n), \\
& x_{ij} \geq 0 \ \ \forall \ \ i \text{ and } \ j,
\end{aligned}$$

where x_{ij} is the decision variable which represents how much amount of goods delivered from the i-th origin to the j-th destination. C_{ij} is the transportation cost per unit commodity. a_i and b_j are supply and demand at the i-th origin and the j-th destination respectively and $\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} b_j$ is the feasibility condition.

Multi-choice transportation problem:

Due to presence of multiple routes of transportation or fluctuation of the market, the transportation parameters like cost, supply and demand may not fixed always. Keeping the points of view, if we consider all or few of cost, supply and demand parameters as multi-choice nature, then the TP becomes a multi-choice TP. In the atmosphere of multi-choice transportation parameters, the mathematical model of the Multi-Choice Transportation Problem (MCTP) is defined as follows:

$$\text{minimize} \quad Z = \sum_{i=1}^{m}\sum_{j=1}^{n} (C_{ij}^1 \text{ or } C_{ij}^2 \text{ or } \ldots \text{ or } C_{ij}^r)x_{ij}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \leq (a_i^1 \text{ or } a_i^2 \text{ or } \ldots \text{ or } a_i^p) \quad (i = 1, 2, \ldots, m),$$

$$\sum_{i=1}^{m} x_{ij} \geq (b_j^1 \text{ or } b_j^2 \text{ or } \ldots \text{ or } b_j^q) \quad (j = 1, 2, \ldots, n),$$

$$x_{ij} \geq 0 \quad \forall \quad i \text{ and } \quad j.$$

Multi-objective transportation problem:

Single objective transportation problem is not enough to formulate all the real-life transportation problems. The transportation problem with multiple objective functions are considered as Multi-Objective Transportation Problem (MOTP). However, we deal with those kind of objective functions, which are conflicting and non commensurable to each other involving TP. If there be more than one objective function in a TP, then it becomes a MOTP, whose mathematical model is as follows:

$$\text{minimize/maximize} \quad Z^t = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}^t x_{ij} \quad (t = 1, 2, \ldots, K)$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \leq a_i \quad (i = 1, 2, \ldots, m),$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j \quad (j = 1, 2, \ldots, n),$$

$$x_{ij} \geq 0 \quad \forall \quad i \text{ and } \quad j.$$

Multi-choice multi-objective transportation problem:

In multi-choice environment, the mathematical model of MOTP, i.e., the mathematical model of Multi-choice Multi-Objective Transportation Problem (MC-MOTP) takes the following form:

$$\text{minimize/maximize} \quad Z^t = \sum_{i=1}^{m} \sum_{j=1}^{n} (C_{ij}^{t1} \text{ or } C_{ij}^{t2} \text{ or } \ldots \text{ or } C_{ij}^{tr}) x_{ij}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \leq (a_i^1 \text{ or } a_i^2 \text{ or } \ldots \text{ or } a_i^p) \quad (i = 1, 2, \ldots, m),$$

$$\sum_{i=1}^{m} x_{ij} \geq (b_j^1 \text{ or } b_j^2 \text{ or } \dots \text{ or } b_j^q) \quad (j = 1, 2, \dots, n),$$
$$x_{ij} \geq 0, \quad \forall \;\; i \text{ and } \;\; j.$$

Interval-valued transportation problem:

Considering the unstable situation of the market or weather condition, the transportation parameters like cost, supply and demand, may not be taken as crisp values. Keeping the points of view, if at least one of the transportation parameters is considered as interval valued then the TP becomes an interval valued TP.

Multi-choice interval-valued transportation problem:

Involvement of multi-choices in the interval valued TP, the transportation problem reduces to a multi-choice interval valued transportation problem. Mathematical model of MCITP is as follows:

$$\begin{aligned}
\text{minimize} \quad & Z = \sum_{i=1}^{m}\sum_{j=1}^{n} (C_{ij}^1 \text{ or } C_{ij}^2 \text{ or } \dots \text{or } C_{ij}^k) x_{ij} \quad (k = 1, 2, \dots, K) \\
\text{subject to} \quad & \sum_{j=1}^{n} x_{ij} \leq (a_i^1 \text{ or } a_i^2 \text{ or } \dots \text{or } a_i^p) \quad (i = 1, 2, \dots, m) && (1.6) \\
& \sum_{i=1}^{m} x_{ij} \geq (b_j^1 \text{ or } b_j^2 \text{ or } \dots \text{or } b_j^q) \quad (j = 1, 2, \dots, n), && (1.7) \\
& x_{ij} \geq 0, \quad \forall \;\; i \text{ and } \;\; j. && (1.8)
\end{aligned}$$

Here, multi-choice parameters C_{ij}^k, a_i^p and b_j^q are interval numbers and these are defined as $C_{ij}^k = [C_{ij}^{k^l}, C_{ij}^{k^u}]$, $a_i^p = [a_i^{p^l}, a_i^{p^u}]$, $b_j^q = [b_j^{q^l}, b_j^{q^u}]$. The feasibility condition in this case is

$$\sum_{i=1}^{m} \max_{p^u}(a_i^{1^u}, a_i^{2^u}, \dots, a_i^{p^u}) \geq \sum_{j=1}^{n} \min_{q^l}(b_j^{1^l}, b_j^{2^l}, \dots, b_j^{q^l}).$$

Fuzzy transportation problem

Many real-life decision making problems, there may occur some cases where we need to optimize the objective function (Z) according to the decision maker's preferences. In this case, the decision variables (x_{ij}) of transportation problem are considered as real variables and the crisp solutions are obtained. In our daily life, many situations occur, where it is not applicable to fit a mathematical model using real variables and if we consider the decision variable in a TP as unknown fuzzy number (x_{ij}), then transportation problem becomes a Fuzzy Transportation Problem (FTP). The mathematical model of FTP can be written as follows:

$$\begin{aligned}
\text{minimize} \quad & Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} \tilde{x}_{ij} \\
\text{subject to} \quad & \sum_{j=1}^{n} \tilde{x}_{ij} \leq a_i \quad (i = 1, 2, \ldots, m), \\
& \sum_{i=1}^{m} \tilde{x}_{ij} \geq b_j \quad (j = 1, 2, \ldots, n), \\
& \tilde{x}_{ij} \geq 0 \;\; \forall \;\; i \text{ and } \; j.
\end{aligned}$$

Using the concept of FTP in MOTP with considering the decision variable as fuzzy variable, it is easy to formulate multi-objective FTP.

Two-stage grey transportation problem

In the classical sense of transportation problem, there are two types of node, one is source node from which the goods are delivered and other is destination node in which the transported goods are gathered. According to the real-life situations, in a TP, sometimes it is also to be considered that before transporting the goods in the destinations from the sources, goods are to be stored at warehouses from the sources and thereafter they are delivered to the destinations. So, the Decision Maker (DM) utilizes the concept for managing Two-Stage transportation in his position which maximizes the profit. A TP is called a Two-Stage transportation problem, if it consists of transporting the goods by two stages, namely, One Stage transportation problem and Another

Stage transportation problem. The TP which is considered in two stages of collecting the goods in warehouses is called an One Stage TP; the TP considered in Another Stage of distributing the goods is referred to as an Another Stage TP. Here, we introduce a new class of TP in which a single commodity of goods is transported to the destination using two times of transportation, and so, it is considered as Two-Stage TP. Considering multiple objective functions in Two-Stage TP, it reduces multi-objective Two-Stage TP. To accommodate the reality, the grey goals are considered into objective functions which make the TP a multi-objective Two-Stage grey transportation problem.

Multi-modal transportation problem

Treating reality in the decision making problem, transportation problem presents the situation such that, there may have origins/destinations in different levels to fulfill the requirements in the final destination points of a transportation network. Due to the factor of multiple routes or multi-modes of transportation in a TP, the TP becomes a Multi-Modal Transportation Problem (MMTP). Multi-modal transport which is also known as combined transport allows to transport the goods under a single contract, but it is performed with at least two modes of transport; the carrier is liable (in a legal sense) for the entire carriage, even though it is used by several different modes of transport such as sea, road, etc. The carrier does not have to possess all the means of transport, and in practice usually it does not valid. The carrier is often performed by sub-carrier which is referred to in legal language as "actual carriers". The carrier responsible for the entire carriage is addressed to as a Multi-Modal Transport Operator (MMTO). In a transportation problem, if there be at least one origin except the ground and final origins which have both receiving and dispatching capacity of goods, then it is called a MMTP.

Transportation problem under cost reliability

In the TP, the completion time of transportation of amount of goods should be finished within the specified time, otherwise there may be created a damage of the items or storing problem and/or the customer may reject the ordered item. In that situation, the transportation cost or the profit may not be considered as crisp value. Then the selection of goals for the objective functions or the solution of the MOTP cannot be made in usual way. To overcome this difficulty for selecting the proper goals to the objective functions, here, we incorporate the concept of reliability for the cost parameters in the TP. In that situation, we introduce a new term "cost reliability" for the transportation cost in the proposed study. Generally, Reliability refers the probability of a machine operating its intended purpose adequately for the period of time desired under the operating conditions encountered. More precisely, reliability is the probability with which the devices will not fail to perform a required operation for a certain period of time. Taking advantage of the reliability function in the real-life decision making problem, we formulate the MOTP where the objective functions are connected with some multi-choice goals. The advantage of MOTP under cost reliability is illustrated broadly in the proposed thesis.

Integrated optimization in inventory transportation

Inventory is the stock of items or resources used in an organization. The study of inventory refers to know how much amount of goods have to be sold by decision maker and how much amount left after sold and how much amount need to order from suppliers to keep stock with enough product. In the classical sense of basic inventory optimization, inventory and transportation are carried out separately and total logistics cost is calculated by summing the separate outcome results. On the other hand, an integrated optimization in inventory transportation is a problem to optimize the combination of transportation cost and inventory cost under the prerequisite assumptions. So, the main objective is to reduce the total logistics cost and determine the transportation and inventory strategies of the system. Considering the uncertain situations in real-life problems, the supply and demand are considered as stochastic and multi-choice type respectively in the proposed study of IOIT.

1.1.3 Fuzzy programming

In the real-life uncertain situations, the fuzzy set theory is an important topic to read. Usually, the fuzzy set theory is used in the field of OP as a tool for solving MOO problems. Nowadays, it is not only used as its classical sense but also plays an important role for accommodating real-life uncertain decision making problems. The fuzzy set theory has been applied in many fields, such as operations research, management science, artificial intelligence, human behavior, etc. The fuzzy mathematical programming has been applied to many disciplines such as advertising, assignment, budgeting, computer section, diet section, location media planing, networks, project selection, transportation, water resource management and many others.

Fuzzy set: A fuzzy set $\tilde{A}$ is a pair $(A, \mu_{\tilde{A}})$ where A is a crisp set belongs to the universal set X and $\mu_{\tilde{A}} : X \to [0, 1]$ is a function, called membership function.

Optimization under fuzziness: The ordinary optimization process generally is used to minimize or maximize the objective function subject to a set of constraints. In the fuzzy optimization, objective function and the constraints are both considered vague and they are expressed by fuzzy set. The process is to find the set of parameter values which maximizes the satisfaction of both the objective and the constraints at the same time. Thus the objective and the constraints perform the same purpose of defining the solution set.

The concept of fuzzy optimization is applied to the classical optimization algorithms, such as Fuzzy Linear Programming Problem (FLLP) problem and fuzzy dynamic programming. In the case of FLLP problem, transportation cost, supply and demand parameters are known only in fuzzy numbers and it determines the amount to be shipped between each demand and supply node.

Fuzzy decision variable: In an optimization problem, usually the unknown variables are considered as real variables. Sometimes there may occur some situations where the decisions are made by selecting fuzzy numbers among a set of fuzzy numbers, in that situations the decision variables are not considered as real and they are taken as fuzzy decision variable. The fuzzy decision variable in the Transportation problem creates a new field of transportation namely fuzzy transportation problem under fuzzy decision variable.

1.1.4 Goal programming

In an optimization problem, if the optimal solution is obtained according to the desired value (namely goal) of an objective function by the decision maker, then the optimization problem becomes a Goal Programming (GP). Goal programming, an analytical approach is devised to address the decision making problem where targets have been assigned to all objective functions which are conflicting and non-commensurable to each other and DM interests to maximize the achievement level of the corresponding goals.

In long back, the main concept of GP was that to minimize the deviation between the achievement goals and the achievement levels. The mathematical model of Multi-Objective Decision Making (MODM) can be considered in the following form:

Model GP

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{K} w_i |Z^i(x) - g_i| \\ \text{subject to} \quad & x \in F, \end{aligned}$$

where F is the feasible set and w_i are the weights attached to the deviation of the achievement function. $Z^i(x)$ is the i-th objective function of the i-th goal and g_i is the aspiration level of the i-th goal. $|Z^i(x) - g_i|$ represents the deviation of the i-th goal. Later on, a modification on GP is provided and is denoted as Weighted Goal Programming (WGP) which can be displayed in the following form:

Model WGP

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{K} w_i (d_i^+ + d_i^-) \\ \text{subject to} \quad & Z^i(x) - d_i^+ + d_i^- = g_i, \\ & d_i^+ \geq 0, d_i^- \geq 0 \quad (i = 1, 2, \ldots, K), \\ & x \in F, \end{aligned}$$

where d_i^+ and d_i^- are over and under achievements of the i-th goal respectively. A vast studies has been developed in GP, but Chang (17) introduced the concept of Revised Multi-choice Goal Programming (RMCGP) for solving MODM, which is more effective than the GP or the WGP. The mathematical model of MODM using RMCGP is defined as follows:

Model RMCGP

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{K} \left[w_i (d_i^+ + d_i^-) + \alpha_i (e_i^+ + e_i^-) \right] \\ \text{subject to} \quad & Z^i(X) - d_i^+ + d_i^- = y_i \quad (i = 1, 2, \ldots, K), \\ & y_i - e_i^+ + e_i^- = g_{i,max} \ \text{ or } \ g_{i,min} \quad (i = 1, 2, \ldots, K), \end{aligned}$$

$$g_{i,min} \leq y_i \leq g_{i,max} \quad (i = 1, 2, \ldots, K),$$
$$d_i^+, d_i^-, e_i^+, e_i^- \geq 0 \quad (i = 1, 2, \ldots, K),$$
$$x \in F.$$

Here, F being the feasible set and y_i is the continuous variable associated with i-th goal which restricted between the upper ($g_{i,max}$) and lower ($g_{i,min}$) bounds and e_i^+ and e_i^- are positive and negative deviations attached to the i-th goal of $|y_i - g_{i,max}|$ and α_i is the weight attached to the sum of the deviations of $|y_i - g_{i,max}|$, other variables are defined as in WGP.

1.1.5 Conic scalarization

In most of the cases for determining optimal solution of the MOO [equations (1.1) to (1.5)], the model is transformed to a scalar-valued optimization problem and on solving it, we obtain the compromise solution. In many research works such as Kim and Weck (71), Koski (73), Zaffaroni (167), using the weight w_t $(t = 1, 2, \ldots, K)$ for the t-th objective function, the MOO problem is reduced to the scalar problem as follows:

$$\text{minimize} \quad \sum_{t=1}^{K} w_t f_t(x), \quad x \in F.$$

By solving the scalar problem for a variety of parameters, for instance, for different weights, several solutions of the MOO problem are generated. Based on much better computer performances, it is now possible to represent the whole efficient set those are not obtained through the old techniques.

Using the classical sense of cone of a set and efficient point, the Conic Scalarization approach is introduced, which is an effective technique to obtain proper efficient solutions for MOO problem. In one of the chapter of this thesis, we use the Conic Scalarization approach for solving MCMTP and to establish the effectiveness of the approach on comparing to the other scalar optimization techniques, like GP and RMCGP.

1.2 Literature survey

The TP was formalized in 1781 by the French mathematician Monge (108), which was considered as the earliest known anticipation of linear programming type of problems. Major advances were made in the field during World War II by the Soviet/Russian mathematician and economist Kantorovich (65). Consequently, the problem as it is now stated is sometimes known as the Monge-Kantorovich transportation problem. Kantorovich (65) published a research work on continuous version of the problem and later on Kantorovich (66) developed a study on planning and production in a transportation problem. Many scientific approaches incorporated toward analyzing real-life problems associated with the transportation problem, including operations research, engineering, economics, geographic information science and geography, etc. It is explored especially in the field of mathematical programming and engineering literatures. Sometimes it is referred to as the facility location and allocation problem, the transportation optimization problem can be modeled as a large-scale mixed integer Linear Programming Problem (LPP). The basic model of transportation problem was originally developed by Hitchcock (47). He first considered the problem of minimizing the cost of distribution of product from several factories to a number of customers. Later on, Koopmans (72) presented an independent study on optimum utilization of the transportation system. Dantzig (24) proposed the simplex method for solving transportation problem which is known as the primal simplex transportation method.

The single objective transportation problem is not enough to handle real-life decision making problem due to our present competitive market scenario. To cover all the real-life situations on TP, we have to introduce here multi-objective TP. Charnes and Cooper (21) first discussed various approaches on the solution of managerial level problems involving multiple conflicting objective functions. Garfinkl and Rao (39) worked out the two objective problems by giving high and low priorities to the objective functions. Verma et al. (156) used fuzzy min operator approach to develop a compromise solution for the multi-objective transportation problem. Ringuest and Rinks (125) proposed two interactive algorithms for generating all non-dominated solutions and identified minimum cost solution as the best compromise solution. Waiel (157) developed a multi-objective transportation problem under fuzziness to get compromise solution. Ebrahimnejad (29) developed a new approach for solving fuzzy transportation problems with generalized trapezoidal fuzzy numbers.

Linear programming problem is not sufficient to form all types of decision making problem in our real-life situations. As a result, non-linear programming problem has been incorporated in the multi-objective TP. In this regard, we present here three important research works on TP with non-linear cost. Shetty (143) discussed a method to solve non-linear transportation problem with non-linear cost. Florian (36) developed a study on non-linear cost network models in transportation analysis. A study on non-linear integer programming transportation models introduced by Yang et al. (164). Recently, Maity and Roy (98) established a study on MOTP introducing a new concept on non-linear cost.

Multi-modal TP is a important class of TP in transportation planning and decision making problems. Murphy (111) pointed out the dual of the transportation problem and its implications for land-use and transport planning in Urban spatial location. James et al. (59) discussed on improving transportation service quality based on information fusion. A good number of works on transportation safety planning were developed by Abdel-Aty et al. (1), Zhi-

Chun et al. (169), Luathep et al. (76), Xu and Cai (163), Sheu and Chen (142), Tootkaleh et al. (152), Ergun et al. (28).

Many approaches are available in literature to deal with the uni-modal transport problem. Nanry and Wesley (112) presented an overview on uni-modal TP. In the multi-modal transportation problem, also some works have been done there, though none of these works solve the complete logistics problem, being centered in other problems associated with multi-modal transportation or in subproblems that do not represent all the constraints. Macharis and Bontekoning (92) discussed the opportunities for OR in intermodal freight transport. The research work (92) was reviewed on OR models which is currently used in this emerging transportation research field and defines the modeling problems which need to be addressed. Eibl et al. (27) introduced a case study applying an interactive vehicle routing and scheduling software to a brewing company in the UK. They explained how a commercial tool was applied to schedule the day-by-day (operational) vehicle routing and scheduling to distribute the goods. This tool was specific for the brewing problem, and the operator that manages the tool needs a previous training process to manage all variables involved. In this case, the solution is quite domain-independent, with less user knowledge requirements. Catalani (13) considered a statistical study to improve the intermodal freight transport in Italy, by using the road-ship and road-train transports. Qu and Chen (121) posed the multi-modal transport problem as a Multi-Criteria Decision Making (MCDM) problem. They proposed a hybrid MCDM by combining a Feed-forward Artificial Neural Network with the Fuzzy Analytic Hierarchy Process. The case study was a network in which nodes represent terminals, and edges represent different transportation modes such as road, ship and train etc. The model can deal with several cost functions and constraints, but they only defined six nodes, while in our proposed model there may have thousand of nodes.

Several research papers [Mahapatra et al. (93), Ropke and Pisinger (129), Veloso (155)] are available in the literature to analyze the transportation problem, but to the best of our knowledge, till now no one has used multi-modal transport systems for solving the transportation problems for decision making problems.
The concept of Grey Numbers emerged as an effective model for systems with partial information known. Liu (88) proposed a study on forming a new algebraic system of grey numbers. Palanci et al. (117) studied on uncertainty under grey goals in a cooperative game. A study on interval grey numbers to solve grey multi-attribute decision making problem was introduced by Honghua and Yong (49). Liu et al. (90) proposed a study on interval grey number which suggested some knowledge on the degree of greyness of grey numbers. A study on general grey number and their operations was given by Liu et al. (91). Xie and Liu (161) presented novel methods on comparing grey numbers. Mi et al. (105) introduced a study on Generic Second Order Macroscopic (GSOM) model based on interval grey number.

Cai and Lei (12), Kutanoglu and Lohiyav (74) studied on integrated optimization in inventory transportation model and they expressed in their study that transportation and storage of the product are the most important aspect in logistics system. Weijun and Cui (160) showed on their study that strategy of inventory and transportation are extremely important when they together come into real inventory problem. When the transportation plan is made up, the volume discounts brought by large quantities of transportation should not be pursuit excessively [cf., Cetinkaya and Lee (14), Xiao-Feng et al. (162)]. As this would bound to increase inventory costs throughout the system, also when the inventory strategy is determined, transportation cost cannot be dealt as a fixed charge, but as the variable cost directly impacting on transportation frequency and inventory distribution [Mason (103)]. Maity and Roy (99) proposed an integrated study on inventory and transportation problem under multi-choice and stochastic environment.

Goal Programming (GP), an analytical approach is devised to address the decision making problem where targets have been assigned to all objective functions which are conflicting and non-commensurable to each other and DM interests to maximize the achievement level of the corresponding goals. Charnes et al. (20) introduced the concept of GP. The interesting philosophy and high applicability of GP in handling real world decision making problems with multi objectives structures made it very useful and widespread. This leads to further development of GP for different decision making problems. The related research can be categorized into two broad classes: goal programming techniques which are proposed for crisp decision making problems and fuzzy goal programming models. Many research papers in the goal programming literature belong to the first class. The research papers by Lee (78), Ignizio (50), Romero (126), and Tamiz et al. (150) belong to this class. Tamiz et al. (151) presented a bibliography of the related researches published during 1990-2000. The second class includes the developed goal programming models for decision making in fuzzy environment. The proposed models in this class used the fuzzy set theory as a modeling tool to deal with the uncertainty of real world decision making problems. The uncertainty of decision making problem may exist because of imprecision aspiration levels, using linguistic variables, vague objective priorities or weights, uncertainty of resources, technological coefficients, etc. In the 1980s, fuzzy sets have been used in GP models to deal with the uncertainty of parameter and as well to represent a satisfaction degree of the decision maker with respect to his/her preference structure. The second class contains the research papers by Narasimhan (113), Hannan (46), Wang et al. (159), and many others. Finally, a comprehensive overview of the state-of-the-art in goal programming can be found in [cf. Ignizio (51; 52)]. Recently, Maity and Roy (97) presented a study to solve MOTP using utility function approach under goal environment.

However, in the real world situations, decision making problems may arise in economics, industry, health care, transportation, agriculture, storing seeds, military purpose, and technology etc. with different structures which cannot be handled using standard decision making approaches. For example, in a multi-choice multi-objective decision making problem, the decision maker presents multi aspiration levels as goals for each objective, the classical models of decision making including goal programming cannot be applied directly. To deal with this type of problems, it is very much essential to develop new decision making models. To do this, Chang (16) proposed a Multi-Choice Goal Programming (MCGP) approach to deal with such problems. Chang (17) revised his approach to make it easier understanding and implementation of linear programming packages for solving such problems. Liao (83) also presented the formulation of multi-segmented goal programming which can be applied to solve multiple decision making problems which have multi-segmented aspiration levels. Recently, Roy et al. (131) presented the formulation and solution procedure on multi-choice transportation problem involving exponential distribution. Mahapatra et al. (95) formulated and solved multi-choice stochastic transportation problem involving extreme value distribution. Of late, Roy (132) described and solved multi-choice stochastic transportation problem involving Weibull distribution which has added a new dimension on real-life TP.

Recently, Maity and Roy (96) studied and discussed the solution procedure on multi-choice multi-objective transportation problem using utility function approach.

Fuzzy programming technique is a well known tool to solve multi-objective optimization problems occurred in real-life situations. Cadenas and Verdegay (11) solved multi-objective linear programming problem by fuzzy ranking function. Waiel (157) developed the MOTP under fuzziness to obtain the compromise solution. Gupta and Kumar (44), Ebrahimnejad and Tavana (30) worked on uncertainty under different fuzzy environments. Jiménez and Verdegay (60) used the fuzzy uncertainty in solving solid transportation problem. Many researchers have shown great interest on uncertain mathematical programming. A few references are presented with their works. In order to deal with human uncertainty, Liu (89) presented a study on uncertainty theory and later, it was modified by Liu et al. (90) based on normality, duality, sub-additivity and product axioms considering degrees of greyness in grey numbers. Gao (38) introduced some properties on continuous uncertain measure in his paper. In practical aspect, Liu et al. (91) proposed an uncertain programming of mathematical programming involving grey numbers. In very recent, Maity et al. (100) presented a study on uncertain TP under the consideration of cost reliability.

A study on Characterization over the Benson proper efficiency and scalarization in a non-convex optimization field proposed by Gasimov (40). Later on, Gasimov (41) introduced a class of monotonically increasing sub-linear functions on partially ordered real normed spaces and showed without any convexity and bounded-ness assumptions that support points of a set obtained by these functions are properly minimal in the sense of Benson (5). Thereafter, Gasimov and Ozturk (42) presented a study on separation via polyhedral conic functions. Of late, Roy et al. (135) established a study on MOTP using Conic

Scalarization approach.

1.3 Objective and scope of the thesis

The main objective of the thesis has been defined after an extension literature survey based on transportation problem under multi-choice environment. The main objectives of the research work are as follows:

1: To incorporate the concept of extra cost in cost parameter of a TP which produces a non-linear TP and analyze in multi-objective ground of TP under multi-choice demands.

2: Applying the utility function approach to solve a MOTP under multi-choice environment, an effective solution is obtained in compare to the existing studies such as GP and RMCGP.

3: The concept of conic scalarization is implemented to solve MOTP in which each objective function has some goals and an extended study is given to justify the efficiency of the study in multi-choice environment.

4: In a transportation problem, transportation time is a key factor, so considering transportation time and transportation cost in a TP we have studied an bi-objective transportation problem and most importantly the bi-objective function is solved through single objective function. The study is developed by considering the multi-choice interval-valued transportation parameters.

5: Here, we have defined transportation problem under fuzzy decision variable. Assuming the expected allocations in the cells of a TP as multi-choice fuzzy numbers, a new technique using multi-choice goal programming is introduced and solve it to get better results in both single objective and multi-objective grounds.

6: Introducing the time in a TP, we have discussed the concept of cost reliability in MOTP. Again, according to the vague phenomenon of real market situations, the supply and demand constraints are treated as uncertain variables and transformed them through a uncertain measure concept. Numerical example is given to justify the proposed study.

7: In a transportation problem there may occur two types of transportation in warehouses where the goods are stored from markets and then delivered to other places. So, there are two stages involving in the transportation and to introduce as Two-Stage TP. Based on real-life situation, the study is formulated under grey environment and multi-objective ground. A new method using utility function approach is discussed to solve and to select the goal of the objective functions in the proposed problem.

8: Again, we formulate the mathematical model of a TP considering different modes of transportation which is termed a multi-modal TP. On solving the MMTP, the optimal solution and corresponding mode of transportation are presented.

9: Transportation and inventory are two different branches of study, here we made a connection between them and formulated a technique IOIT which gives better solution in compare to traditional inventory and transportation optimization procedure. The study is extended for multi-item of goods in uncertain environments which make it more realistic.

1.4 Organization of the thesis

The whole thesis contains eleven chapters. A brief introduction related to the proposed research work is presented to Chapter-1. In Chapter-2, we develop a multi-objective transportation problem with non-linear cost and multi-choice demand. The Chapter-3 is devoted to solve multi-objective transportation problem using utility function approach. In Chapter-4, we extend the concept of Conic Scalarization approach and is used to solve multi-objective transportation problem. The Chapter-5, present the study of solving bi-objective optimization problem under the environment of multi-choice and interval valued transportation parameters. In Chapter-6, we provide the concept of transportation problem under fuzzy decision variable in both single objective and multi-objective cases. Chapter-7 is introduced the concept of cost reliability in multi-objective transportation problem under uncertain environment. In Chapter-8, we introduce the Two-Stage grey transportation problem using utility function approach. The Chapter-9 is devoted the study on multi-modal transportation problem. In Chapter-10, we introduce the study of integrated optimization in inventory and transportation problem. In the last Chapter, the conclusions and scope of future works are presented regarding our research work.

The chapter wise summary of the proposed research works is given below:
Chapter 1 introduces the study of optimization problem, especially in the field of transportation problem under several environments. A brief survey on optimization problem, transportation problem, fuzzy programming, goal programming, conic scalarization approach is furnished. We discuss in short the cost reliability, two-stage transportation problem, multi-modal transportation problem, integrated study on inventory transportation problem. Finally, we present the objective, scope and organization of the thesis.

In **Chapter** 2, we develop a mathematical model of multi-objective transportation problem with non-linear cost and multi-choice demand. The objective functions of the proposed transportation problem are non-commensurable and conflict with each other. Furthermore, the objective functions are non-linear type which are occurred due to extra cost for supplying the remaining goods from origin to destinations and demand parameters are treated as multi-choice type. Thus, the mathematical model is formulated by considering the non-linear cost and multi-choice demand. A general transformation technique is developed to tractable the multi-choice demand with the help of binary variables. Therefore, an equivalent multi-objective decision making model is established in order to find the optimal solution of the problem. The outcome from numerical example demonstrates the feasibility of the proposed method [A part of this chapter has been published in ***International Journal of Management Science and Engineering Management,*** Taylor & Francis, ESCI, 11(1), 62-70, (2016)].

Chapter 3 contains two parts, in first part, we present the study of Transportation Problem (TP) with interval goal under multi-objective environment. In most of the cases, Multi-Objective Transportation Problems (MOTPs) are solved by Goal Programming (GP) approach. Using GP, the solution of MOTP may not be satisfied always by the Decision Maker (DM) when the proposed problem contains interval-valued aspiration level. To overcome this difficulty, here we propose the approaches of Revised Multi-Choice Goal Programming (RMCGP) and utility function into the MOTP, and then compare the solutions. A real-life example is presented to justify and to test reality of the proposed concept. In second part, the study of first part has been extended in the multi-choice environment. An example is presented in multi-choice environment to justify the concept [First part of this chapter has been published in ***International Journal of Operational Research***, Inderscience, Scopus, 27(4), 513-529, (2016), and second part of this chapter has been published in ***Journal of Uncertainty Analysis and Applications***, Springer Open, 2:11, doi:10.1186/2195-5468-2-11, (2014)].

Chapter 4 explores the concept of Multi-Choice Multi-Objective Transportation Problem (MCMTP) under the light of Conic Scalarizing function. The MCMTP is a multi-objective transportation problem where the parameters such as cost, demand and supply are treated as multi-choice parameters. Most of the MOTPs are solved by goal programming approach, but the solution of MOTP may not be satisfied always by the decision maker when the objective functions of the proposed problem contains interval-valued aspiration levels. To remove this difficulty, here we propose the approaches of revised multi-choice goal programming and conic scalarizing function into the MOTP, and then we compare among the obtained solutions. Two numerical examples are presented to show the feasibility and usefulness of the discussion topic in the chapter [A part of this chapter has been published in ***Annals of Operations Research,*** **Springer, SCI, IF: 1.406,** DOI 10.1007/s10479-016-2283-4.]

In **Chapter** 5 we consider the study of Transportation Problem (TP) in the light of multi-Choice environment with interval analysis. The parameters of TP follow multi-choice interval valued type so this form of TP is called Multi-Choice Interval Transportation Problem (MCITP). Introduction of time is an important notion in TP of this chapter. Transportation time and cost, both are minimized through single objective function of TP, which is the main aim of this chapter. A procedure is shown for converting from MCITP to deterministic TP and then solve it. Finally, a case study is presented to illustrate the usefulness of the proposed study [A part of this chapter has appeared in **Journal of Intelligent & Fuzzy Systems**, IOS Press, SCIE, IF: 1.004].

Chapter 6 develops a study on TP under fuzzy decision variable. This chapter is divided into two parts. In first part, we consider the study of a single objective TP with fuzzy decision variable. Generally, the decision variable in a transportation problem is considered as real variable. But, here the decision variable in each node is chosen from a set of multi-choice fuzzy numbers. A new formulation of mathematical model of Fuzzy Transportation Problem (FTP) with fuzzy goal to the objective function is designed. After that, the solution technique of the proposed model is included through multi-choice goal programming approach. The proposed approach is not only improved the applicability of goal programming in real world situations but is also provided useful insight about the solution of a new class of the TP. Finally, a real-life example is incorporated to analyze the feasibility and usefulness of the study. The last part of this chapter considered the study of first part extending the concept of single objective TP to a MOTP. The usefulness is justified through the numerical examples [First part of this chapter is communicated in **International Journal**; and second part part of this chapter has accepted for publish in ***International Journal of Operations Research and Information Systems (IJORIS), IGI Global, Info-SCI,*** Vol 8, No. 2].

Chapter 7 initiates the study of cost reliability in the multi-objective transportation problem under uncertain environment. Assuming the uncertainty in real-life decision making problems, the concept of reliability is incorporated in the transportation cost and the effectiveness is justified through the proposed MOTP. Again, considering the real phenomenon in the MOTP, we consider the transportation parameters, like supply and demand as uncertain variables. Also, we consider the fuzzy multi-choice goals to the objective functions of the MOTP; and Fuzzy Multi-Choice Goal Programming (FMCGP) is used to select the proper goals to the objective functions of the proposed MOTP. A numerical example is presented to illustrate and to justify the proposed study [A part of this chapter has been published in ***International Journal of Computational Intelligence Systems (IJCIS)***, Atlantis Press and Taylor & Francis, SCIE, IF: 0.391, Vol. **9**, No. 5, pp. 839-849].

In **Chapter** 8 we define Multi-Objective Goal Programming (MOGP) is applied to solve problems in many application areas of real-life decision making problems. This chapter attempts to formulate the mathematical model of Two-Stage Multi-Objective Transportation Problem (MOTP) where we design the feasibility space based on the selection of goal values. Considering the uncertainty in real-life situations, we incorporate the interval grey parameters for supply and demands in the Two-Stage MOTP, and a procedure is applied to reduce the interval grey numbers into real numbers. Thereafter, we present a solution procedure to the proposed problem by introducing an algorithm and using the approach of Revised Multi-Choice Goal Programming (RMCGP). In the proposed algorithm, we introduce a utility function for selecting the goals of the objective functions. Finally, a case study is encountered to justify the reality and feasibility of the proposed study [A part of this chapter is submitted after revision in ***Central European Journal of Operations Research,*** **Springer, SCI, IF. 0.978**].

In **Chapter** 9, a new method is designed for solving Transportation Problem (TP) by considering the multi-modal transport systems. This new method is a combination of TP and multi-modal systems and here it is referred to as Multi-Modal Transportation Problem (MMTP). To analyze the proposed method a case study is included and solved which reveals a better impact for analyzing the real-life decision making problems [A part of this chapter is Communicated to **International Journal**].

In **Chapter** 10, we investigate the study of multi-item multi-choice transportation problem in the ground of inventory optimization. We study the basic inventory optimization and then we develop a methodology for Integrated Optimization in Inventory Transportation (IOIT) to reduce the logistic cost of a system. To accommodate the present status of real-life TP, the stochastic supply is taken into consideration in the study. We describe a technique to reduce stochastic supply constraint to deterministic supply constraint with the help of stochastic programming. An algorithm is presented to solve the proposed problem using MATLAB. Then the proposed problem is solved by well known optimization technique and the obtained solution is compared with the solution of basic inventory optimization method. An example is encountered to verify the effectiveness of the study in the chapter [A part of this chapter has been accepted for publication in **International Journal of Operational Research**, Inderscience, Scopus].

Finally, the concluding remarks on the work carried out in Chapters 2 to 10 are described in **Chapter 11**. Future scope of further research works on the presented topic is also discussed.

Chapter 2

Solving MOTP with Non-linear Cost and Multi-choice Demand*

In this chapter, we develop a mathematical model of multi-objective transportation problem with non-linear cost and multi-choice demand. Here, we introduce the concept of non-linear cost by developing a sense of extra cost for supplying the remaining goods from origin to destinations. A general transformation technique is developed to tractable the multi-choice demand with the help of binary variables. Thereafter, an algorithm presents the solution procedure of proposed model using fuzzy programming approach. Usefulness of the study is justified through a real-life example in comparing with traditional method.

2.1 Introduction

Transportation problems (TPs) sustain economic and social activity and are central nerve system to operations research and management science. The TP can be delineated as a special case of a linear programming problem and its model formulated to determine an optimal solution of the TP. The TP determines how many units of a commodity are to be shipped from each point of origin to various destinations, satisfying source availabilities and destination demands, while minimizing the total cost of transportation along with cutting

*A part of this chapter has appeared in ***International Journal of Management Science and Engineering Management,*** Taylor & Francis, ESCI, 11(1), 62-70, (2016).

down the costs per unit of items for the purchasers.

The single-objective TP is not adequate to handle real-life decision making problems owing to our present competitive market scenario. To cover all real-life situation of TPs, here we introduce the multi-objective TP. Again, Linear programming problems are not enough for formulating all types of decision making problem in real-life situations. As a result, non-linear programming problems are incorporated into multi-objective TPs. Here, in our problem, the non-linear objective function is occurred in the TP due to some goods are left after distributing the goods from the origin to the destination points.

Due to fluctuations in the competitive market scenario, we consider the demands as multi-choice rather than single-choice type. In this circumstance, we should require that the total amount of goods at all supply points must be equal to or greater than the total amount of goods at all demand points.

In recent years, method of multi-choice programming has become increasingly important in scientifically based decision making involved in practical problem arising in economics, industry, health care, transportation, agriculture, storing seeds, military purpose, and technology etc.

The main aim of this chapter is to formulate the multi-objective non-linear transportation model with multi-choice demand and to solve the proposed model using fuzzy programming approach which leads compromise solution of the proposed problem.

2.2 Mathematical model

In a typical transportation problem, a homogenous product is to be transported from several origins (or sources) to numerous destinations in such way that the total transportation cost is minimum. Suppose there are m origins $(i = 1, 2, \ldots, m)$ and n destinations $(j = 1, 2, \ldots, n)$. The sources may be production facilities, warehouses etc. and these are characterized by available supplies $a_1, a_2, \ldots, a_m$. The destinations may be warehouses and sales outlets etc., and these are noted by demand levels $b_1, b_2, \ldots, b_n$. The transportation cost C_{ij} is associated with transporting a unit of product from origin i to destination j. A variable x_{ij} is used to represent the unknown quantity to be transported from origin O_i to destination D_j. The mathematical model of a classical transportation problem is as follows:

Model 2.1

$$\begin{aligned} \text{minimize} \quad & Z = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}x_{ij} \\ \text{subject to} \quad & \sum_{j=1}^{n} x_{ij} \le a_i \quad (i = 1, 2, \ldots, m), \\ & \sum_{i=1}^{m} x_{ij} \ge b_j \quad (j = 1, 2, \ldots, n), \\ & x_{ij} \ge 0 \;\; \forall \;\; i \;\; \text{and} \;\; j. \end{aligned}$$

Due to some unstable situations of market or for the cause of some special discounts to the customers in business ground, there may exist some cases that the cost parameter per unit commodity in transportation problem changes in such a way that it depends on the number of goods are delivered to destination from the origin along with the source capacity of supply. Assuming that the amount of goods remains in the origin after transportation which lead an extra cost according to the following rule:

$$\text{Extra Cost} = \frac{a_i - \text{no. of goods supplied to the } j^{th} \text{ destination from } i^{th}\text{origin}}{\text{total capacity of supply at } i^{th} \text{ origin}} \times C_{ij}$$

If C_{ij} along with this extra cost represents the cost parameter in the transportation problem then the mathematical model reduces to the following form (i.e., Model 2.2):

Model 2.2

$$\begin{aligned} \text{minimize} \quad & Z = \sum_{i=1}^{m}\sum_{j=1}^{n} \left[C_{ij} + \frac{a_i - x_{ij}}{a_i} C_{ij} \right] x_{ij} \\ \text{subject to} \quad & \sum_{j=1}^{n} x_{ij} \le a_i \quad (i = 1, 2, \ldots, m), \\ & \sum_{i=1}^{m} x_{ij} \ge b_j \quad (j = 1, 2, \ldots, n), \end{aligned}$$

$$\text{and} \quad x_{ij} \geq 0 \quad \forall \; i \; \text{ and } \; j.$$

The objective function in Model 2.2 is of non-linear type and non-linearity occurs due to the effect of extra cost in the cost parameter in TP. In this situation, the transportation problem (i.e., Model 2.2) is treated as TP with non-linear cost.

Single objective transportation problem is not enough to formulate all the real-life transportation problems. The transportation problem with multiple objective functions are considered as multi-objective transportation problem (MOTP). However, we deal with those kind of objective functions, which are conflicting and non commensurable to each other.

Demands in the destinations may not be fixed always. Due to weather condition, instability in share market, unpredictable expectation in the market etc., the demands may be treated as multi-choice rather than single choice. The decision maker always tries to distribute the goods in such a way that the customers get it easily with profitable in both point of view.

In this respect, the mathematical model of the multi-objective non-linear transportation problem with multi-choice demand is written as follows:

Model 2.3

$$\text{minimize} \quad Z^k = \sum_{i=1}^{m}\sum_{j=1}^{n}\left[C_{ij}^k + \frac{a_i - x_{ij}}{a_i}C_{ij}^k\right]x_{ij} \; (k = 1, 2, \ldots, K)$$

$$Or, \text{minimize} \quad Z^k = \sum_{i=1}^{m}\sum_{j=1}^{n}\left[2C_{ij}^k x_{ij} - \frac{C_{ij}^k}{a_i}x_{ij}^2\right], \tag{2.1}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \leq a_i \;\; \forall \; i, \tag{2.2}$$

$$\sum_{i=1}^{m} x_{ij} \geq (b_j^1 \text{ or } b_j^2 \text{ or} \ldots \text{or } b_j^{s_j}) \;\; \forall \; j, \tag{2.3}$$

$$\text{and} \quad x_{ij} \geq 0 \quad \forall \;\; i \text{ and } j. \tag{2.4}$$

In Model 2.3, decision maker would like to minimize the set of K objectives simultaneously. In addition, the objective functions are also conflicting to each other. We assume that $a_i \;\; (i = 1, 2, \ldots, m)$ is the capacity of supply at the i^{th}

origin and $b_j = (b_j^1 \text{ or } b_j^2 \text{ or } \ldots, \text{ or } b_j^{s_j})$ $(j = 1, 2, \ldots, n)$ is the multi-choice demand at the j^{th} destination and $C_{ij}^k = (C_{ij}^1, C_{ij}^2, \ldots, C_{ij}^K)$ cost per unit goods for each objective function when the total amount is transported from the i^{th} origin to the j^{th} destination. Here Z^k (X) $= (Z^1(X), Z^2(X), \ldots, Z^k(X))$ is a vector of K objective functions and superscript on both $Z^k(X)$ and C_{ij}^k are used to identify the number of objective functions i.e., $k = 1, 2, \ldots, K$. Without loss of generality, it is assumed here that $a_i > 0$ and $b_j > 0$ for all i and j along with feasibility condition $\sum_{i=1}^m a_i \geq \sum_{j=1}^n \min_{s_j}(b_j^1, b_j^2, \ldots, b_j^{s_j})$, and $C_{ij}^k \geq 0 \; \forall \; i$ and j. The feasibility condition may differ according to the decision maker's point of view.

However, in the mathematical Model 2.3, we construct a TP with K objectives each of them are non-linear types, in real-life problems there may present some non-linear as well as linear objective functions in a TP. As the objective functions are conflicting to each other, so, non-linearity of an objective function does not effect on linearity of an another objective function.

2.3 Solution procedure

In Model 2.3, we consider that the constraints (2.3) are multi-choice constraints. These constraints cannot be handled directly unless convert those to deterministic constraints. A transformation technique has been considered by Mahapatra et al. (95) for converting from multi-choice constraints to deterministic constraints in a TP. Here, a general transformation technique is developed for implementing the same, which is shown in the following subsection. Again in Model 2.3, the objective functions are multi-objective type. To convert in a single objective problem from multi-objective programming problem, we describe fuzzy programming approach which is shown in later subsection.

2.3.1 General transformation technique is used to convert multi-choice demand constraints into deterministic constraints

Here we present the generalized function for selecting a single choice from a set of multi-choice of parameters using binary variables. If we have to choose one among t number of possibilities then, we use p number of binary variables where $2^{p-1} < t \leq 2^p$.

Let $t =^p C_0 +^p C_1 +^p C_2 + \ldots +^p C_d + k$, for some d satisfying $1 \leq d \leq p, 0 \leq k <^p C_{d+1}$. In the selection procedure ${}^pC_i$ refers the number of possibilities with value zero of i binary variables among p variables to select a single choice among multi-choice elements.

Let us take p binary variables $z_j^1, z_j^2, \ldots, z_j^p$ to deduce a formula which will select one among the t values $c_j^1, c_j^2, \ldots, c_j^t$.

Let us further construct a function with p binary variables,

$f_0(z) = (z_j^1 z_j^2 \ldots z_j^p)c_j^1$, where $z = (z_j^1, z_j^2, \ldots, z_j^p)$, when each z_j^i=1 for $i = 1, 2, \ldots, p$. Thus, $f_0(z)$= c_j^1, when $z_j^1 + z_j^2 + \ldots + z_j^p = p$. Again, let us assume a function

$f_1(z) = (1-z_j^1)z_j^2 \ldots z_j^p c_j^2 + (1-z_j^2)z_j^1 z_j^3 \ldots z_j^p c_j^3 + \ldots + (1-z_j^p)z_j^1 \ldots z_j^{p-1} c_j^{1+^pC_1}$.

If $z_j^1 + z_j^2 + \ldots + z_j^p = p-1$, $f_1(z)$ gives as its an output any value among the following c_j^t's: $c_j^2, c_j^3, \ldots, c_j^{1+^pC_1}$. Similarly, we consider

$$\begin{aligned} f_2(z) \quad = \quad & (1-z_j^1)(1-z_j^2)z_j^3 \ldots z_j^p c_j^{1+^pC_1+1} + (1-z_j^1)(1-z_j^3)z_j^2 z_j^4 \ldots z_j^p c_j^{1+^pC_1+2} \\ & + \ldots + (1-z_j^1)(1-z_j^p)z_j^2 \ldots z_j^{p-1} c_j^{1+^pC_1+(p-2)} + \\ & (1-z_j^2)(1-z_j^3)z_j^1 z_j^4 \ldots z_j^p)c_j^{1+^pC_1+(p-2)+1} + \\ & \vdots \\ & +(1-z_j^{p-1})(1-z_j^p)z_j^1 \ldots z_j^{p-2} c_j^{1+^pC_1+^pC_2}. \end{aligned}$$

If $z_j^1 + z_j^2 + \ldots + z_j^p = p-2$, the above function $f_2(z)$ gives anyone among the following c_j^t's: $c_j^{1+^pC_1+1}, c_j^{1+^pC_1+2}, \ldots, c_j^{1+^pC_1+^pC_2}$.

Proceeding in the same manner, we find,

$$f_d(z) = (1-z_j^1)(1-z_j^2) \ldots (1-z_j^d) z_j^{d+1} \ldots z_j^p c_j^{1+^pC_1+^pC_2+\ldots+^pC_{d-1}+1} +$$

$$(1-z_j^1)(1-z_j^2)\dots(1-z_j^{d-1})(1-z_j^{d+1})z_j^d z_j^{d+2}\dots z_j^p c_j^{1+{}^pC_1+{}^pC_2+\dots+{}^pC_{d-1}+2}$$
$$\vdots$$
$$+(1-z_j^{p-d+1})(1-z_j^{p-d+2})(1-z_j^p)z_j^1\dots z_j^{p-d}c_j^{1+{}^pC_1+{}^pC_2+\dots+{}^pC_d}.$$

If $z_j^1+z_j^2+\dots+z_j^p=p-d$, the above function gives anyone among the following c_j^t's: $c_j^{1+{}^pC_1+{}^pC_2+\dots{}^pC_{d-1}+1}, c_j^{1+{}^pC_1+{}^pC_2+\dots+{}^pC_{d-1}+2}, \dots, c_j^{1+{}^pC_1+{}^pC_2+\dots+{}^pC_{d-1}+{}^pC_d}$.
If $k=0$, the function $f(z)=f_0(z)+f_1(z)+\dots+f_d(z)$ gives anyone of the value c_j^t for all z satisfying $p-d\leq z_j^1+z_j^2+\dots+z_j^p\leq p$.
If $k\neq 0$, then $k<^p C_{d+1}$ and we formulate the function

$$\begin{aligned} f_{d+1}(z) &= (1-z_j^{i_1})(1-z_j^{i_2})\dots(1-z_j^{i_d})(1-z_j^{i_{d+1}})z_j^{d+2}\dots z_j^n c_j^{t-k+1}+\\ &\quad (1-z_j^{i_1})(1-z_j^{i_2})\dots(1-z_j^{i_d})(1-z_j^{i_{d+2}})z_j^{d+1}z_j^{d+3}\dots z_j^p c_j^{t-k+2}+\dots\\ &\quad +(\text{terms up-to } c_j^t). \end{aligned}$$

Whenever $z_j^1+z_j^2+\dots+z_j^p=p-(d+1)$, $f_{d+1}(z)$ can give ${}^pC_{d+1}$ a number of outputs in maximally. Here we use ${}^pC_{d+1}-k$ restrictions to diminish its possible outputs in k numbers. Let k-th term occurred at $i_1=i_1^{'}, i_2=i_2^{'},\dots,i_{d+1}=i_{d+1}^{'}$, then the restrictions are $p-(d+1)\leq z_j^{i_1}+z_j^{i_2}+\dots+z_j^{i_p}\leq p$;
$z_j^{i_1}+z_j^{i_2}+\dots+z_j^{i_{d+1}}\geq 1$, for all $i_1=i_1^{'}, i_2=i_2^{'},\dots,i_d=i_d^{'}, i_p\geq i_{d+1}>i_{d+1}^{'}$;
$z_j^{i_1}+z_j^{i_2}+\dots+z_j^{i_{d+1}}\geq 1$, for all $i_1=i_1^{'}, i_2=i_2^{'},\dots,i_{d-1}=i_{d-1}^{'}, i_{p-1}\geq i_d>i_d^{'}$;
$\vdots$
$z_j^{i_1}+z_j^{i_2}+\dots+z_j^{i_{d+1}}\geq 1$, for all $i_{p-d-1}\geq i_1>i_1^{'}$.
Thus, $f(z)=f_0(z)+f_1(z)+\dots+f_d(z)+f_{d+1}(z)$ gives the generalized selection function of the multi-choice c_j^t's.
Without loss of generality, treating the value of $c_j^t=1$ and using the product and summation notation, we formulate the following formula to select the crisp value of multi-choice parameters:

$$\prod_{i=1}^{p} z_j^i+\sum_{i_1=1}^{p}\left[(1-z_j^{i_1})\prod_{i=1,i\neq i_1}^{p} z_j^i\right]+\sum_{\substack{i_2=2\\ i_2>i_1}}^{p}\sum_{i_1=1}^{p}\left[(1-z_j^{i_1})(1-z_j^{i_2})\prod_{i=1,\ i\neq i_1,i_2}^{p} z_j^i\right]$$
$$+\dots+\sum_{\substack{i_d=d\\ i_d>i_{(d-1)}}}^{p}\ \sum_{\substack{i_{d-1}=d-1\\ i_{d-1}>i_{d-2}}}^{p}\dots\sum_{i_1=1}^{p}\left[(1-z_j^{i_1})(1-z_j^{i_2})\dots(1-z_j^{i_d})\prod_{i=1,i\neq i_1,..,i_d}^{p} z_j^i\right],$$

where $p-d \leq z_j^{i_1}+z_j^{i_2}+\ldots+z_j^{i_p} \leq p$, for all $i_1<i_2<\ldots<i_p$.
If $k \neq 0$, we add first k terms with the above function from the following formula:

$$\begin{aligned}
&(1-z_j^{i_1})(1-z_j^{i_2})...(1-z_j^{i_d})(1-z_j^{i_{d+1}}) \prod_{i=1, i\neq i_1,..,i_d,i_{d+1}}^{p} z_j^i \\
&+(1-z_j^{i_1})(1-z_j^{i_2})\ldots(1-z_j^{i_d})(1-z_j^{i_{d+2}}) \prod_{i=1, i\neq i_1,..,i_d,i_{d+2}}^{p} z_j^i \\
&+\ldots+(1-z_j^{i_1})(1-z_j^{i_2})\ldots(1-z_j^{i_d})(1-z_j^{i_p}) \prod_{i=1, i\neq i_1,..,i_d,i_p}^{p} z_j^i \\
&+(1-z_j^{i_1})(1-z_j^{i_2})\ldots(1-z_j^{i_{d+1}})(1-z_j^{i_{d+2}}) \prod_{i=1, i\neq i_1,..,i_{d+1},i_{d+2}}^{p} z_j^i \\
&+(1-z_j^{i_1})(1-z_j^{i_2})\ldots(1-z_j^{i_{d+1}})(1-z_j^{i_{d+3}}) \prod_{i=1, i\neq i_1,..,i_{d+1}}^{p} z_j^i+\ldots \\
&+(1-z_j^{i_{p-(d+1)}})(1-z_j^{i_{p-(d-1)}})\ldots(1-z_j^{i_{p-1}})(1-z_j^{i_p}) \prod_{\substack{i=1,\\ i\neq i_{p-d-1},i_{p-d+1},\ldots,i_p}}^{p} z_j^i.
\end{aligned}$$

Assuming that $i_1<i_2<\ldots<i_p$ and let k-th term occur at
$i_1^{'}, i_2^{'}, \ldots, i_{d+1}^{'}$, then the restrictions are
$p-(d+1) \leq z_j^{i_1}+z_j^{i_2}+\ldots+z_j^{i_p} \leq p$;
$z_j^{i_1}+z_j^{i_2}+...+z_j^{i_{d+1}} \geq 1$, for all $i_1=i_1^{'}, i_2=i_2^{'}, \ldots, i_d=i_d^{'}, i_p \geq i_{d+1}>i_{d+1}^{'}$;
$z_j^{i_1}+z_j^{i_2}+\ldots+z_j^{i_{d+1}} \geq 1$, for all $i_1=i_1^{'}, i_2=i_2^{'}, \ldots, i_{d-1}=i_{d-1}^{'}, i_{p-1} \geq i_d>i_d^{'}$;
$\vdots$
$z_j^{i_1}+z_j^{i_2}+\ldots+z_j^{i_{d+1}} \geq 1$, for all $i_{p-d-1} \geq i_1>i_1^{'}$.
In the proposed mathematical model, we consider the multi-choice demands $b_j=(b_j^1, b_j^2, \ldots, b_j^{s_j})$. For different values of j, the values of s_j are different. Using the above functions, the constraints (2.3) in Model 2.3 becomes as:

$$\tilde{b}_j = \sum_{g=1}^{t}(term)^g b_j^g \quad (j=1,2,\ldots,n), \tag{2.5}$$

where $(term)^g$ (for $g=1,2,\ldots,t$) are the t number of terms in the functions of the binary variables mentioned in above to reduce the t number of choices b_j^g to single choice $\tilde{b}_j$.

2.3.2 Fuzzy programming approach to convert non-linear MOTP into single objective non-linear TP

To solve the non-linear MOTP in this chapter, first, we use the fuzzy programming approach to reduce the multi-objective problem to a single objective problem then single objective problem is solved to find the compromise solution of the multi-objective problem. The steps to be followed for transforming a multi-objective problem to a single objective problem are as follows:

Step 1: First, we convert the constraints (2.3) from Model 2.3 involving multi-choice demand to deterministic constraints (2.5) using the technique discussed in the previous subsection.

Step 2: Solve the multi-objective deterministic problem obtained from Step 1, using only one objective at a time and ignoring the others. Repeat the process K times for the K different objective functions. Let $X_1^*, X_2^*, \ldots, X_K^*$ be the respective ideal solutions of the K objectives.

Step 3: Evaluate all these objective functions of all the ideal solutions obtained in Step 2 and formulate a pay-off matrix of order $K \times K$.

Pay-off Matrix

$$\begin{bmatrix} Z^1(X_1^*) & Z^2(X_1^*) & \cdots & Z^K(X_1^*) \\ Z^1(X_2^*) & Z^2(X_2^*) & \cdots & Z^K(X_2^*) \\ Z^1(X_3^*) & Z^2(X_3^*) & \cdots & Z^K(X_3^*) \\ \vdots & \vdots & \vdots & \vdots \\ Z^1(X_K^*) & Z^2(X_K^*) & \cdots & Z^K(X_K^*) \end{bmatrix}$$

Step 4: Obtain the lower bound L_k and upper bound U_k corresponding to the k^{th} objective function, $\forall\ k$. Then formulate the membership function using Zimmermann (171) approach for each objective function $Z^k(X)$ $(k = 1, 2, \ldots, K)$ as follows:

$$\mu(Z^k(X)) = \begin{cases} 0, & \text{if } Z^k(X) \geq U_k \\ \frac{U_k - Z^k(X)}{U_k - L_k}, & \text{if } L_k < Z^k(X) < U_K \quad k = 1, 2, \ldots, K \\ 1, & \text{if } Z^k(X) \leq L_k. \end{cases}$$

Step 5: By introducing an auxiliary variable λ, formulate an equivalent fuzzy non-linear programming problem as:

$$\begin{aligned} \text{maximize} \quad & \lambda \\ \text{subject to} \quad & \lambda \leq \mu(Z^k(X)) \ \ (k = 1, 2, \ldots, K), \\ \text{and} \quad & \text{the constraints } (2.2), (2.4) \ \& \ (2.5). \end{aligned}$$

Here, $\mu(Z^k(X))$ is the membership function of the k^{th} objective function for $(k = 1, 2, \ldots, K)$ as given in Step 4.

Step 6: Solve the equivalent crisp model obtained in Step 5 and derive the compromise solution.

2.4 Numerical example

The reputed computer company delivers laptop from four stores Kolkata (O_1), Durgapur (O_2), Darjeeling (O_3), Bhubaneswar (O_4) of India, to the dealers of four cities namely Burdwan (D_1), Kalyani (D_2), Agartala (D_3), Maldaha (D_4) in India. The transportation cost, production cost and servicing cost per unit product of laptops are provided in Tables 2.1, 2.2 and 2.3, respectively. All the supplied costs are minimum in Tables 2.1, 2.2 and 2.3 when the total number of laptops purchased by the dealers from a source otherwise the extra cost has to be paid according to the desired rule of the company in each case. Tables 2.1, 2.2 and 2.3 represent the costs (Rupees 1000/unit item) are given as follows:

Table 2.1: Transportation cost per unit product of laptop.

	D_1	D_2	D_3	D_4
O_1	2.08	1.9	2.5	3.5
O_2	2.2	1.8	3.8	2.0
O_3	2.0	2.5	1.7	1.6
O_4	2.2	2.9	2.8	2.4

Table 2.2: Production cost for unit product of laptop.

	D_1	D_2	D_3	D_4
O_1	25	23.5	24	25
O_2	24	26	25	23
O_3	24.5	25	25.5	24.5
O_4	25.5	27	26.5	25

Table 2.3: Servicing cost per laptop.

	D_1	D_2	D_3	D_4
O_1	0.4	0.5	1.3	0.7
O_2	0.9	0.8	1.5	0.6
O_3	0.8	0.75	0 .5	0.8
O_4	1.2	0.9	0.8	2.95

The capacities of store in the origins (O_i) are $a_1 = 29, a_2 = 26, a_3 = 32, a_4 = 28$ maximum in numbers and the multi-choice demands in the destinations (D_j) are $b_1 = (22, 24, 25, 27, 26), b_2 = (24, 23, 22), b_3 = (28, 30, 29), b_4 = 34$. The company wishes to find a compromise solution which minimizes the cost of each objective function at a time according to the supplied cost matrices. To justify the effectiveness of multi-choices demands, let us introduce an extra constraint assuming that the demand points D_2 and D_3 are maintained by one person and total demands in D_2 and D_3 is 51 or more than 51. Since, the proposed problem is a minimization problem, so, the value of the objective functions are always minimized and the minimum demand value will be taken for giving the optimal solution. But, when this extra condition imposed in the problem then the demand value at D_2 and D_3 cannot be precisely determined. The solution of formulated problem will suggest the selection of the demand from the multi-choice options.
The mathematical model is formulated corresponding to available data, as follows:

minimize

$$\begin{aligned} Z^1(X) &= 2(2.08x_{11} + 1.9x_{12} + 2.5x_{13} + 3.5x_{14} + 2.2x_{21} + 1.8x_{22} + 3.8x_{23} \\ &+ 2x_{24} + 2x_{31} + 2.5x_{32} + 1.7x_{33} + 1.6x_{34} + 2.2x_{41} + 2.9x_{42} + 2.8x_{43} \\ &+ 2.4x_{44}) - \frac{1}{29}(2.08x_{11}^2 + 1.9x_{12}^2 + 2.5x_{13}^2 + 3.5x_{14}^2) - \frac{1}{26}(2.2x_{21}^2 \\ &+ 1.8x_{22}^2 + 3.8x_{23}^2 + 2x_{24}^2) - \frac{1}{32}(2x_{31}^2 + 2.5x_{32}^2 + 1.7x_{33}^2 + 1.6x_{34}^2) \\ &- \frac{1}{28}(2.2x_{41}^2 + 2.9x_{42}^2 + 2.8x_{43}^2 + 2.4x_{44}^2) \qquad (2.6) \end{aligned}$$

minimize

$$Z^2(X) = 2(25x_{11} + 23.5x_{12} + 24x_{13} + 25x_{14} + 24x_{21} + 26x_{22} + 25x_{23} + 23x_{24}$$

$$
\begin{aligned}
&+ \quad 24.5x_{31} + 25x_{32} + 25.5x_{33} + 24.5x_{34} + 25.5x_{41} + 27x_{42} + 26.5x_{43} \\
&+ \quad 25x_{44}) - \frac{1}{29}(25x_{11}^2 + 23.5x_{12}^2 + 24x_{13}^2 + 25x_{14}^2) - \frac{1}{26}(24x_{21}^2 + 26x_{22}^2 \\
&+ \quad 25x_{23}^2 + 23x_{24}^2) - \frac{1}{32}(24.5x_{31}^2 + 25x_{32}^2 + 25.5x_{33}^2 + 24.5x_{34}^2) \\
&- \quad \frac{1}{28}(25.5x_{41}^2 + 27x_{42}^2 + 26.5x_{43}^2 + 25x_{44}^2) \qquad (2.7)
\end{aligned}
$$

minimize

$$
\begin{aligned}
Z^3(X) \;=\;& 2(.4x_{11} + 0.5x_{12} + 1.3x_{13} + .7x_{14} + .9x_{21} + .8x_{22} + 1.5x_{23} + .6x_{24} \\
&+ \quad .8x_{31} + .75x_{32} + 0.5x_{33} + .8x_{34} + 1.2x_{41} + .9x_{42} + .8x_{43} + 2.95x_{44}) \\
&- \quad \frac{1}{29}(.4x_{11}^2 + 3.8x_{12}^2 + 1.3x_{13}^2 + .7x_{14}^2) - \frac{1}{26}(.9x_{21}^2 + .8x_{22}^2 + 1.5x_{23}^2 \\
&+ \quad .6x_{24}^2) - \frac{1}{32}(.8x_{31}^2 + .75x_{32}^2 + 14.7x_{33}^2 + .8x_{34}^2) - \frac{1}{28}(1.2x_{41}^2 \\
&+ \quad .9x_{42}^2 + .8x_{43}^2 + 2.95x_{44}^2) \qquad (2.8)
\end{aligned}
$$

subject to

$$\sum_{j=1}^{4} x_{1j} \leq 29 \qquad (2.9)$$

$$\sum_{j=1}^{4} x_{2j} \leq 26 \qquad (2.10)$$

$$\sum_{j=1}^{4} x_{3j} \leq 32 \qquad (2.11)$$

$$\sum_{j=1}^{4} x_{4j} \leq 28 \qquad (2.12)$$

$$\sum_{i=1}^{4} x_{i1} \geq (25 \text{ or } 24 \text{ or } 22 \text{ or } 27 \text{ or } 26) \qquad (2.13)$$

$$\sum_{i=1}^{4} x_{i2} \geq (24 \text{ or } 23 \text{ or } 22) \qquad (2.14)$$

$$\sum_{i=1}^{4} x_{i3} \geq (28 \text{ or } 30 \text{ or } 29) \qquad (2.15)$$

$$\sum_{i=1}^{4} x_{i4} \geq 34 \qquad (2.16)$$

$$\sum_{i=1}^{4} x_{i2} + \sum_{i=1}^{4} x_{i3} \geq 51 \qquad (2.17)$$

$$\text{and} \qquad x_{ij} \geq 0 \;\; \forall \;\; i \;\; \text{and} \;\; j. \tag{2.18}$$

Using the procedure described in subsection 2.3.1, the constraints (2.13)- (2.15) transformed to deterministic constraints as follows:

$$\sum_{i=1}^{4} x_{i1} \geq 25z_1^1z_1^2z_1^3 + 24(1-z_1^1)z_1^2z_1^3 + 22(1-z_1^2)z_1^1z_1^3$$
$$+27(1-z_1^3)z_1^1z_1^2 + 26(1-z_1^1)(1-z_1^2)z_1^3 \tag{2.19}$$
$$\sum_{i=1}^{4} x_{i2} \geq 24z_2^1z_2^2 + 23(1-z_2^2)z_2^1 + 22(1-z_2^1)z_2^2 \tag{2.20}$$
$$\sum_{i=1}^{4} x_{i3} \geq 28z_3^1z_3^2 + 30(1-z_3^2)z_3^1 + 29(1-z_3^1)z_3^2 \tag{2.21}$$
$$1 \leq z_1^1 + z_1^2 + z_1^3 \leq 3 \tag{2.22}$$
$$z_1^1 + z_1^3 \geq 1 \tag{2.23}$$
$$z_1^2 + z_1^3 \geq 1 \tag{2.24}$$
$$1 \leq z_2^1 + z_2^2 \leq 2 \tag{2.25}$$
$$1 \leq z_3^1 + z_3^2 \leq 2 \tag{2.26}$$
$$z_p^q = 0 \text{ or } 1 \;\; \text{for} \;\; p, q = 1, 2, 3 \tag{2.27}$$

The Ideal solutions obtained by solving the objective functions (2.6)-(2.8) separately subject to the constraints (2.9)-(2.12) and (2.16)-(2.27) are given below:
$[X_1^*]$ = [22,0,0,0,0,22,0,2,0,0,0,32,0,0,29,0]
$[X_2^*]$ = [22,0,0,0,0,23,0,2,0,0,0,32,0,0,28,0]
$[X_3^*]$ = [0,23,0,0,22,0,0,2,0,0,0,32,0,0,28,0]
Using the above solution, we formulate the payoff matrix which is shown in Table 2.4.

Table 2.4: Payoff Matrix.

	Z^1	Z^2	Z^3
X_1^*	239.69	2956.00	82.50
X_2^*	245.87	2864.00	87.03
X_3^*	240.27	2964.00	81.75

Using above payoff matrix, we formulate the following membership functions corresponding to each objective function of our proposed problem as:

$$\mu(Z^1(X)) = \begin{cases} 0, & \text{if } Z^1(X) > U_1 \\ \frac{245.87 - Z^1(X)}{245.87 - 239.69}, & \text{if } L_1 \leq Z^1(X) \leq U_1 \\ 1, & \text{if } Z^1(X) < L_1 \end{cases}$$

$$\mu(Z^2(X)) = \begin{cases} 0, & \text{if } Z^2(X) > U_2 \\ \frac{2964 - Z^2(X)}{2964 - 2864}, & \text{if } L_2 \leq Z^2(X) \leq U_2 \\ 1, & \text{if } Z^2(X) < L_2 \end{cases}$$

$$\mu(Z^3(X)) = \begin{cases} 0, & \text{if } Z^3(X) > U_3 \\ \frac{87.03 - Z^3(X)}{87.03 - 81.75}, & \text{if } L_3 \leq Z^3(X) \leq U_3 \\ 1, & \text{if } Z^3(X) < L_3 \end{cases}$$

Using the procedure described in the subsection 2.3.2, finally we formulate the following model:

Model 2.4

maximize λ

subject to,

$6.18\lambda \leq 245.87 - (2(2.08x_{11} + 1.9x_{12} + 2.5x_{13} + 3.5x_{14} + 2.2x_{21} + 1.8x_{22} + 3.8x_{23} + 2x_{24} + 2x_{31} + 2.5x_{32} + 1.7x_{33} + 1.6x_{34} + 2.2x_{41} + 2.9x_{42} + 2.8x_{43} + 2.4x_{44}) - \frac{1}{29}(2.08x_{11}^2 + 1.9x_{12}^2 + 2.5x_{13}^2 + 3.5x_{14}^2) - \frac{1}{26}(2.2x_{21}^2 + 1.8x_{22}^2 + 3.8x_{23}^2 + 2x_{24}^2) - \frac{1}{32}(2x_{31}^2 + 2.5x_{32}^2 + 1.7x_{33}^2 + 1.6x_{34}^2) - \frac{1}{28}(2.2x_{41}^2 + 2.9x_{42}^2 + 2.8x_{43}^2 + 2.4x_{44}^2))$

$100.00\lambda \leq 2964 - (2(25x_{11} + 23.5x_{12} + 24x_{13} + 25x_{14} + 24x_{21} + 26x_{22} + 25x_{23} + 23x_{24} + 24.5x_{31} + 25x_{32} + 25.5x_{33} + 24.5x_{34} + 25.5x_{41} + 27x_{42} + 26.5x_{43} + 25x_{44}) - \frac{1}{29}(25x_{11}^2 + 23.5x_{12}^2 + 24x_{13}^2 + 25x_{14}^2) - \frac{1}{26}(24x_{21}^2 + 26x_{22}^2 + 25x_{23}^2 + 23x_{24}^2) - \frac{1}{32}(24.5x_{31}^2 + 25x_{32}^2 + 25.5x_{33}^2 + 24.5x_{34}^2) - \frac{1}{28}(25.5x_{41}^2 + 27x_{42}^2 + 26.5x_{43}^2 + 25x_{44}^2))$

$5.28\lambda \leq 87.03 - (2(0.4x_{11} + 0.5x_{12} + 1.3x_{13} + .7x_{14} + 0.9x_{21} + 0.8x_{22} + 1.5x_{23} + 0.6x_{24} + 0.8x_{31} + 0.75x_{32} + 0.5x_{33} + 0.8x_{34} + 1.2x_{41} + 0.9x_{42} + 0.8x_{43} + 2.95x_{44}) - \frac{1}{29}(0.4x_{11}^2 + 0.5x_{12}^2 + 1.3x_{13}^2 + 0.7x_{14}^2) - \frac{1}{26}(0.9x_{21}^2 + 0.8x_{22}^2 + 1.5x_{23}^2 + 0.6x_{24}^2) - \frac{1}{32}(0.8x_{31}^2 + 0.75x_{32}^2 + 0.5x_{33}^2 + 0.8x_{34}^2) - \frac{1}{28}(1.2x_{41}^2 + 0.9x_{42}^2 + 0.8x_{43}^2 + 2.95x_{44}^2))$

$2(2.08x_{11} + 1.9x_{12} + 2.5x_{13} + 3.5x_{14} + 2.2x_{21} + 1.8x_{22} + 3.8x_{23} + 2x_{24} + 2x_{31} + 2.5x_{32} + 1.7x_{33} + 1.6x_{34} + 2.2x_{41} + 2.9x_{42} + 2.8x_{43} + 2.4x_{44}) - \frac{1}{29}(2.08x_{11}^2 + 1.9x_{12}^2 + 2.5x_{13}^2 + 3.5x_{14}^2) - \frac{1}{26}(2.2x_{21}^2 + 1.8x_{22}^2 + 3.8x_{23}^2 + 2x_{24}^2) - \frac{1}{32}(2x_{31}^2 + 2.5x_{32}^2 + 1.7x_{33}^2 + 1.6x_{34}^2) - \frac{1}{28}(2.2x_{41}^2 + 2.9x_{42}^2 + 2.8x_{43}^2 + 2.4x_{44}^2) \leq 245.87$

$2(25x_{11} + 23.5x_{12} + 24x_{13} + 25x_{14} + 24x_{21} + 26x_{22} + 25x_{23} + 23x_{24} + 24.5x_{31} + 25x_{32} + 25.5x_{33} + 24.5x_{34} + 25.5x_{41} + 27x_{42} + 26.5x_{43} + 25x_{44}) - \frac{1}{29}(25x_{11}^2 + 23.5x_{12}^2 + 24x_{13}^2 + 25x_{14}^2) - \frac{1}{26}(24x_{21}^2 + 26x_{22}^2 + 25x_{23}^2 + 23x_{24}^2) - \frac{1}{32}(24.5x_{31}^2 + 25x_{32}^2 + 25.5x_{33}^2 + 24.5x_{34}^2) - \frac{1}{28}(25.5x_{41}^2 + 27x_{42}^2 + 26.5x_{43}^2 + 25x_{44}^2) \leq 2964.00$

$2(0.4x_{11} + 0.5x_{12} + 1.3x_{13} + 0.7x_{14} + 0.9x_{21} + 0.8x_{22} + 1.5x_{23} + 0.6x_{24} + 0.8x_{31} + 0.75x_{32} + 14.7x_{33} + 0.8x_{34} + 1.2x_{41} + 0.9x_{42} + 0.8x_{43} + 2.95x_{44}) - \frac{1}{29}(0.4x_{11}^2 + 0.5x_{12}^2 + 1.3x_{13}^2 + 0.7x_{14}^2) - \frac{1}{26}(0.9x_{21}^2 + 0.8x_{22}^2 + 1.5x_{23}^2 + 0.6x_{24}^2) - \frac{1}{32}(0.8x_{31}^2 + 0.75x_{32}^2 + 0.5x_{33}^2 + 0.8x_{34}^2) - \frac{1}{28}(1.2x_{41}^2 + 0.9x_{42}^2 + 0.8x_{43}^2 + 2.95x_{44}^2) \leq 87.03$

and (2.9)- 2.12), (2.16)- (2.27).

Result and discussion:

Model 2.4 is a non-linear programming problem. Using LINGO10 software, we list the following compromise optimal solution:

$[X^*]$ =[0,22,0,0,22,0,0,2,0,0,0,32,0,0,29,0]. The minimum values of the objective functions are $Z^1(X^*)$=244.52, $Z^2(X^*)$=2864.00, $Z^3(X^*)$=86.18 and the demands for each destination are represented as: $b_1 = 22, b_2 = 22, b_3 = 29, b_4 = 34$. The value of aspiration level is $\lambda = 0.28$. For minimization problem, the selection of multi-choice demands tends to the minimum value as much as possible. One can choose the demands as $b_1 = 22, b_2 = 23, b_3 = 28, b_4 = 34$ to satisfy the constraint (2.17) and then the compromised optimal solution becomes $[X^*]$ =[0,23,0,0,22,0,0,2,0,0,0,32,0,0,28,0] and in this case the minimum values of the objective functions are $Z^1(X^*)$=245.87, $Z^2(X^*)$=2876.00, $Z^3(X^*)$=87.03 and value of $\lambda = 0$. So as per our satisfactory level is concerned, selection of multi-choice done in the first solution is more better than the later.

2.5 Sensitivity analysis and comparison

Main intention of this study is to formulate and solve the multi-objective transportation problem with multi-choice demands and non linear cost. Let us discuss why we have considered such study and what is the contribution of this study in compare to other research works have been studied by many researchers in this direction. Study of non-linearity in TP have been studied by several researchers, Walther (158), Dahiya and Verma (22), Yang et. al. (164), Fanrong and Renan (34) and many others. Most of them are treated as non-linearity on TP by considering the objective function is non-linear or constraints are non-linear or both. Due to globalization of market or other real-phenomena, we have assumed the demands are multi-choice type and the non-linearity occurs in the TP. But here, we have presented the situation for transporting goods in such a way that if the number of transporting goods be the total amount of supply at a point then the transportation cost is minimum and it is equal to a fixed cost C_{ij} for the node (i, j), otherwise an extra cost has to be paid by the customer. In this regard, we have introduced the concept of an extra cost which makes the problem as non-linear TP. To understand the phenomena, let us consider Figure 2.1 which plots the transportation cost per unit item is varying with the supply amount (a_i) either linearly or non-linearly. Generally, we see that when the transportation cost per unit item is C_{ij} or $2C_{ij}$ or any value between C_{ij} and $2C_{ij}$ i.e., it follows from the graph that either dealer or customer will suffer for transporting the goods. But in our proposed model, the transportation cost per unit item follows a nonlinear graph because it depends on the amount of goods transported and the available supply of goods. In this situation, our proposed model works well from both points of view (both the supplier and the dealer).

2.6 Conclusion

In this chapter, a solution procedure for multi-objective transportation problem with non-linear cost and multi-choice demands are considered. Initially,

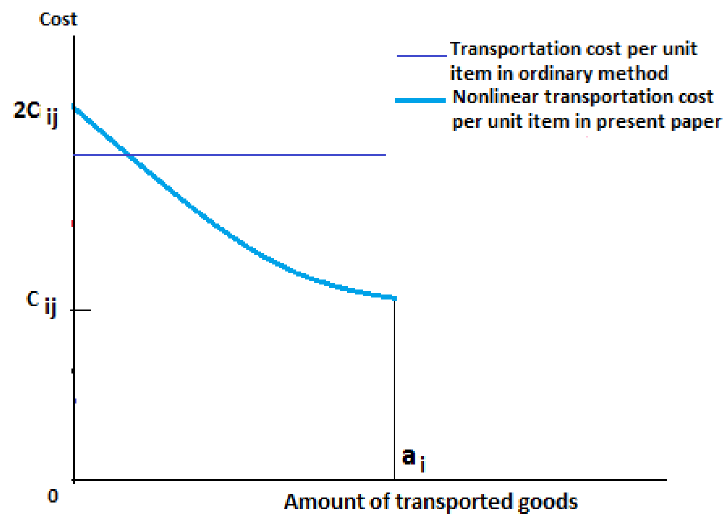

Figure 2.1: Transportation cost per unit item.

the demand constraints involving multi-choice demands have been converted into an equivalent deterministic demand constraints using binary variables. Fuzzy programming approach has been applied to solve the proposed problem and to obtain compromise solution from the multi-objective transportation problem. The selection of single choice of demand from multi-choices of demand for each destination has been calculated through binary variables.

The transportation problems have wide applications in many real-life problems of practical importance which reduce the cost specially in business environment. Multi-objective transportation problem with non-linear cost still exists in so many cases of managerial decision making problem such as planning of many complex resource allocation systems in the areas of industrial production, storing of foods, in which demands are of multi-choice type in practical situation. The content of this chapter may be a source of producing better results in such kind of complex decision making situations.

Chapter 3

Transportation Problem using Utility Function*

Here, we present the utility function approach to solve MOTP on the environment of interval goals to the objective functions of MOTP. We justify our concept of utility function which produces a better result in compare to GP and RMCGP for solving MOTP and MOTP with multi-choice of transportation parameters.

3.1 Introduction

MOTP plays an important role for decision making problem to cover the real-life situations. Goal programming, an analytical approach is devised to solve MOTP, where targets have been assigned to all objective functions. The objective functions are conflicting and non-commensurable to each other and the DM is interested to minimize the non-achievement of the corresponding goals. In other word, the DM derives an optimal solution with this strategy of GP which is satisfactory. However, using GP, the solution procedure for MOTP has some limitations. The main limitation behind GP is that the priority of goals for the DM is not easily considered. Based on practical situation, a new

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approach using utility function to solve MOTP is presented in this chapter. In the past, the notion of utility function introduced by several researchers cf., Al-nowaihi et al. (2), Yu et al. (165), Podinovski (119), Maity and Roy (96), and many others.

In earlier days, transportation problem was developed with the assumption that the supply, demand and cost parameters are exactly known. But in real-life applications, all the parameters of the transportation problem are not generally defined precisely. Keeping this point of view, we incorporate with MCMTP considering the parameters of transportation problem as multi-choice type.

Instead of single choice, if there may be involved several choices associated with the transportation parameters like cost, supply or demand, then decision maker confuses to select the proper choice for these parameters. In this circumstance, the study of transportation problem creates a new direction which is called multi-choice multi-objective transportation problem.

However, to the best of our knowledge, no work has been done on utility function to solve MOTP with DM's preferences. The main motivation of this study is to investigate a better solution of MOTP and MCMTP by using utility function approach and, then compared solution to other methods such as GP and RMCGP.

3.2 Mathematical model

The mathematical model of multi-objective transportation problem (MOTP) can be considered as follows:

Model 3.1

$$\text{minimize/maximize} \quad Z^t = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}^t x_{ij} \quad (t = 1, 2, \ldots, K) \tag{3.1}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \le a_i \quad (i = 1, 2, \ldots, m), \tag{3.2}$$

$$\sum_{i=1}^{m} x_{ij} \ge b_j \quad (j = 1, 2, \ldots, n), \tag{3.3}$$

$$\text{and} \quad x_{ij} \geq 0, \quad \forall \; i \; \text{ and } \; j. \tag{3.4}$$

Here C_{ij}^t, a_i, b_j are the cost, supply and demand parameters of t-th objective function in MOTP respectively and $\sum_{i=1}^m a_i \geq \sum_{j=1}^n b_j$ is the feasibility condition. According to the nature of the problem, the decision maker has right to choice the goals of the objective functions. Assuming that these goals are g_t $(t = 1, 2, \cdots, K)$ of K objective functions and the goals are defined as interval valued as $g_t = [g_{t,min}, g_{t,max}]$, $(t = 1, 2, \ldots, K)$.

In many real-life situation, the multiple choices in the transportation parameters like cost, demand and source create complexity to take decision to the DM. Multi-choice costs may occur due to several routes for transporting the goods. Due to weather condition or different seasons the demand or the supply becomes multi-choices in nature. In the atmosphere of multi-choice transportation parameters, the mathematical model of MCMTP is defined as follows:

Model 3.2

$$\text{minimize/maximize} \quad Z^t = \sum_{i=1}^m \sum_{j=1}^n (C_{ij}^{t1} \text{ or } C_{ij}^{t2} \text{ or...or } C_{ij}^{tr}) x_{ij} \quad \forall \; t \tag{3.5}$$

$$\text{subject to} \quad \sum_{j=1}^n x_{ij} \leq (a_i^1 \text{ or } a_i^2 \text{ or } \ldots \text{ or } a_i^p) \;\; \forall \; i, \tag{3.6}$$

$$\sum_{i=1}^m x_{ij} \geq (b_j^1 \text{ or } b_j^2 \text{ or } \ldots \text{ or } b_j^q) \;\; \forall \; j, \tag{3.7}$$

$$\text{and} \quad x_{ij} \geq 0, \quad \forall \; i \text{ and } j. \tag{3.8}$$

Here $(C_{ij}^{t1}$ or C_{ij}^{t2} or $\ldots$ or $C_{ij}^{tr})$, $(a_i^1$ or a_i^2 or $\ldots$ or $a_i^p)$ and $(b_j^1$ or b_j^2 or $\ldots$ or $b_j^q)$ are the multi-choice cost, supply and demand parameters for the t-th objective function. In a transportation problem, the total demands should be less or equal to the total capacity of supply to get a feasible solution. In present case for multi-choice of supply and demands the information of total capacity of supply in the origins and demands in the destinations are not precisely calculated. So, we select the maximum possible supply in the origins and consequently the minimum demand in the destinations and then formulated the feasibility condition as:

$\sum_{i=1}^{m} \max(a_i^1 \text{ or } a_i^2 \text{ or} \ldots \text{or } a_i^p) \geq \sum_{j=1}^{n} \min(\tilde{b}_j^1 \text{ or } \tilde{b}_j^2 \text{ or} \ldots \text{or } \tilde{b}_j^q)$. This feasibility condition is the best possible widely range of feasible region of the MCMTP. However, the feasibility condition can be remodeled according to choice of decision maker.

3.2.1 Reduction of MCMTP to MOTP

Due to the presence of multi-choice parameters in the objective functions and in the constraints, the MCMTP model is not in deterministic form. So, we reduce the MCMTP to a MOTP by the reduction procedure using binary variables, as described in subsection 2.3.1 of Chapter 2.

$$\text{Let} \quad \tilde{C}_{ij}^t = \sum_{g=1}^{T} (term)^g C_{ij}^{tg} \quad (i = 1, 2, \ldots, m; j = 1, 2, \ldots, n), \tag{3.9}$$

where $(term)^g$ (for $g = 1, 2, \ldots, T$) are the T number of terms in the functions of the binary variables mentioned in above. Similarly,

$$\tilde{a}_i = \sum_{g=1}^{P} (term)^g a_i^g \quad (i = 1, 2, \ldots, m), \tag{3.10}$$

$$\text{and} \quad \tilde{b}_j = \sum_{g=1}^{Q} (term)^g b_j^g \quad (j = 1, 2, \ldots, n), \tag{3.11}$$

where $(term)^g$ (for $g = 1, 2, \ldots, P$) are the P number of terms in the functions of the binary variables mentioned in above to reduce the P number of choices a_i^g to single choice $a_i^{'}$ and $(term)^g$ (for $g = 1, 2, \ldots, Q$) are the Q number of terms in the functions of the binary variables mentioned in above to reduce the Q number of choices b_j^g to single choice $b_j^{'}$.

Thus the equivalent MOTP of Model 3.2 is given in the following model:
Model 3.3

$$\text{minimize/maximize} \qquad Z^t = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}^{\prime t} x_{ij} \quad (t = 1, 2, \ldots, K) \tag{3.12}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \leq a'_i \quad (i = 1, 2, \ldots, m), \tag{3.13}$$

$$\sum_{i=1}^{n} x_{ij} \geq b'_j \quad (j = 1, 2, \ldots, n), \tag{3.14}$$

$$\text{and} \quad x_{ij} \geq 0 \quad \forall\, i \text{ and } j. \tag{3.15}$$

Here C''^{t}_{ij}, a'_i, b'_j are the reduced cost, supply and demand parameters of t-th objective function in MOTP respectively. The transportation problem i.e., Model 3.3 is same as the problem described in Model 3.1. We can solve Model 3.3 with the procedure described using the different techniques to solve Model 3.1.

3.3 Solution procedure

The approaches such as goal programming and revised multi-choice goal programming are used to solve the MOTP, which are defined as follows:

A. Goal programming approach

Let us briefly discuss the goal programming approach for solving MOTP (see Model 3.4). If d_t^+ and d_t^- be positive and negative deviations corresponding to the t-th goal of the objective function. Then the mathematical model is defined as follows:

Model 3.4

$$\text{minimize} \quad \sum_{t=1}^{K} w_t(d_t^+ + d_t^-) \tag{3.16}$$

$$\text{subject to} \quad Z^t(X) - d_t^+ + d_t^- = y_t \quad (t = 1, 2, \ldots, K), \tag{3.17}$$

$$g_{t,min} \leq y_t \leq g_{t,max} \quad (t = 1, 2, \ldots, K), \tag{3.18}$$

$$d_t^+, d_t^- \geq 0 \quad (t = 1, 2, \ldots, K), \tag{3.19}$$

and the constraints (3.2) – (3.4).

B. Revised multi-choice goal programming approach

In the similar way, the RMCGP is introduced to solve the MOTP. Let us assume that the multiple goals be considered to the objective functions and, then this can be achieved by considering the following model (see Model 3.5) as:

Model 3.5

$$\text{minimize} \quad \sum_{t=1}^{K} \left[w_t(d_t^+ + d_t^-) + \alpha_t(e_t^+ + e_t^-) \right] \tag{3.20}$$

$$\text{subject to} \quad Z^t(X) - d_t^+ + d_t^- = y_t \quad (t = 1, 2, \ldots, K), \tag{3.21}$$

$$y_t - e_t^+ + e_t^- = g_{t,max} \text{ or } g_{t,min} \quad (t = 1, 2, \ldots, K), \tag{3.22}$$

$$g_{t,min} \leq y_t \leq g_{t,max} \quad (t = 1, 2, \ldots, K), \tag{3.23}$$

$$d_t^+, d_t^-, e_t^+, e_t^- \geq 0 \quad (t = 1, 2, \ldots, K), \tag{3.24}$$

$$\text{and} \quad \text{the constraints } (3.2) - (3.4),$$

where t-th aspiration level defined as y_t which is the continuous variable lies between the upper ($g_{t,max}$) and the lower ($g_{t,min}$) bounds. Again e_t^+ and e_t^- are positive and negative deviations attached to t-th goal of $|y_t - g_{t,max}|$ and α_t is the weight attached to the sum of the deviations of $|y_t - g_{t,max}|$.

3.3.1 Utility function approach to solve MOTP

Here, the concept of utility function has been addressed to solve the MOTP. A short introduction is presented here and then we discuss methodology for solving MOTP using utility function.

Utility function

Introduction of utility is taken to be correlative to 'Desire' or 'Want'. It has been already argued that desire cannot be measured directly, but only indirectly, by the outward phenomena in which the context is presented.

Definition 3.3.1 *The utility function describes a function $U : X \longrightarrow \mathbb{R}$ which assigns a real number to every outcome in such a way that it captures the DM's preferences over the desired goals of the objectives, where X is the set of feasible points and $\mathbb{R}$ is the set of real numbers.*

The purpose of this study is to derive the achievement function of MOTP under the light of utility function for DM according to the priority of goals. In our proposed approach, the DM wants to maximize his/her expected utility.

For the sake of simplicity, two popular utility functions (linear and S-shaped) are considered as follows.

Linear utility function $u_i(y_i)$ for decision making (management) problems can be found in Lai and Hwang (75) and S-shaped utility function (for the same purpose) has been proposed by Chang (19). The utility function is generally considered in three cases as follows:

Case 1: Left Linear Utility Function(LLUF)

$$u_i(y_i) = \begin{cases} 1, & \text{if } y_i \leq g_{i,min} \\ \frac{g_{i,max} - y_i}{g_{i,max} - g_{i,min}}, & \text{if } g_{i,min} \leq y_i \leq g_{i,max}, \\ 0, & \text{if if } y_i \geq g_{i,max} \end{cases} \quad i = 1, 2, \ldots, K$$

Case 2: Right Linear Utility Function(RLUF)

$$u_i(y_i) = \begin{cases} 1, & \text{if } y_i \geq g_{i,max} \\ \frac{y_i - g_{i,min}}{g_{i,max} - g_{i,min}}, & \text{if } g_{i,min} \leq y_i \leq g_{i,max}, \\ 0, & \text{if if } y_i \leq g_{i,min} \end{cases} \quad i = 1, 2, \ldots, K$$

Case 3: S-shaped Utility Function

$$u_i(y_i) = \begin{cases} 0, & \text{if } y_i \leq g_{i2} \\ \frac{y_i - g_{i2}}{g_{i8} - g_{i2}}, & \text{if } g_{i2} \leq y_i \leq g_{i4} \\ \frac{y_i - g_{i3}}{g_{i6} - g_{i3}}, & \text{if } g_{i4} \leq y_i \leq g_{i5} \\ \frac{y_i - g_{i1}}{g_{i7} - g_{i1}}, & \text{if } g_{i5} \leq y_i \leq g_{i7} \end{cases} , i = 1, 2, \ldots, K$$

where $g_{i,min}$ and $g_{i,max}$ are lower and upper bounds corresponding to the i-th

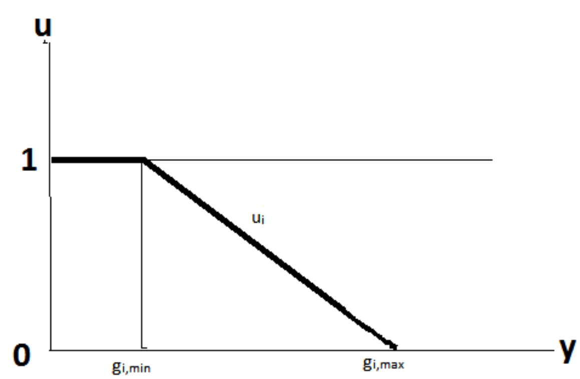

Figure 3.1: Graph of LLUF.

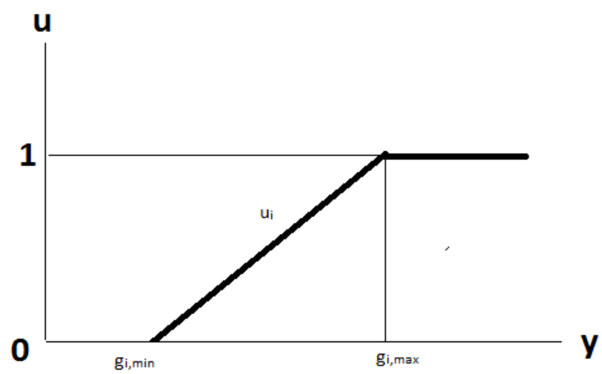

Figure 3.2: Graph of RLUF.

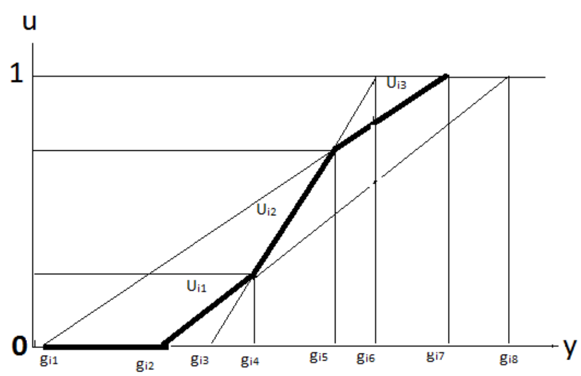

Figure 3.3: Graph of S-shaped utility function.

goal respectively. The graphs of above utility functions are drawn in Figures (cf., Figures 3.1, 3.2 and 3.3). **Model formulation for Case 1**
The DM would like to increase the utility value $u_t(y_t)$ as much as possible in case of LLUF (Figure 3.1). In order to achieve this goal, the value of y_t should be as close to the target value $g_{t,min}$ as possible. The MOTP from Model 3.4 can be reformulated using the proposed LLUF as follows:

Model 3.6

$$\text{minimize} \quad \sum_{t=1}^{K} \left[w_t(d_t^+ + d_t^-) + \beta_t f_t^- \right] \tag{3.25}$$

$$\text{subject to} \quad Z^t(X) - d_t^+ + d_t^- = y_t \quad (t = 1, 2, \ldots, K), \tag{3.26}$$

$$g_{t,min} \leq y_t \leq g_{t,max} \quad (t = 1, 2, \ldots, K), \tag{3.27}$$

$$u_t \leq \frac{g_{t,max} - y_t}{g_{t,max} - g_{t,min}} \quad (t = 1, 2, \ldots, K), \tag{3.28}$$

$$u_t + f_t^- = 1 \quad (t = 1, 2, \ldots, K), \tag{3.29}$$

$$u_t, f_t^- \geq 0 \quad (t = 1, 2, \ldots, K), \tag{3.30}$$

and the constraints (3.2) – (3.4),

where β_t is the weight attached to deviation f_t^-. The role of weight β_t can be seen as the preferential component for the utility value u_t.

Proposition 3.1: Achievement of optimal utility in the LLUF (Figure 3.1) is equivalent to the optimal solution of Model 3.6.

Proof: When u_t approaches to the highest value 1, then the deviation $f_t^- \to 0$ of the utility function [from Eq. (3.29)], because f_t^- should be minimized in the objective function and hence to obtain the optimal solution of Model 3.6. This represents y_t approach to $g_{t,min}$ [from Eq. (3.28)] and $Z_t(X)$ is also closer to $g_{t,min}$ [from Eq. (3.26)] because d_t^+ and d_t^- should also be minimized in the objective function. It is obvious that the behavior of Model 3.6 and the level of utility achieved. This completes the proof of the proposition.

Model formulation for Case 2

The DM would like to increase the utility value $u_t(y_t)$ as much as possible in the case of RLUF (Figure 3.2). In order to achieve this goal, the value of y_t should be as close to the target value $g_{t,max}$ as possible. The MOTP from Model 3.4 can be reformulated using the proposed RLUF as follows:

Model 3.7

$$\begin{aligned}
\text{minimize} \quad & \sum_{t=1}^{K} \left[w_t(d_t^+ + d_t^-) + \beta_t f_t^- \right] \\
\text{subject to} \quad & Z^t(X) - d_t^+ + d_t^- = y_t \quad (t = 1, 2, \ldots, K) \\
& g_{t,min} \leq y_t \leq g_{t,max} \quad (t = 1, 2, \ldots, K), \\
& u_t \leq \frac{y_t - g_{t,min}}{g_{t,max} - g_{t,min}} \\
& u_t + f_t^- = 1 \\
& u_t, f_t^- \geq 0 \\
\text{and} \quad & \text{the constraints } (3.2) - (3.4),
\end{aligned}$$

where β_t is the weight attached to the deviation f_t^-. The role of weight β_t can be seen as a preferential component for the utility value u_t.

Proposition 3.2: Achievement of optimal utility in the RLUF (Figure 3.2) is equivalent to the optimal solution of Model 3.7.

Proof: Similar way can be followed as we have done in Proposition 3.1.

The advantages of use the LLUF and RLUF in the decision making problems are as follows:

(1) DM can easily formulate their MOTP by taking into account their preference mappings with utility functions in real situation, and

(2) The two linear utility models represented as linear form which can be easily solved using software.

Due to variation of deviation variables d_t^+, d_t^- and f^- in different ranges, biasness may be occurred towards the objective functions with larger magnitude. Normalization technique may help to remove this bias-ness. Several normalization approaches such as Percentage, Euclidean, Summation and Zero-one notarizations [Tamiz et al. (151), Kettani et al. (70)] are available to execute this. According to the normalization technique proposed by Tamiz et al. (151), Model 3.6 can be redesigned as follows:

$$
\begin{aligned}
\text{minimize} \quad & \sum_{t=1}^{K} \left[\frac{w_t(d_t^+ + d_t^-) + \beta_t f_t^-}{\phi_t} \right] \\
\text{subject to} \quad & \text{the constraints } (3.2) - (3.4) \ \& \ (3.26) - (3.30),
\end{aligned}
$$

where ϕ_t is the normalization constant for t-th goal.

In order to solve this problem, utility normalization concept is introduced as follows: Let $d_t^+, d_t^- \in [0, \bar{u}_t]$ and $f_t^- \in [0, 1]$ where $\bar{u}_t$ is the upper bound of d_t^+ and d_t^-. The normalized weights w_t and β_t can be easily obtained as $w_t = \frac{1}{1+\bar{u}_t}$ and $\beta_t = \frac{\bar{u}_t}{1+\bar{u}_t}$. This technique of normalization ensures that deviation variables d_t^+, d_t^- and f_t^- approximated the same magnitude.

Similarly, the same methodology can be applied to the Model 3.7.

Utility value for S-shaped utility function can be expressed as a sum of linear utility functions (RLUF or LLUF) by introducing binary variables [Chang (15)]. But Chang (19) proposed that the utility value for S-shaped utility function which can be considered without using the binary variables and this is shown in the following model (i.e., Model 3.8).

Model 3.8

$$
\begin{aligned}
\text{minimize} \quad & \sum_{t=1}^{K} \left[w_t \left(p_{t1} + p_{t2} + p_{t3} \right) + \beta_t f_t^- \right] \\
\text{subject to} \quad & Z^t(X) - d_t^+ + d_t^- = y_t \quad (t = 1, 2, \ldots, K), \\
& g_{t,min} \leq y_t \leq g_{t,max} \quad (t = 1, 2, \ldots, K), \\
& u_t = \left[u_t(g_{t4}) - u_t(g_{t2}) \right] \frac{p_{t1} - p_{t2}}{g_{t4} - g_{t2}} + \left[u_t(g_{t5}) - u_t(g_{t4}) \right] \frac{p_{t2} - p_{t3}}{g_{t5} - g_{t4}} \\
& + \left[u_t(g_{t7}) - u_t(g_{t5}) \right] \frac{p_{t3}}{g_{t7} - g_{t5}} \quad (t = 1, 2, \ldots, K), \\
& y_t - p_{t1} + n_{t1} = g_{t2} \quad (t = 1, 2, \ldots, K), \\
& y_t - p_{t2} + n_{t2} = g_{t4} \quad (t = 1, 2, \ldots, K), \\
& y_t - p_{t3} + n_{t3} = g_{t5} \quad (t = 1, 2, \ldots, K), \\
& u_t + f_t^- = 1 \quad (t = 1, 2, \ldots, K), \\
& u_t, p_{tl}, n_{tl} \geq 0 \quad (t = 1, 2, \ldots, K; l = 1, 2, 3), \\
\text{and} \quad & \text{the constraints (3.2)} - \text{(3.4)}.
\end{aligned}
$$

The MCMTP occurred in many real-life situations can be reduced to MOTP and then the problem can be reduced to the models such as Models 3.6, 3.7 and 3.8, with interval goals under the consideration of utility functions related to these goals. Solving the formulated problem, the DM obtains the satisfactory solution.

3.4 Numerical examples

In this section, we present two numerical examples, the first example presents the applicability of utility function approach to solve MOTP and the second example establishes the same for the MCMTP.

3.4.1 Example 3.4.1

A reputed network company have three towers T1, T2 and T3 in a particular city with different types of unit range capacity. The towers can provide network to the areas A1, A2 and A3 in the city. Each tower has also some fixed unit capacity and each unit has some capacity to maintain the telephone calls at a time. Also, there are generating systems which can force to activate the units as per utility of the people connected by the network and they are activated as per desire of the system manager assigned by the company.

The company should keep on mind the following things in their business deal: high quality of network supply to the customers, a satisfactory profit and minimized the service providing cost. According to the capacity of the company, there are some costs, which needed to run each unit of a tower. In a certain period, the expectation of public needed the units in the areas are given in that case. Sometimes, some units may be stopped to make more profit but in that situation the customers faced some difficulties and the company may fail to keep the popularity. Here, Tables 3.1, 3.2 and 3.3 represent the amount of profit per each unit of a tower, the maintenance cost of a unit of a network tower and the number of phone calls (maximum capacity) per minute provide by each tower is defined, respectively.

Table 3.1: Profit per unit ($).

	A1	A2	A3
T1	70	80	78
T2	80	72	84
T3	90	80	76

Table 3.2: Unit maintaining cost ($).

	A1	A2	A3
T1	500	600	620
T2	600	500	550
T3	650	600	580

Table 3.3: Number of calls (maximum) per minute.

	A1	A2	A3
T1	100	80	90
T2	80	95	85
T3	90	88	98

The demands at the areas A1, A2 and A3 are 9, 8 and 10 units respectively. The supply of the towers T1, T2 and T3 are 10, 9 and 11 respectively. The goals corresponding to the objectives are

(1) Profit of at least $2300 and expected more, where the upper bound of the profit goal is $2500.

(2) For maintenance the network service, the required amount of cost belongs to the interval [17500, 20000], in this case less is better.

(3) Total number of calls by the towers at a time belong to the interval [2400, 2850], more is better but follow the S-shaped utility function.

Assume that each goal is equally importance to the company. Goal of the third objective function in the given problem keeps the demand high of the network in the city. Obviously, company tries to earn more money and wants more customers connecting to his network at a time, so, Goal 3 tends to reach maximum value. When Goal 3 tended to its maximum value of the specified problem, sometimes customers face problems connecting peoples through their network. Then to keep the good reputation in the market, company runs the towers in such a way that Goal 3 tends to a value surrounding 2700 and the priority of goal value follows S-shaped utility function (cf., Figure 3.4)

Using the data provided in the Tables 3.1, 3.2 and 3.3, we formulate the following MOTP model:

Model 3.9

Goal 1: $Z^1 = 70x_{11}+80x_{12}+78x_{13}+80x_{21}+72x_{22}+84x_{23}+90x_{31}+80x_{32}+76x_{33}$

with interval goal [2300, 2500], more is better, follows RLUF

Goal 2: $Z^2 = 500x_{11} + 600x_{12} + 620x_{13} + 600x_{21} + 500x_{22} + 550x_{23} + 650x_{31} + 600x_{32} + 580x_{33}$

with interval goal [17500, 20000], less is better, follows LLUF

Goal 3: $Z^3 = 100x_{11}+80x_{12}+90x_{13}+80x_{21}+95x_{22}+85x_{23}+90x_{31}+88x_{32}+98x_{33}$

with interval goal [2400, 2850], more is better, follows S-shaped utility function as given in Figure 3.4.

$$\text{subject to} \quad x_{11} + x_{12} + x_{13} \leq 10 \tag{3.31}$$

$$x_{21} + x_{22} + x_{23} \leq 9 \tag{3.32}$$

$$x_{31} + x_{32} + x_{33} \leq 11 \tag{3.33}$$

$$x_{11} + x_{21} + x_{31} \geq 9 \tag{3.34}$$

$$x_{12} + x_{22} + x_{32} \geq 8 \tag{3.35}$$

$$x_{13} + x_{23} + x_{33} \geq 10 \tag{3.36}$$

$$x_{ij} \geq 0 \;\; \forall \;\; i, j = 1, 2, 3. \tag{3.37}$$

To achieve the goals in the proposed problem (see Model 3.9), we may formulate the following models:
In the proposed problem, the deviations of goals 1, 2, 3 are 200, 2500, 450 respectively. By considering the weights $w_1 = \frac{1}{200}, w_2 = \frac{1}{2500}, w_3 = \frac{1}{450}$ for the Model 3.4, Model 3.9 reduces to the following model (i.e., Model 3.10) as:
Model 3.10

$$\begin{aligned}
\text{minimize} \quad & \frac{1}{200}(d_1^+ + d_1^-) + \frac{1}{2500}(d_2^+ + d_2^-) + \frac{1}{450}(d_3^+ + d_3^-) \\
\text{subject to} \quad & 70x_{11} + 80x_{12} + 78x_{13} + 80x_{21} + 72x_{22} + 84x_{23} \\
& +90x_{31} + 80x_{32} + 76x_{33} - d_1^+ + d_1^- = y_1 \\
& 2300 \leq y_1 \leq 2500 \\
& 500x_{11} + 600x_{12} + 620x_{13} + 600x_{21} + 500x_{22} + \\
& 550x_{23} + 650x_{31} + 600x_{32} + 580x_{33} - d_2^+ + d_2^- = y_2 \\
& 17500 \leq y_2 \leq 20000 \\
& 100x_{11} + 80x_{12} + 90x_{13} + 80x_{21} + 95x_{22} + 85x_{23} \\
& +90x_{31} + 88x_{32} + 98x_{33} - d_3^+ + d_3^- = y_3 \\
& 2400 \leq y_3 \leq 2850 \\
& d_t^+, d_t^- \geq 0, \quad t = 1, 2, 3 \\
\text{and} \quad & \text{the constraints } (3.31) - (3.37).
\end{aligned}$$

Again, considering the same weights w_t as used in Model 3.10 for all t=1, 2, 3 and setting α_t=w_t for t=1, 2, 3 for deviation of goals and using the Model 3.5, the Model 3.9 reduces to the following model (i.e., Model 3.11) as:

Model 3.11

$$
\begin{aligned}
\text{minimize} \quad & \frac{1}{200}(d_1^+ + d_1^-) + \frac{1}{2500}(d_2^+ + d_2^-) + \frac{1}{450}(d_3^+ + d_3^-) \\
& + \frac{1}{200}(e_1^+ + e_1^-) + \frac{1}{2500}(e_2^+ + e_2^-) + \frac{1}{450}(e_3^+ + e_3^-) \\
\text{subject to} \quad & 70x_{11} + 80x_{12} + 78x_{13} + 80x_{21} + 72x_{22} + 84x_{23} \\
& +90x_{31} + 80x_{32} + 76x_{33} - d_1^+ + d_1^- = y_1 \\
& y_1 - e_1^+ + e_1^- = 2500 \\
& 2300 \le y_1 \le 2500 \\
& 500x_{11} + 600x_{12} + 620x_{13} + 600x_{21} + 500x_{22} + \\
& 550x_{23} + 650x_{31} + 600x_{32} + 580x_{33} - d_2^+ + d_2^- = y_2 \\
& y_2 - e_2^+ + e_2^- = 17500 \\
& 17500 \le y_2 \le 20000 \\
& 100x_{11} + 80x_{12} + 90x_{13} + 80x_{21} + 95x_{22} + 85x_{23} \\
& +90x_{31} + 88x_{32} + 98x_{33} - d_3^+ + d_3^- = y_3 \\
& y_3 - e_3^+ + e_3^- = 2850 \\
& 2400 \le y_3 \le 2850 \\
& d_t^+, d_t^-, e_t^+, e_t^- \ge 0, \quad t = 1, 2, 3 \\
\text{and} \quad & \text{the constraints } (3.31) - (3.37).
\end{aligned}
$$

Using the concept of utility function described in subsection 3.3.1, Model 3.9 can be reformulated as follows:
The consideration of utility function depends on the DM. Here, we assume that Goals 1, 2 and 3 follow the utility function LLUF (Figure 3.1), RLUF (Figure 3.2) and S-shaped utility function as given in Figure 3.4 respectively. In the given example, the upper bound of variations $d_1^+, d_1^-, d_2^+, d_2^-, d_3^+, d_3^-$ are 200, 200, 2500, 2500, 400, 400 respectively and the upper bounds of f_1^-, f_2^-, f_3^- are 1. We find the weights as described in subsection 3.3 as follows: $w_1 = \frac{1}{200}$, $w_2 = \frac{1}{2500}$, $w_3 = \frac{1}{450}$,$\beta_1 = \frac{200}{201}$, $\beta_2 = \frac{2500}{2501}$, $\beta_3 = \frac{450}{451}$.
With these supplied data, Model 3.9 can be reformulated as follows:

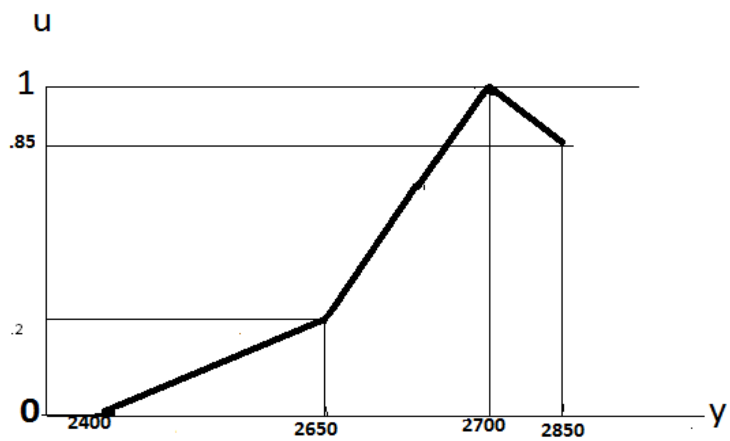

Figure 3.4: S-shaped utility function for the goal 3 of proposed example.

Model 3.12

$$\begin{aligned}
\text{minimize} \quad & w_1(d_1^+ + d_1^-) + \beta_1 f_1^- + w_2(d_2^+ + d_2^-) + \\
& \beta_2 f_2^- + w_3(d_{31}^+ + d_{32}^+ + d_{32}^+) + \beta_3 f_3^- \\
\text{subject to} \quad & 70x_{11} + 80x_{12} + 78x_{13} + 80x_{21} + 72x_{22} + 84x_{23} \\
& +90x_{31} + 80x_{32} + 76x_{33} - d_1^+ + d_1^- = y_1, \\
& u_1 \leq \frac{2500 - y_1}{200}, \\
& f_1^- + u_1 = 1, \\
& 2300 \leq y_1 \leq 2500 \\
& 500x_{11} + 600x_{12} + 620x_{13} + 600x_{21} + 500x_{22} + \\
& 550x_{23} + 650x_{31} + 600x_{32} + 580x_{33} - d_2^+ + d_2^- = y_2, \\
& u_2 \leq \frac{y_2 - 17500}{2500}, \\
& f_2^- + u_2 = 1 \\
& 17500 \leq y_2 \leq 20000 \\
& 100x_{11} + 80x_{12} + 90x_{13} + 80x_{21} + 95x_{22} + 85x_{23} \\
& +90x_{31} + 88x_{32} + 98x_{33} - d_3^+ + d_3^- = y_3 \\
& u_3 = (.2 - 0)\frac{d_{31}^+ - d_{32}^+}{250} + (1 - .2)\frac{d_{32}^+ - d_{33}^+}{50} + (.85 - 1)\frac{d_{33}^+}{150}
\end{aligned}$$

$$y_3 - d_{31}^+ + dn_{31} = 2650, y_3 - d_{32}^+ + dn_{32} = 2700,$$
$$y_3 - d_{33}^+ + dn_{33} = 2850, d_{31}^+ dn_{31} = 0, d_{32}^+ dn_{32} = 0,$$
$$d_{33}^+ dn_{33} = 0,\ f_3^- + u_3 = 1$$
$$u_t \geq 0, f_t^- \geq 0 \ \ \forall \ \ t = 1, 2, 3$$

and the constraints (3.31) – (3.37).

Result and discussion

Using LINGO software, solve Models 3.10, 3.11 and 3.12 and report the solution as follows:

The optimal solution of Model 3.10 is reported as: $x_{11} = 0, x_{12} = 10, x_{13} = 0, x_{21} = 0, x_{22} = 0, x_{23} = 9, x_{31} = 3, x_{32} = 0, x_{33} = 8$;
$Z^1 = 2434, Z^2 = 17540, Z^3 = 2619$.

The optimal solution of Model 3.11 is as follows:
$x_{11} = 4, x_{12} = 6, x_{13} = 0, x_{21} = 0, x_{22} = 0, x_{23} = 9, x_{31} = 8, x_{32} = 2, x_{33} = 1$;
$Z^1 = 2472, Z^2 = 17530, Z^3 = 2639$.

The optimal solution of Model 3.12 is also as follows:
$x_{11} = 2, x_{12} = 0, x_{13} = 8, x_{21} = 0, x_{22} = 7, x_{23} = 2, x_{31} = 8, x_{32} = 1, x_{33} = 2$;
$Z^1 = 2388, Z^2 = 17520, Z^3 = 2763$.

Table 3.4: Comparison among the achieved goals obtained from different methods.

Method	Achievement of Goal 1 in(%)	Achievement of Goal 2 in(%)	Achievement of Goal 3 in(%)
GP	60	93	40
RMCGP	80	95	45
Utility Approach	45	98	95

From Table 3.4, we conclude that the solution obtained from the Model 3.11 is better than the solution of Model 3.10, but DM is not satisfied, because in the proposed problem, satisfying the goal is not only the important criteria, there is a utility factor to DM which is an important concept for the decision making (management) problem. When the utility value is more important rather than the benefit, then the solutions obtained from Model 3.10 and from Model 3.11 are not satisfied by DM for taking the appropriate decision. The marketing survey indicates that the higher utility value of Goal 3 will increase the number of customers to the network service provider company. The solution obtained from Model 3.12 provided high utility value of Goal 3, whenever the other two methods GP and RMCGP fail to provide so. In this context, we may suggest that the utility function approach is provided better result in comparing with other results obtained from the approaches such as GP and RMCGP.

3.4.2 Example 3.4.2

Let us consider the following MCMTP (i.e., Model 3.9) with two objectives:
Model 3.13
Goal 1: $Z_1 = (5 \text{ or } 7)x_{11} + 8x_{12} + (7 \text{ or } 6 \text{ or } 10)x_{13} + (6 \text{ or } 8)x_{21} + 8x_{22} + 10x_{23}$ with goal as [150,200], more is better, but follows S-shape utility function (Figure 3.5)
Goal 2: $Z_2 = 15x_{11} + (18 \text{ or } 16)x_{12} + 17x_{13} + 16x_{21} + (18 \text{ or } 20)x_{22} + 20x_{23}$ with goal as [400,500], less is better, follows LLUF.

$$\text{subject to} \quad x_{11} + x_{12} + x_{13} \leq (11 \text{ or } 13 \text{ or } 12 \text{ or } 16) \tag{3.38}$$
$$x_{21} + x_{22} + x_{23} \leq (14 \text{ or } 13) \tag{3.39}$$
$$x_{11} + x_{21} \geq (8 \text{ or } 7) \tag{3.40}$$
$$x_{12} + x_{22} \geq (7 \text{ or } 8 \text{ or } 6) \tag{3.41}$$
$$x_{13} + x_{23} \geq 9 \tag{3.42}$$
$$x_{ij} \geq \ 0 \ \forall \ \ i = 1, 2 \text{ and } j = 1, 2, 3. \tag{3.43}$$

The Model 3.13 is equivalent to the following model (i.e., Model 3.14)
Model 3.14
Goal 1: $Z_1 = (5z_{11}^{11} + 7(1 - z_{11}^{11}))x_{11} + 8x_{12} + (7z_{11}^{12}z_{11}^{13} + 6z_{11}^{12}(1 - z_{11}^{13})$
$+10z_{11}^{13}(1 - z_{11}^{12}))x_{13} + (6z_{11}^{21} + 8(1 - z_{11}^{21}))x_{21} + 8x_{22} + 10x_{23}$
with goal as [150,200], more is better, but follows S-shape utility function (Figure 3.5)
Goal 2: $Z_2 = 15x_{11} + (18z_{12}^{11} + 16(1 - z_{12}^{11}))x_{12} + 17x_{13} + 16x_{21}$
$+(18z_{12}^{12} + 20(1 - z_{12}^{12}))x_{22} + 20x_{23}$

with goal as [400,500], less is better, follows LLUF.

$$\text{subject to} \quad x_{11} + x_{12} + x_{13} \leq (11z_1^{11}z_1^{12} + 13z_1^{11}(1 - z_1^{12})$$
$$+12z_1^{12}(1 - z_1^{11}) + 16(1 - z_1^{11})(1 - z_1^{12})) \tag{3.44}$$
$$x_{21} + x_{22} + x_{23} \leq (14z_2^{11} + 16(1 - z_2^{11})) \tag{3.45}$$
$$x_{11} + x_{21} \geq (8z^{11} + 7(1 - z^{11})) \tag{3.46}$$
$$x_{12} + x_{22} \geq (7z^{21}z^{22} + 8z^{21}(1 - z^{22}) + 6z^{22}(1 - z^{21})) \tag{3.47}$$
$$x_{13} + x_{23} \geq 9 \tag{3.48}$$
$$x_{ij} \geq 0 \;\; \forall \;\; i = 1, 2 \;\; \text{and} \;\; j = 1, 2, 3. \tag{3.49}$$
$$z_{11}^{12} + z_{11}^{13} \geq 1 \tag{3.50}$$
$$z^{21} + z^{22} \geq 1 \tag{3.51}$$
$$z_{11}^{11}, z_{11}^{12}, z_{11}^{13}, z_{12}^{11}, z_{12}^{12} = 0 \text{ or } 1 \tag{3.52}$$
$$z_1^{11}, z_1^{12}, z_2^{11} = 0 \text{ or } 1 \tag{3.53}$$
$$z^{11}, z^{21}, z^{22} = 0 \text{ or } 1 \tag{3.54}$$

In Model 3.14, the deviations of Goal 1 and Goal 2 are 50 and 100 respectively. By considering the weights $w_1 = \frac{1}{50}, w_2 = \frac{1}{100}$ for Model 3.14, we find the following model (i.e., Model 3.15):

Model 3.15

$$\text{minimize} \quad \frac{1}{50}(d_1^+ + d_1^-) + \frac{1}{100}(d_2^+ + d_2^-)$$
$$\text{subject to} \quad (5z_{11}^{11} + 7(1 - z_{11}^{11}))x_{11} + 8x_{12} + (7z_{11}^{12}z_{11}^{13} + 6z_{11}^{12}(1 - z_{11}^{13})$$
$$+10z_{11}^{13}(1 - z_{11}^{12}))x_{13} + (6z_{11}^{21} + 8(1 - z_{11}^{21}))x_{21} + 8x_{22}$$
$$+10x_{23} - d_1^+ + d_1^- = y_1,$$
$$150 \leq y_1 \leq 200,$$
$$15x_{11} + (18z_{12}^{11} + 16(1 - z_{12}^{11}))x_{12} + 17x_{13} + 16x_{21}$$
$$+(18z_{12}^{12} + 20(1 - z_{12}^{12}))x_{22} + 20x_{23} - d_2^+ + d_2^- = y_2,$$
$$400 \leq y_2 \leq 500,$$
$$d_t^+, d_t^- \geq 0, \quad t = 1, 2$$
$$\text{and} \quad \text{the constraints } (3.44) - (3.54).$$

Again, considering the same weights w_t as used in Model 3.10 for all $t = 1, 2$ and the weights α_t=w_t for $t = 1, 2$ for deviation of goals and using Model 3.5, Model 3.14, reduce to the model (i.e., Model 3.16) as:

Model 3.16

$$\begin{aligned}
\text{minimize} \quad & \frac{1}{50}(d_1^+ + d_1^-) + \frac{1}{100}(d_2^+ + d_2^-) + \\
& + \frac{1}{50}(e_1^+ + e_1^-) + \frac{1}{100}(e_2^+ + e_2^-) \\
\text{subject to} \quad & (5z_{11}^{11} + 7(1 - z_{11}^{11}))x_{11} + 8x_{12} + (7z_{11}^{12}z_{11}^{13} + 6z_{11}^{12}(1 - z_{11}^{13}) \\
& +10z_{11}^{13}(1 - z_{11}^{12}))x_{13} + (6z_{11}^{21} + 8(1 - z_{11}^{21}))x_{21} \\
& +8x_{22} + 10x_{23} - d_1^+ + d_1^- = y_1, \\
& y_1 - e_1^+ + e_1^- = 200, \\
& 150 \leq y_1 \leq 200, \\
& 15x_{11} + (18z_{12}^{11} + 16(1 - z_{12}^{11}))x_{12} + 17x_{13} + 16x_{21} \\
& +(18z_{12}^{12} + 20(1 - z_{12}^{12}))x_{22} + 20x_{23} - d_2^+ + d_2^- = y_2, \\
& y_2 - e_2^+ + e_2^- = 400, \\
& 400 \leq y_2 \leq 500, \\
& d_t^+, d_t^-, e_t^+, e_t^- \geq 0, \quad t = 1, 2 \\
\text{and} \quad & \text{the constraints } (3.44) - (3.54).
\end{aligned}$$

Let us solve the proposed problem (i.e., Model 3.14) using the concept of utility function. The consideration of utility function depends on the DM. Here we assume that Goal 1 and Goal 2 follow S-shaped utility function as given in Figure 3.5 and the utility functions LLUF (i.e., Figure 3.1) respectively. In given example, the upper bound of variations $d_1^+, d_1^-; d_2^+, d_2^-$ are 50, 100 respectively and the upper bounds of f_1^- and f_2^- are 1. We find the weights as per suggested in the subsection 3.3 as follows: $w_1 = \frac{1}{50}$, $\beta_1 = \frac{50}{51}$, $w_2 = \frac{1}{100}$, $\beta_2 = \frac{100}{101}$. With these supplied data, Model 3.14 can be formulated as follows:

Model 3.17

$$\begin{aligned}
\text{minimize} \quad & w_1(d_{11}^+ + d_{12}^+ + d_{13}^+) + \beta_1 f_1^- + w_2(d_2^+ + d_2^-) + \beta_2 f_2^- \\
\text{subject to} \quad & (5z_{11}^{11} + 7(1 - z_{11}^{11}))x_{11} + 8x_{12} + (7z_{11}^{12}z_{11}^{13} + 6z_{11}^{12}(1 - z_{11}^{13}) + 10z_{11}^{13}
\end{aligned}$$

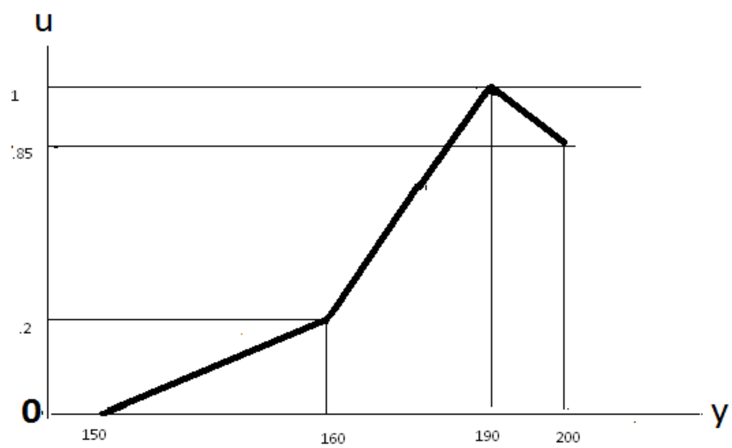

Figure 3.5: S-shaped utility function for goal 2 in Case 2.

$$
\begin{aligned}
& (1 - z_{11}^{12}))x_{13} + (6z_{11}^{21} + 8(1 - z_{11}^{21}))x_{21} \\
& +8x_{22} + 10x_{23} - d_1^+ + d_1^- = y_1, \\
& u_1 = (0.2 - 0)\frac{d_{11}^+ - d_{12}^+}{10} + (1 - 0.2)\frac{d_{12}^+ - d_{13}^+}{30} + (0.85 - 1)\frac{d_{13}^+}{10} \\
& y_1 - d_{11}^+ + d_{11}^- = 160, y_1 - d_{12}^+ + d_{12}^- = 190, y_1 - d_{13}^+ + d_{13}^- = 200, \\
& d_{11}^+ d_{11}^- = 0, d_{12}^+ dn_{12}^- = 0, d_{13}^+ d_{13}^- = 0, f_1^- + u_1 = 1, \\
& 15x_{11} + (18z_{12}^{11} + 16(1 - z_{12}^{11}))x_{12} + 17x_{13} + 16x_{21} \\
& +(18z_{12}^{12} + 20(1 - z_{12}^{12}))x_{22} + 20x_{23} - d_2^+ + d_2^- = y_2, \\
& u_2 \leq \frac{200 - y_2}{100}, \\
& f_2 + u_2 = 1, \\
& 400 \leq y_2 \leq 500, \\
& u_t \geq 0, f_t \geq 0 \;\; \forall \;\; t = 1, 2 \\
\text{and} \quad & \text{the constraints } (3.44) - (3.54).
\end{aligned}
$$

Result and discussion

Solving the model presented in Model 3.15, The optimal solution of Model 3.15 is reported as: $x_{11} = 7, x_{12} = 5, x_{13} = 0, x_{21} = 0, x_{22} = 1, x_{23} = 10$ and the values of the objective functions are $Z^1 = 197, Z^2 = 405$.

The selection of the choices corresponding to the optimal solution is as follows:

$c_{11}^1 = 7, c_{12}^1 = 8, c_{13}^1 = 10, c_{21}^1 = 8, c_{22}^1 = 8, c_{23}^1 = 10$
$c_{11}^2 = 15, c_{12}^2 = 16, c_{13}^2 = 17, c_{21}^2 = 16, c_{22}^2 = 18, c_{23}^2 = 20$
$a_1 = 16, a_2 = 14, b_1 = 7, b_2 = 6, b_3 = 9.$
Solving the model presented in Model 3.16, we list the following solution as: $x_{11} = 7, x_{12} = 6, x_{13} = 0, x_{21} = 0, x_{22} = 0, x_{23} = 10$ and the values of the objective functions are $Z^1 = 197, Z^2 = 401$.
The selection of the choices corresponding to the optimal solution is as follows:
$c_{11}^1 = 7, c_{12}^1 = 8, c_{13}^1 = 10, c_{21}^1 = 6, c_{22}^1 = 8, c_{23}^1 = 10$
$c_{11}^2 = 15, c_{12}^2 = 16, c_{13}^2 = 17, c_{21}^2 = 16, c_{22}^2 = 18, c_{23}^2 = 20$
$a_1 = 16, a_2 = 14, b_1 = 7, b_2 = 8, b_3 = 9.$
Solving the model presented in Model 3.17, we obtain the solution as follows: $x_{11} = 6, x_{12} = 3, x_{13} = 0, x_{21} = 0, x_{22} = 3, x_{23} = 10$ and the values of the objective functions are $Z^1 = 190, Z^2 = 404$.
The selection of the choices corresponding the optimal solution is as follows:
$c_{11}^1 = 7, c_{12}^1 = 8, c_{13}^1 = 10, c_{21}^1 = 6, c_{22}^1 = 8, c_{23}^1 = 10$
$c_{11}^2 = 15, c_{12}^2 = 18, c_{13}^2 = 17, c_{21}^2 = 16, c_{22}^2 = 20, c_{23}^2 = 20$
$a_1 = 12, a_2 = 14, b_1 = 7, b_2 = 6, b_3 = 9.$

Table 3.5: Comparison related to achieved goals obtained by different methods.

Method	Achievement of Goal 1 in(%)	Achievement of Goal 2 in(%)
GP	85	96
RMCGP	85	98
Utility Approach	100	95

Table 3.5 helps us to conclude that, the solution of the MCMTP obtained in Model 3.16 is better in compare with the solution of Model 3.15, but DM is not satisfied, because in the proposed problem satisfying the goal is not only the important notion, there is an utility factor to DM which is an important factor for the decision making problem. When the utility value is more important rather than the benefit, then the solutions obtained in Model 3.15 or in Model 3.16 are not satisfied by DM to take appropriate decision. The solution obtained in Model 3.17 demonstrated the high utility value of Goal 2, whenever

the other two models are failed to give satisfactory results. In this context, we may suggest that the utility function approach provided a better result in compare with other results.

3.5 Conclusion

This chapter explores the study of MOTP with interval goals of each objective functions. GP and RMCGP are well known methods to formulate the mathematical model and solve multi-objective decision making problem having interval goals to each of the objective functions. According to the models of GP or RMCGP, it is almost impossible to find a mathematical model in which the DM preferred a crisp goal value for objective (objectives) lies (lie) in the interval of proposed goal corresponding the objective function. To tractable this type of situation, here, we incorporate the concept of utility function. The notion of utility function approach is used to form a mathematical model of MOTP with interval goal for each of the objective functions in the light of goal preferences by the DM.

Again, accommodating the modern daily-life real phenomenons, we consider the MCMTP where the cost, demand and supply coefficients are multi-choice type. Concept of utility, in this chapter, propose a new approach for extending the utilization of real-life MOTP and MCMTP and improves the skill for representing the DM's preferences.

Chapter 4

Conic Scalarization Approach in MOTP *

This chapter explores the study of Multi-Choice Multi-objective Transportation Problem (MCMTP) under the light of Conic Scalarizing function. Solving MCMTP by RMCGP or utility function approach includes a large number of auxiliary variables, whereas the Conic Scalarization approach produces a better result with a less number of auxiliary variables in comparing to RMCGP or utility function approach. In this chapter, the way to solve MCMTP using Conic Scalarization approach is introduced and the feasibility and usefulness of the study are drawn through numerical examples.

4.1 Introduction

Goal Programming (GP), an analytical approach, is devised to address the decision making problem where targets have been assigned to all objective functions which are conflicting and non-commensurable to each other and the decision maker interests to maximize the achievement level of the corresponding goals.

The main concept of GP was that to minimize the deviation between the achievement goals and the achievement levels. Our mathematical model of

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multi-objective decision making can be considered in the following form:

$$\begin{aligned} \text{minimize} \quad & \sum_{t=1}^{K} w_t |Z^t(x) - g_t| \\ \text{subject to} \quad & x \in F, \end{aligned}$$

where F is the feasible set and w_t are the weights attached to the deviation of the achievement function, $Z^t(x)$ is the t-th objective function of the t-th goal and g_t is the aspiration level of the t-th goal, and $|Z^t(x) - g_t|$ represents the deviation of the t-th goal. Later on, a modification on GP is provided and is denoted as Weighted Goal Programming (WGP) which can be displayed in the following form:

WGP

$$\begin{aligned} \text{minimize} \quad & \sum_{t=1}^{K} w_t (d_t^+ + d_t^-) \\ \text{subject to} \quad & Z^t(x) - d_t^+ + d_t^- = g_t, \\ & d_t^+ \geq 0, d_t^- \geq 0 \quad (t = 1, 2, \ldots, K), \\ & x \in F, \end{aligned}$$

where d_t^+ and d_t^- are over- and under- achievements of the t-th goal, respectively.

However, the conflicts of resources and the incompleteness of available information make it almost impossible for decision makers to set the specific aspiration levels and choose the better decision. To overcome this situation, multi-choice goal programming (MCGP) approach has been presented by Chang (16) with a new direction to solve Multi-Objective Decision Making (MODM) problem. The mathematical model of MODM employing RMCGP is defined as follows:

RMCGP

$$\begin{aligned} \text{minimize} \quad & \sum_{t=1}^{K} \left[w_t (d_t^+ + d_t^-) + \alpha_t (e_t^+ + e_t^-) \right] \\ \text{subject to} \quad & Z^t(X) - d_t^+ + d_t^- = y_t \quad (t = 1, 2, \ldots, K), \\ & y_t - e_t^+ + e_t^- = g_{t,\max} \ \text{ or } \ g_{t,\min} \quad (t = 1, 2, \ldots, K), \end{aligned}$$

$$g_{t,\min} \leq y_t \leq g_{t,\max} \quad (t = 1, 2, \ldots, K),$$
$$d_t^+, d_t^-, e_t^+, e_t^- \geq 0 \quad (t = 1, 2, \ldots, K),$$
$$x \in F.$$

Here, y_t is the continuous variable associated with t-th goal which restricted between the upper ($g_{t,\max}$) and lower ($g_{t,\min}$) bounds and e_t^+ and e_t^- are positive and negative deviations attached to the t-th goal of $|y_t - g_{t,\max}|$ and α_t is the weight attached to the sum of the deviations of $|y_t - g_{t,\max}|$; other variables are defined as in WGP.

In general, scalarization means the replacement of a multi-objective optimization problem by a suitable scalar optimization problem which is also an optimization problem with a real-valued objective function. Since the scalar optimization problem is widely developed, the scalarization turns out to be of great importance for multi-objective optimization.

In this study, we consider the multi-choice reference points (aspiration levels) corresponding to objective functions and introduce the approach of conic scalarizing function to obtain a more satisfactory solution of an MCMTP and, then, we compare among the solutions obtained from three methods such as GP, RMCGP and Conic Scalarization approach. Again, the study is developed in the environment of multi-choice parameters, and we present their selection procedure for the achievement of a better aspiration level.

4.2 Mathematical model

In this section, we propose two mathematical models (Model 4.1 and Model 4.2), between them the first one is for MOTP and the second one is for MCMTP. The Model 4.1 is defined as follows:

Model 4.1

$$\text{minimize} \quad Z^t = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}^t x_{ij} \quad (t = 1, 2, \ldots, K) \tag{4.1}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \leq a_i \quad (i = 1, 2, \ldots, m), \tag{4.2}$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j \quad (j = 1, 2, \ldots, n), \tag{4.3}$$

$$\text{and} \qquad x_{ij} \geq 0, \quad \forall \ i \ \text{ and } \ j. \tag{4.4}$$

Here, C_{ij}^t, a_i and b_j are the transportation parameters (cost, supply and demand) of t-th objective function in multi-objective transportation problem respectively and $\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} b_j$ is the feasibility condition. According to the nature of the problem, the decision maker has the right to choice the goals of the objective functions. We assume that these goals are g_t $(t = 1, 2, \ldots, K)$, namely, K objective functions, and these goals are defined as interval-valued $g_t = [g_{t,\min}, g_{t,\max}]$ $(t = 1, 2, \ldots, K)$.

In many real-life situations, the multiple choices in transportation parameters create complexities in order to take right decision by decision maker. Multi-choice costs may occur due to several routes for transporting the goods. Due to weather condition or different seasonal, demands or supply parameters become multi choices in nature. In the atmosphere of multi-choice transportation parameters, the Model 4.2 is defined as follows:

Model 4.2

$$\text{minimize} \qquad Z^t = \sum_{i=1}^{m} \sum_{j=1}^{n} (C_{ij}^{t1} \text{ or } C_{ij}^{t2} \text{ or } \ldots \text{ or } C_{ij}^{tr}) x_{ij} \quad (t = 1, 2, \ldots, K) \tag{4.5}$$

$$\text{subject to} \qquad \sum_{j=1}^{n} x_{ij} \leq (a_i^1 \text{ or } a_i^2 \text{ or } \ldots \text{ or } a_i^p) \quad (i = 1, 2, \ldots, m), \tag{4.6}$$

$$\sum_{i=1}^{m} x_{ij} \geq (b_j^1 \text{ or } b_j^2 \text{ or } \ldots \text{ or } b_j^q) \quad (j = 1, 2, \ldots, n), \tag{4.7}$$

$$\text{and} \qquad x_{ij} \geq 0, \quad \forall \ i \text{ and } \ j. \tag{4.8}$$

Here, $(C_{ij}^{t1}$ or C_{ij}^{t2} or $\ldots$ or $C_{ij}^{tr})$, $(a_i^1$ or a_i^2 or $\ldots$ or $a_i^p)$ and $(b_j^1$ or b_j^2 or $\ldots$ or $b_j^q)$ are the multi-choices cost, supply and demand parameters for the t-th objective function. In a TP, the total demand should be less or equal to the total capacity of supply to get a feasible solution. In case for multi-choice supply and demand, the information of total capacity of supply in origins and demands in the destinations are not precisely calculated. So, we select here the maximum possible supply in the origins and consequently the minimum demand in the destinations, and then we rewrite the feasibility condition as:

$\sum_{i=1}^{m}\max(a_i^1, a_i^2, \ldots, a_i^p) \geq \sum_{j=1}^{n}\min(b_j^1, b_j^2, \ldots, b_j^q)$. (4.$A$)

This feasibility condition means the best possible wide range of feasible region regarding the MCMTP. However, the feasibility condition can be remodeled as per as the choice of decision maker.

Transformation technique for multi-choice parameters like cost, supply and demand to an equivalent form

When there are multiple choice of parameters such as cost, supply and demand, we should select a single choice satisfying supply and demand restrictions. The selection of choices should be done in such a way that the whole problem will be optimized. Introduction of binary variables is an important concept to select choice within the problem. Using the general transformation technique stated in subsection 2.3.1 of Chapter 2, we reduce the multi-choice constraints to single valued deterministic constraints in the following way:

$$\text{Let} \quad \tilde{C}_{ij}^{t} = \sum_{g=1}^{T}(term)^g C_{ij}^{tg} \qquad (i = 1, 2, \ldots, m; j = 1, 2, \ldots, n), \tag{4.9}$$

where $(term)^g$ (for $g = 1, 2, \ldots, T$) stands for T many terms in the functions of the binary variables mentioned above. Similarly,

$$\tilde{a}_i = \sum_{g=1}^{P}(term)^g a_i^g \quad (i = 1, 2, \ldots, m), \tag{4.10}$$

$$\text{and} \quad \tilde{b}_j = \sum_{g=1}^{Q}(term)^g b_j^g \quad (j = 1, 2, \ldots, n). \tag{4.11}$$

Here, $(term)^g$ $(g = 1, 2, \ldots, P)$ stands for P many terms in the functions of the binary variables mentioned above to reduce the P number of choices a_i^g to single choice $a_i^{'}$, and $(term)^g$ $(g = 1, 2, \ldots, Q)$ stands for Q many terms in the functions of the binary variables mentioned above to reduce the Q number of choices b_j^g to single choice $b_j^{'}$.

4.2.1 Reduction of MCMTP to MOTP

The Model 4.2 of MCMTP is converted to a MOTP by transforming the multi-choice parameters to real numbers which involved in the objective function (4.5) and the supply and demand constraints (4.6) and (4.7) through the equations (4.9)-(4.11). Thereafter, the reduced MOTP model is encountered in Model 4.3.

Model 4.3

$$\text{minimize} \quad Z^t = \sum_{i=1}^{m} \sum_{j=1}^{n} C''^t_{ij} x_{ij} \quad (t = 1, 2, \ldots, K) \tag{4.12}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \leq a'_i \quad (i = 1, 2, \ldots, m), \tag{4.13}$$

$$\sum_{i=1}^{m} x_{ij} \geq b'_j \quad (j = 1, 2, \ldots, n), \tag{4.14}$$

$$\text{and} \quad x_{ij} \geq 0 \quad \forall \; i \; \text{ and } \; j. \tag{4.15}$$

Here, C''^t_{ij}, a'_i, b'_j are reduced cost, supply and demand parameters of the t-th objective function in MOTP, respectively, and the same feasibility condition is provided in condition (4.A). The transportation problem i.e., Model 4.3 is the same as the problem described in Model 4.1.

4.2.2 Solution procedure

The approaches such as goal programming and revised multi-choice goal programming are used to solve MOTP, and they are defined as stated subsequently.

Goal programming approach:

Let us briefly discuss the goal programming approach for solving MOTP (Model 4.4). Furthermore, let d_t^+ and d_t^- be positive and negative deviations corresponding to the t-th goal of the objective function, respectively. Then the mathematical model is defined as follows:

Model 4.4

$$\text{minimize} \quad \sum_{t=1}^{K} w_t(d_t^+ + d_t^-) \tag{4.16}$$

$$\text{subject to} \quad Z^t(X) - d_t^+ + d_t^- = y_t \quad (t = 1, 2, \ldots, K), \tag{4.17}$$

$$g_{t,\text{min}} \leq y_t \leq g_{t,\text{max}} \quad (t = 1, 2, \ldots, K), \tag{4.18}$$

$$d_t^+, d_t^- \geq 0 \quad (t = 1, 2, \ldots, K), \tag{4.19}$$

$$\text{and} \quad \text{the constraints (4.13)} - \text{(4.15)}.$$

Revised multi-choice goal programming approach:
Similarly, the RMCGP is introduced to solve the MOTP. Let us assume that the multiple goals are considered by the objective functions. This can be achieved by referring to the following model (Model 4.5) as:
Model 4.5

$$\text{minimize} \quad \sum_{t=1}^{K} \left[w_t(d_t^+ + d_t^-) + \alpha_t(e_t^+ + e_t^-) \right] \tag{4.20}$$

$$\text{subject to} \quad Z^t(X) - d_t^+ + d_t^- = y_t \quad (t = 1, 2, \ldots, K), \tag{4.21}$$

$$y_t - e_t^+ + e_t^- = g_{t,\text{max}} \text{ or } g_{t,\text{min}} \quad (t = 1, 2, \ldots, K), \tag{4.22}$$

$$g_{t,\text{min}} \leq y_t \leq g_{t,\text{max}} \quad (t = 1, 2, \ldots, K), \tag{4.23}$$

$$d_t^+, d_t^-, e_t^+, e_t^- \geq 0 \quad (t = 1, 2, \ldots, K), \tag{4.24}$$

$$\text{and} \quad \text{the constraints (4.13)} - \text{(4.15)},$$

where the t-th aspiration level is defined as y_t which is the continuous variable lying between upper ($g_{t,\text{max}}$) and lower ($g_{t,\text{min}}$) bounds. Again, e_t^+ and e_t^- are positive and negative deviations attached to the t-th goal of $|y_t - g_{t,\text{max}}|$, and α_t is the weight attached to the sum of the deviations of $|y_t - g_{t,\text{max}}|$.

Conic Scalarization approach to solve MOTP

Here, the concept of Conic Scalarization is addressed to solve the MOTP. A short introduction and the related definitions of Conic Scalarization approach are presented here, and then we discuss the methodology for solving MOTP using it.

Reference point methodology provides the foundation for many methods in multiple objective programming (MOP) (Ustun (153)). The Conic Scalarizing function is also treated as "Conic Scalarization" that a general characterization for Benson proper efficient point set, which was firstly proposed by Gasimov (40). An MOP can be written as follows:

Model 4.6

$$\begin{array}{ll} \text{minimize} & Z^t = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^t x_{ij} \quad (t = 1, 2, \ldots, K) \\ \text{subject to} & x \in X, \end{array}$$

where $X \subset \mathbb{R}^{mn}$ is the feasible region corresponding to the problem, and $Z : \mathbb{R}^{mn} \longrightarrow \mathbb{R}^K$ is a vector valued objective function mapping a feasible solution x to the point $Z = (Z^1(x), Z^2(x), \ldots, Z^K(x))$ in the objective space $\mathbb{R}^K$.

Here, we present some useful relations to define the definitions followed by them.

Let $\mathbb{R}^K_{\geq}$=$\{y \in \mathbb{R}^K : y_j \geq 0, \quad j = 1, 2, \ldots, K\}$ and $\mathbb{R}^K_{>}$=$\{y \in \mathbb{R}^K : y_j > 0, \quad j = 1, 2, \ldots K\}$. For all $y^1, y^2 \in \mathbb{R}^K$,

$y^1 \leqq y^2$ if $y^2 - y^1 \in \mathbb{R}^K_{\geqq}$,

$y^1 \leq y^2$ if $y^1 \leqq y^2$ and $y^1 \neq y^2$,

$y^1 < y^2$ if $y^2 - y^1 \in \text{int } (\mathbb{R}^K_{\geqq})$=$\mathbb{R}^K_{>}$.

Definition 4.2.1 ***(Pareto minimal element):*** *Considering a nonempty subset S of $\mathbb{R}^K$, an element $s \in S$ is said to be a Pareto minimal element of the set S, written as $s \in min(S)$, if $\{s - \mathbb{R}^K_{\geq}\} \bigcap S = \{s\}$, where $s - \mathbb{R}^K_{\geq} = \{s - t : \ t \in \mathbb{R}^K_{\geq}\}$.*

Definition 4.2.2 ***(Efficient point):*** *Let Y be a non empty subset of $\mathbb{R}^K$. An element $y \in Y$ is called an efficient point if $\{y - \mathbb{R}^K_{\geq}\} \bigcap Y = \{y\}$, i.e., there is no $y^* \in \mathbb{R}^K$ such that $y^* \leq y$.*

Definition 4.2.3 ***(Cone of a set):*** *The cone of a set Y is denoted by cone(Y) and is defined as cone(Y)=$\{\alpha y : \alpha \geq 0, y \in Y\}$.*

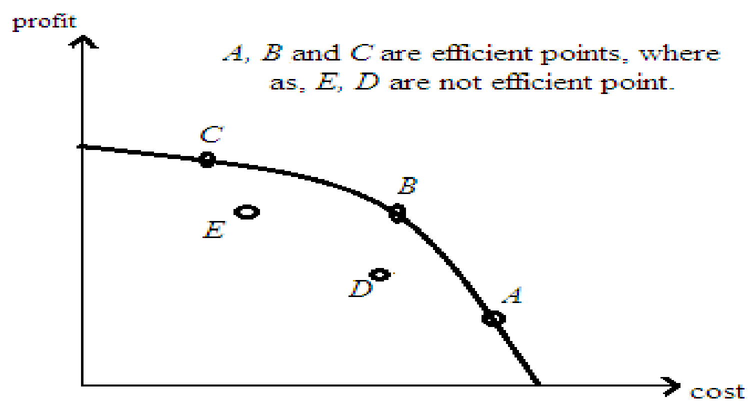

Figure 4.1: Efficient point.

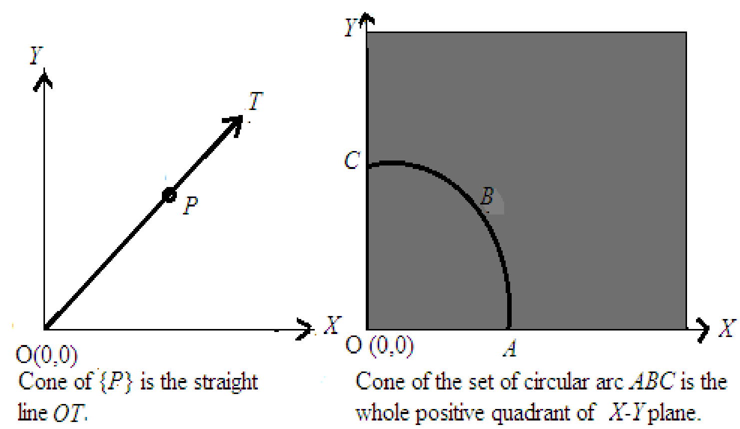

Figure 4.2: Cone of a set.

Definition 4.2.4 ***(Properly efficient point):*** *An element $y \in Y$ is said to be a properly efficient point (in the sense of Benson) if y is an efficient point of Y and the zero element of $\mathbb{R}^K$ is an efficient point of cl(cone $(Y + \mathbb{R}^K_{\geqq} - y)$), where cl$(Y)$ denotes the closure of a set Y.*

Figures 4.1, 4.2 and 4.3 show the graphs of Efficient point, Cone of a set and Properly efficient point respectively. The set of all efficient points of Y is denoted by Y_N, and the set of all properly efficient points is denoted by Y_{pN}. A feasible solution $x \in X$ is called (properly) efficient solution if $y = y(x)$ is a (properly) efficient point of Y. The set of (properly) efficient solutions of

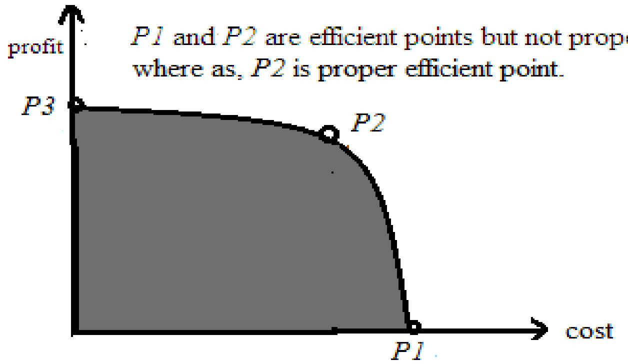

Figure 4.3: Properly efficient point.

MOP is denoted by X_{pE} (X_E), respectively. Conic Scalarization Gasimov (40) can be represented in the following manner:
Let $W = \{(\beta, w) \in \mathbb{R} \times \mathbb{R}^K_{\geq} : 0 < \beta < \min\{w_1, w_2, \ldots, w_K\}$, where $w_i \geq 0$ for all $i\}$.

Proposition 4.1 [Ustun (153)]
Suppose that for some $(\beta, w) \in W$, a feasible solution $\hat{x} \in X$ is an optimal solution of minimization problem as stated in Model 4.7.

Model 4.7

$$\begin{aligned} \text{minimize} \quad & \left[\beta \sum_{t=1}^{K} |Z^t(x)| + \sum_{t=1}^{K} w_t \; Z^t(x)\right] \\ \text{subject to} \quad & x \in X, \end{aligned}$$

where X is the feasible set, then $\hat{x}$ is a Benson proper efficient solution to Model 4.6.

Proposition 4.2 [Ustun (153)]
Let $\hat{x} \in X$ is a Benson proper efficient solution to Model 4.6. Then there exists a vector $(\beta, w) \in W$ such that $\hat{x}$ is an optimal solution to the minimization problem described in Model 4.8.

Model 4.8

$$\begin{aligned} \text{minimize} \quad & \left[\beta \sum_{t=1}^{K} |Z^t(x) - Z^t(\hat{x})| + \sum_{t=1}^{K} \; w_t \; (Z^t(x) - Z^t(\hat{x}))\right] \\ \text{subject to} \quad & x \in X. \end{aligned}$$

In non-convex MOP, the difference between supported and unsupported efficient solutions is important. An efficient solution $\hat{x} \in X_E$ is called supported, if there is an $w \in \mathbb{R}^K_>$ such that $\hat{x}$ is an optimal solution to following Model 4.9.

Model 4.9

$$\begin{array}{ll} \text{minimize} & \sum_{t=1}^{K} w_t Z^t(x) \\ \text{subject to} & x \in X. \end{array}$$

Now, it is evident that if $\hat{x} \in X$ is an efficient solution to Model 4.6, then it is also an efficient solution to the related MOP:

Model 4.10

$$\begin{array}{ll} \text{minimize} & \left[Z^1(x) - g_1, Z^2(x) - g_2, \ldots, Z^K(x) - g_K\right] \\ \text{subject to} & x \in X, \end{array}$$

where $g = (g_1, g_2, \ldots, g_K) \in \mathbb{R}^K$ is an arbitrary vector. Such a shifting can be occured in situations when objective functions do not change sign on whole efficient solution set X_E in order to make the absolute value used in the scalarized problem in Model 4.7. In this case, we can formulate the following scalarized problem, which is similar to that in Model 4.8 and can be used even if we do not know any efficient solution. Furthermore, we introduce Model 4.11.

Model 4.11

$$\begin{array}{ll} \text{minimize} & \left[\beta \sum_{t=1}^{K} |Z^t(x) - g_i)| + \sum_{t=1}^{K} \ w_t \ (Z^t(x) - g_t)\right] \\ \text{subject to} & x \in X. \end{array}$$

We can therefore completely characterize Benson proper efficient solutions through scalarization of Gasimov (40).

Proposition 4.3

A feasible solution $\hat{x} \in X$ is a Benson proper efficient if and only if there are $g \in \mathbb{R}^K$ and $(\beta, w) \in W$ such that it is an optimal solution to the scalar minimization problem of Model 4.11.

Again, Model 4.11 is equivalent to following Model 4.12:

Model 4.12

$$\begin{aligned}
\text{minimize} \quad & \left[\sum_{t=1}^{K} \big((\beta + w_t)d_t^+ + (\beta - w_t)d_t^-\big)\right] \\
\text{subject to} \quad & Z^t(x) - d_t^+ + d_t^- = g_t \quad (t = 1, 2, \ldots, K), \\
& d_t^+, d_t^- \geq 0 \;\; (t = 1, 2, \ldots, K), \\
& x \in X.
\end{aligned}$$

Here, $d_t^+ = \max(0, Z^t - g_t)$ and $d_t^- = \max(0, g_t - Z^t)$ are, over- and under-achievements of the t-th goal, respectively, and g_t is the aspiration or target level for the t-th goal.

If the MOTP has a feasible region, then minimization problem is connected to convex conditional and the solution will be obtained according to a Benson efficient point. Finally, to solve MOTP using Conic Scalarization Function (CSF), we formulate the following Model 4.13, defined as:

Model 4.13

$$\begin{aligned}
\text{minimize} \quad & \left[\sum_{t=1}^{K} \big((\beta + w_t)d_t^+ + (\beta - w_t)d_t^-\big)\right] \\
\text{subject to} \quad & Z'^t(x) - d_t^+ + d_t^- = g_t, && (4.25) \\
& g_{t,\text{min}} \leq g_t \leq g_{t,\text{max}}, && (4.26) \\
& d_t^+, d_t^- \geq 0 \;\; (t = 1, 2, \ldots, K), \\
& \sum_{j=1}^{n} x_{ij} \leq a_i' \;\; (i = 1, 2, \ldots, m), \\
& \sum_{i=1}^{m} x_{ij} \geq b_j' \;\; (j = 1, 2, \ldots, n), \\
\text{and} \quad & x_{ij} \geq 0 \quad \forall \;\; i \;\; \text{and} \;\; j.
\end{aligned}$$

If for the value of objective function, more is better, we consider the constraints (4.25) and (4.26) in the following form without changing other equations in

Model 4.13:

$$-Z'^{t}(x) - d_t^+ + d_t^- = g_t,$$
$$-g_{t,\text{min}} \geq g_t \geq -g_{t,\text{max}}.$$

4.3 Numerical examples

To test the feasibility of the proposed method such as MOTP and MCMTP, we consider two numerical examples. The first example is on MOTP and the second one is on MCMTP. We solve both the problems to justify a better solution of our proposed model using Conic Scalarization approach, and then we compare the solution with the solutions of the existing methods like GP and RMCGP.

4.3.1 Example 4.1

A coal company transports three types of coals namely Anthracite (AN), Bituminus (BI), Pit (PI) to a Thermal Power (THP) and to the Open Market (OM). The different types of coals create pollution after used and the amounts of gas outputted after used in different objectives are given in Table 4.1. The costs of different types of supplied coals to the destinations THP and OM are given in Table 4.2. Also, the transportation costs for supplying to THP and OM are placed at Table 4.3. The company wishes to maximize the coal cost, to minimize the air pollution, i.e., to minimize the outputted gas after used of coal along with the aim to minimize the transportation cost. The company again wishes that the goals corresponding to cost of coal will not be less than \$3000 and in maximum it may take \$3300. The outputted poisonous gas (in litre) will take a value within the interval [800, 850] in which less is better. The total transportation cost will not exceed \$450 and it is greater than \$430, so that minimum transportation cost is preferable. The company also decides to find a compromise solution with priority to the goals as 50% for coal cost, 20% for pollution control, 30% for transportation cost, satisfying all the respective goals depicted in the problem. Here, the priorities are considered as weights and they are $w_1 = 0.5,\ w_2 = 0.2$ and $w_3 = 0.3$ for goals 1, 2 and 3, respectively.

Table 4.1: Coal cost for supplying THP and OM.

	THP	OM
AN	7.5	8.0
BI	6.5	6.7
PI	4.75	4.5

Table 4.2: Outputted poisonous gas in litre/ton.

	THP	OM
AN	1.0	0.5
BI	1.2	1.1
PI	3.2	2.4

Table 4.3: Transportation cost in \$.

	THP	OM
AN	0.6	0.8
BI	0.5	1.5
PI	0.9	1.5

Let us formulate MOTP model utilizing the following data. In this problem, the goals of objective functions are interval-valued and they are
$g_1 = [3000, 3300]$ (more is better), $g_2 = [800, 850]$ (less is better), and $g_3 = [430, 450]$ (less is better).
The demands in the destinations are 200 ton and 250 ton, respectively and supplies at the origins are 200 ton, 125 ton and 175 ton, respectively. Using GP, we formulate the problem as follows:

Model 4.P1

$$\begin{aligned}
\text{minimize} \quad & \frac{0.5}{300}(d_1^+ + d_1^-) + \frac{0.2}{50}(d_2^+ + d_2^-) + \frac{0.3}{20}(d_3^+ + d_3^-) \\
\text{subject to} \quad & Z^1 = 7.5x_{11} + 8x_{12} + 6.5x_{21} + 6.7x_{22} + 4.75x_{31} + 4.5x_{32}, \\
& Z^2 = 1.0x_{11} + 0.5x_{12} + 1.2x_{21} + 1.1x_{22} + 3.2x_{31} + 2.4x_{32}, \\
& Z^3 = 0.6x_{11} + 0.8x_{12} + 0.5x_{21} + 1.5x_{22} + 0.9x_{31} + 1.5x_{32}, \\
& Z^1 - d_1^+ + d_1^- = g_1, \\
& 3000 \leq g_1 \leq 3300,
\end{aligned}$$

$$Z^2 - d_2^+ + d_2^- = g_2,$$
$$800 \leq g_2 \leq 850,$$
$$Z^3 - d_3^+ + d_3^- = g_3,$$
$$430 \leq g_3 \leq 450,$$
$$x_{11} + x_{12} \leq 200, \tag{4.27}$$
$$x_{21} + x_{22} \leq 125, \tag{4.28}$$
$$x_{31} + x_{32} \leq 175, \tag{4.29}$$
$$x_{11} + x_{21} + x_{31} \geq 220, \tag{4.30}$$
$$x_{12} + x_{22} + x_{32} \geq 250, \tag{4.31}$$
$$d_i^+ \geq 0, d_i^- \geq 0, \quad \text{for} \quad i = 1, 2, 3.$$

Using RMCGP, we formulate the same problem as follows:

Model 4.P2

$$\text{minimize} \quad \frac{0.5}{300}(d_1^+ + d_1^-) + \frac{0.2}{50}(d_2^+ + d_2^-) + \frac{0.3}{20}(d_3^+ + d_3^-)$$
$$+\frac{0.5}{300}(e_1^+ + e_1^-) + \frac{0.2}{50}(e_2^+ + e_2^-) + \frac{0.3}{20}(e_3^+ + e_3^-)$$

$$\text{subject to} \quad Z^1 = 7.5x_{11} + 8x_{12} + 6.5x_{21} + 6.7x_{22} + 4.75x_{31} + 4.5x_{32},$$
$$Z^2 = 1.0x_{11} + 0.5x_{12} + 1.2x_{21} + 1.1x_{22} + 3.2x_{31} + 2.4x_{32},$$
$$Z^3 = 0.6x_{11} + 0.8x_{12} + 0.5x_{21} + 1.5x_{22} + 0.9x_{31} + 1.5x_{32},$$
$$Z^1 - d_1^+ + d_1^- = g_1,$$
$$g_1 - e_1^+ + e_1^- = 3300,$$
$$3000 \leq g_1 \leq 3300,$$
$$Z^2 - d_2^+ + d_2^- = g_2,$$
$$g_2 - e_2^+ + e_2^- = 800,$$
$$800 \leq g_2 \leq 850,$$
$$Z^3 - d_3^+ + d_3^- = g_3,$$
$$g_3 - e_3^+ + e_3^- = 430,$$
$$430 \leq g_3 \leq 450,$$

$$d_i^+ \geq 0, d_i^- \geq 0, e_i^+ \geq 0, e_i^- \geq 0, \text{ for } i = 1, 2, 3,$$
$$\text{and} \quad \text{the constraints } (4.27) - (4.31).$$

Using Conic Sclarization approach, we formulate the following mathematical model:

Model 4.P3

$$\begin{aligned}
\text{minimize} \quad & \frac{1}{300}(0.5 + 0.15)d_1^+ + \frac{1}{300}(0.15 - 0.5)d_1^- + \frac{1}{50}(0.2 + 0.15)d_2^+ \\
& + \frac{1}{50}(0.15 - 0.2)d_2^- + \frac{1}{20}(0.3 + 0.15)d_3^+ + \frac{1}{20}(0.15 - 0.3)d_3^- \\
\text{subject to} \quad & -7.5x_{11} - 8x_{12} - 6.5x_{21} - 6.7x_{22} - 4.75x_{31} - 4.5x_{32} - d_1^+ + d_1^- \\
& = g_1, \quad -3300 \leq g_1 \leq -3000, \\
& 1.0x_{11} + 0.5x_{12} + 1.2x_{21} + 1.1x_{22} + 3.2x_{31} + 2.4x_{32} - d_2^+ + d_2^- \\
& = g_2, \quad 800 \leq g_2 \leq 850, \\
& 0.6x_{11} + 0.8x_{12} + 0.5x_{21} + 1.5x_{22} + 0.9x_{31} + 1.5x_{32} - d_3^+ + d_3^- \\
& = g_3, \quad 430 \leq g_3 \leq 450, \\
& d_i^+ \geq 0, d_i^- \geq 0; \text{ for } i = 1, 2, 3, \\
\text{and} \quad & \text{the constraints } (4.27) - (4.31).
\end{aligned}$$

Solving the models, i.e., Models 4.P1, 4.P2 and 4.P3, by LINGO software, we list the solutions in Table 4.4.

Table 4.4: Solutions for Example 4.1 by different methods.

Method (Model)	Optimal Value of Z^1	Optimal Value of Z^2	Optimal Value of Z^3	Optimal Solution
GP (P1)	3140.65	800.0	450.0	$x_{11} = 34.87, x_{12} = 165.14, x_{21} = 24.34,$ $x_{22} = 84.87, x_{31} = 175.0, x_{32} = 0.0$
RMCGP (P2)	3251.25	800.0	430.0	$x_{11} = 0, x_{12} = 200, x_{21} = 79.05,$ $x_{22} = 45.95, x_{31} = 168.25, x_{32} = 6.75$
CSF (P3)	3251.25	800.0	430.0	$x_{11} = 0, x_{12} = 200, x_{21} = 79.05,$ $x_{22} = 45.95, x_{31} = 168.25, x_{32} = 6.75$

According to the obtained solutions of Models 4.P1, 4.P2 and 4.P3, it is clear that RMCGP and Conic Scalarization approach produce better solutions than GP method. Although optimal solutions and optimal values are same from RMCGP and Conic Scalarization approach, we see that the number of auxiliary variables in Conic Scalarization approach is half the number of variable in

RMCGP. In this regard, we may say that the Conic Scalarization approach is used with less effort for solving MCMTP and any number of goals may be accommodated in MCMTP.

4.3.2 Example 4.2

Example 4.2 is designed with the following assumptions in addition to the Example 4.1. Due to uncertain demands in the market, sometime the demand or the supply may not be fixed and they may be multi-choices. Also, cost penalties could be multi-choices due to road conditions or weather conditions, etc. Under these circumstances, decision makers usually wish to consider the best fit of parameters by guessing which may not give a better solution for the problem always. Here, we construct the mathematical model which considers all choices and to find a better choice of parameters and a better solution of the objective function. Instead of single choice, multi-choice options are considered for cost, demand and supply parameters; then we design the MCMTP as follows:

$$\begin{aligned}
Z^1 &= (7.5 \text{ or } 8)x_{11} + 8x_{12} + (6.5 \text{ or } 7.5 \text{ or } 8.4 \text{ or } 8.6)x_{21} + 6.7x_{22} \\
&\quad +4.75x_{31} + 4.5x_{32}, \qquad [3000,\ 3300]\text{: more is better}, \\
Z^2 &= 1.0x_{11} + 0.5x_{12} + (1.2 \text{ or } 2.7)x_{21} + 1.1x_{22} + 3.2x_{31} \\
&\quad +(2.4 \text{ or } 5)x_{32}, \qquad [800,\ 850]\text{: less is better}, \\
Z^3 &= (0.6 \text{ or } 0.7)x_{11} + 0.8x_{12} + 0.5x_{21} + 1.5x_{22} + 0.9x_{31} \\
&\quad +(2.5 \text{ or } 1.5)x_{32}, \qquad [430,\ 450]\text{: less is better}, \\
\text{subject to} \quad & x_{11} + x_{12} \leq \ (200 \ \text{ or } \ 150 \ \text{ or } \ 175), \\
& x_{21} + x_{22} \leq \ (125 \ \text{ or } \ 140 \ \text{ or } \ 130), \\
& x_{31} + x_{32} \leq 175, \\
& x_{11} + x_{21} + x_{31} \geq \ (220 \ \text{ or } \ 150), \\
& x_{12} + x_{22} + x_{32} \geq \ (250 \ \text{ or } \ 200 \ \text{ or } \ 230), \\
& x_{ij} \geq 0, i = 1, 2, \ j = 1, 2, 3.
\end{aligned}$$

To justify the effictiveness of multi-choice parameters, let us introduce the extra constraint that the total demands in two locations is not less 450. The MCMTP can be reduced to the following MOTP problem using binary variables, which can be highlighted as follows:

Model 4.P4

$$\begin{aligned}
Z^1 &= (7.5z_{111}^1 + 8(1 - z_{111}^1))x_{11} + 8x_{12} + \\
&\quad (6.5z_{121}^1 z_{121}^2 + 7.5(1 - z_{121}^1)z_{121}^2 + 8.4(1 - z_{121}^2)z_{121}^1 \\
&\quad +8.6(1 - z_{121}^1)(1 - z_{121}^2))x_{21} + 6.7x_{22} + 4.75x_{31} + 4.5x_{32}, \\
&\quad [3000,\ 3300]\text{: more is better,} \\
Z^2 &= 1.0x_{11} + 0.5x_{12} + (1.2z_{221}^1 + 2.7(1 - z_{221}^1))x_{21} \\
&\quad +1.1x_{22} + 3.2x_{31} + (2.4z_{232}^1 + 5(1 - z_{232}^1))x_{32}, \\
&\quad [800,\ 850]\text{: less is better,} \\
Z^3 &= (0.6z_{311}^1 + 0.7(1 - z_{311}^1))x_{11} + 0.8x_{12} \\
&\quad +0.5x_{21} + 1.5x_{22} + 0.9x_{31} + (2.5z_{332}^1 + 1.5(1 - z_{332}^1))x_{32}, \\
&\quad [430,\ 450]\text{: less is better,}
\end{aligned}$$

subject to $x_{11} + x_{12} \leq 200z_1^1 z_1^2 + 150(1 - z_1^1)z_1^2 + 175(1 - z_1^2)z_1^1,$ (4.32)

$$x_{21} + x_{22} \leq 125z_2^1 z_2^2 + 140(1 - z_2^1)z_2^2 + 130(1 - z_2^2)z_2^1, \tag{4.33}$$

$$x_{31} + x_{32} \leq 175, \tag{4.34}$$

$$x_{11} + x_{21} + x_{31} \geq 220z^1 + 150(1 - z^1), \tag{4.35}$$

$$x_{12} + x_{22} + x_{32} \geq 250z^2 z^3 + 200(1 - z^2)z^3 + 230(1 - z^3)z^2, \tag{4.36}$$

$$x_{11} + x_{12} + x_{21} + x_{22} + x_{31} + x_{32} \geq 450, \tag{4.37}$$

$$1 \leq z_1^1 + z_1^2 \leq 2, \tag{4.38}$$

$$1 \leq z_2^1 + z_2^2 \leq 2, \tag{4.39}$$

$$1 \leq z^2 + z^3 \leq 2, \tag{4.40}$$

$$z_{111}^1, z_{121}^1, z_{121}^2, z_{221}^1, z_{232}^1, z_{311}^1, z_{332}^1 = 0/1, \tag{4.41}$$

$$z_1^1, z_1^2, z_2^1, z_2^2 = 0/1, \tag{4.42}$$

$$z^1, z^2, z^3 = 0/1, \tag{4.43}$$

$$x_{ij} \geq 0 \quad (i = 1, 2,\ j = 1, 2, 3). \tag{4.44}$$

Using GP, Model 4.P4 reduces to Model 4.P5 as follows:

Model 4.P5

$$\begin{aligned}
\text{minimize} \quad & \frac{0.5}{300}(d_1^+ + d_1^-) + \frac{0.2}{50}(d_2^+ + d_2^-) + \frac{0.3}{20}(d_3^+ + d_3^-) \\
\text{subject to} \quad & Z^1 = (7.5z_{111}^1 + 8(1 - z_{111}^1))x_{11} + 8x_{12} + \\
& (6.5z_{121}^1 z_{121}^2 + 7.5(1 - z_{121}^1)z_{121}^2 + 8.4(1 - z_{121}^2)z_{121}^1 \\
& +8.6(1 - z_{121}^1)(1 - z_{121}^2))x_{21} + 6.7x_{22} + 4.75x_{31} + 4.5x_{32}, \\
& Z^2 = 1.0x_{11} + 0.5x_{12} + (1.2z_{221}^1 + 2.7(1 - z_{221}^1))x_{21} + 1.1x_{22} \\
& +3.2x_{31} + (2.4z_{232}^1 + 5(1 - z_{232}^1))x_{32}, \\
& Z^3 = (0.6z_{311}^1 + 0.7(1 - z_{311}^1))x_{11} + 0.8x_{12} + \\
& 0.5x_{21} + 1.5x_{22} + 0.9x_{31} + (2.5z_{332}^1 + 1.5(1 - z_{332}^1))x_{32}, \\
& Z^1 - d_1^+ + d_1^- = g_1, \\
& 3000 \leq g_1 \leq 3300, \\
& Z^2 - d_2^+ + d_2^- = g_2, \\
& 800 \leq g_2 \leq 850, \\
& Z^3 - d_3^+ + d_3^- = g_3, \\
& 430 \leq g_3 \leq 450, \\
& d_i^+ \geq 0, d_i^- \geq 0 \quad \text{for } i = 1, 2, 3, \\
\text{and} \quad & \text{the constraints (4.32) -- (4.44).}
\end{aligned}$$

Using RMCGP, Model 4.P4 reduces to Model 4.P6 as follows:

Model 4.P6

$$\begin{aligned}
\text{minimize} \quad & \frac{0.5}{300}(d_1^+ + d_1^-) + \frac{0.2}{50}(d_2^+ + d_2^-) + \frac{0.3}{20}(d_3^+ + d_3^-) \\
& + \frac{0.5}{300}(e_1^+ + e_1^-) + \frac{0.2}{50}(e_2^+ + e_2^-) + \frac{0.3}{20}(e_3^+ + e_3^-)
\end{aligned}$$

subject to
$$Z^1 = (7.5z^1_{111} + 8(1 - z^1_{111}))x_{11} + 8x_{12} +$$
$$(6.5z^1_{121}z^2_{121} + 7.5(1 - z^1_{121})z^2_{121} + 8.4(1 - z^2_{121})z^1_{121}$$
$$+8.6(1 - z^1_{121})(1 - z^2_{121}))x_{21} + 6.7x_{22} + 4.75x_{31} + 4.5x_{32},$$
$$Z^2 = 1.0x_{11} + 0.5x_{12} + (1.2z^1_{221} + 2.7(1 - z^1_{221}))x_{21}$$
$$+1.1x_{22} + 3.2x_{31} + (2.4z^1_{232} + 5(1 - z^1_{232}))x_{32},$$
$$Z^3 = (0.6z^1_{311} + 0.7(1 - z^1_{311}))x_{11} + 0.8x_{12} +$$
$$0.5x_{21} + 1.5x_{22} + 0.9x_{31} + (2.5z^1_{332} + 1.5(1 - z^1_{332}))x_{32},$$
$$Z^1 - d^+_1 + d^-_1 = g_1, \quad g_1 - e^+_1 + e^-_1 = 3300, \quad 3000 \le g_1 \le 3300,$$
$$Z^2 - d^+_2 + d^-_2 = g_2, \quad g_2 - e^+_2 + e^-_2 = 800, \quad 800 \le g_2 \le 850,$$
$$Z^3 - d^+_3 + d^-_3 = g_3, \quad g_3 - e^+_3 + e^-_3 = 430, \quad 430 \le g_3 \le 450,$$
$$e^+_i \ge 0, e^-_i \ge 0, \ d^+_i \ge 0, d^-_i \ge 0, \ \text{for} \ \ i = 1, 2, 3,$$
and the constraints (4.32) − (4.44).

Using Conic Sclarization approach, we formulate the following model:

Model 4.P7

minimize
$$\frac{1}{300}(0.5 + 0.15)d^+_1 + \frac{1}{300}(0.15 - 0.5)d^-_1 + \frac{1}{50}(0.2 + 0.15)d^+_2$$
$$+\frac{1}{50}(0.15 - 0.2)d^-_2 + \frac{1}{20}(0.3 + 0.15)d^+_3 + \frac{1}{20}(0.15 - 0.3)d^-_3$$

subject to
$$-((7.5z^1_{111} + 8(1 - z^1_{111}))x_{11} + 8x_{12} +$$
$$(6.5z^1_{121}z^2_{121} + 7.5(1 - z^1_{121})z^2_{121} + 8.4(1 - z^2_{121})z^1_{121}$$
$$+8.6(1 - z^1_{121})(1 - z^2_{121}))x_{21} + 6.7x_{22} + 4.75x_{31} + 4.5x_{32})$$
$$-d^+_1 + d^-_1 = g_1, \quad -3300 \le g_1 \le -3000,$$
$$1.0x_{11} + 0.5x_{12} + (1.2z^1_{221} + 2.7(1 - z^1_{221}))x_{21} + 1.1x_{22} +$$
$$3.2x_{31} + (2.4z^1_{232} + 5(1 - z^1_{232}))x_{32} - d^+_2 + d^-_2 = g_2,$$
$$800 \le g_2 \le 850,$$
$$(0.6z^1_{311} + 0.7(1 - z^1_{311}))x_{11} + 0.8x_{12} + 0.5x_{21} + 1.5x_{22}$$

$$+0.9x_{31} + (2.5z_{332}^1 + 1.5(1 - z_{332}^1))x_{32} - d_3^+ + d_3^- = g_3,$$

$$430 \leq g_3 \leq 450,$$

and the constraints (4.32) – (4.44).

Solving the models (Models 4.P5, 4.P6 and 4.P7) by LINGO software, we derive the solutions given in Tables 4.5 and 4.6.

Table 4.5: Solutions for Example 4.2 by different methods.

Method (Model)	Optimal Value of Z^1	Optimal Value of Z^2	Optimal Value of Z^3	Optimal Solution	Choice of Demand	Choice of Supply
GP (P5)	3000.0	800.0	450.0	$x_{11} = 3.0, x_{12} = 168.4,$ $x_{21} = 16.5, x_{22} = 98.5,$ $x_{31} = 174.5, x_{32} = 0$	150 for THP, 250 for OM	175 for A, 125 for B, 175 for C
RMCGP (P6)	3300.0	800.0	430.0	$x_{11} = 32.45, x_{12} = 167.55,$ $x_{21} = 53.33, x_{22} = 63.43,$ $x_{31} = 171.88, x_{32} = 0$	220 for THP, 250 for OM	200 for A, 125 for B, 175 for C
CSF (P7)	3300.0	800.0	425.0	$x_{11} = 25.19, x_{12} = 174.81,$ $x_{21} = 57.73, x_{22} = 56.45,$ $x_{31} = 173.76, x_{32} = 0.0$	220 for THP, 250 for OM	200 for A, 125 for B, 175 for C

Table 4.6: Choice of cost penalties in solutions for Example 4.2 by different methods.

Method	Choice of Cost penalties
GP	$C_{11}^1 = 8, C_{12}^1 = 8, C_{21}^1 = 8.6, C_{22}^1 = 6.7, C_{31}^1 = 4.75, C_{32}^1 = 4.5,$ $C_{11}^2 = 1.0, C_{12}^2 = 0.5, C_{21}^2 = 2.7, C_{22}^2 = 1.1, C_{31}^2 = 3.2, C_{32}^2 = 5,$ $C_{11}^3 = 0.7, C_{12}^3 = 0.8, C_{21}^3 = 0.5, C_{22}^3 = 1.5, C_{31}^3 = 0.9, C_{32}^3 = 2.5$
RMCGP	$C_{11}^1 = 8, C_{12}^1 = 8, C_{21}^1 = 8.6, C_{22}^1 = 6.7, C_{31}^1 = 4.75, C_{32}^1 = 4.5,$ $C_{11}^2 = 1.0, C_{12}^2 = 0.5, C_{21}^2 = 1.2, C_{22}^2 = 1.1, C_{31}^2 = 3.2, C_{32}^2 = 5,$ $C_{11}^3 = 0.6, C_{12}^3 = 0.8, C_{21}^3 = 0.5, C_{22}^3 = 1.5, C_{31}^3 = 0.9, C_{32}^3 = 1.5$
CSF	$C_{11}^1 = 8, C_{12}^1 = 8, C_{21}^1 = 8.6, C_{22}^1 = 6.7, C_{31}^1 = 4.75, C_{32}^1 = 4.5,$ $C_{11}^2 = 1.0, C_{12}^2 = 0.5, C_{21}^2 = 1.2, C_{22}^2 = 1.1, C_{31}^2 = 3.2, C_{32}^2 = 5,$ $C_{11}^3 = 0.6, C_{12}^3 = 0.8, C_{21}^3 = 0.5, C_{22}^3 = 1.5, C_{31}^3 = 0.9, C_{32}^3 = 1.5$

According to the obtained solutions of Models 4.P5, 4.P6 and 4.P7, it is clear that Conic Scalarization approach produces a better result than GP and RMCGP method. Also, we see that the number of auxiliary variables in Conic Scalarization approach is half the number of variables in RMCGP method. In this regard, we can say that Conic Scalarization approach is used to less effort for solving MCMTP and any number of goals may be accommodated in MCMTP.

Again, from last two columns of Table 4.5, it is seen that the selection of demands and supply is different in GP and RMCGP or CSF technique which creates a good effect in the solution for three objective functions satisfying the goals. Selection of multi-choice transportation parameters for optimal solution is presented in Table 4.6. Under the consideration of multi-choice transportation parameters in MOTP, it is seen that the profit of coal cost is highly achieved in Example 4.2 whenever the other goals are also satisfactory to the decision maker. Based on the solutions of our numerical examples, we justify that Conic Scalarization function is a better approach to solve a real-life MOTP involving multi-choice transportation parameters in comparison to others.

4.4 Sensitivity analysis

The mathematical model of GP is a special structure of RMCGP model since the value of $\alpha_i = 0$ in RMCGP model generates a GP model. GP tries to optimize the goal values but it does not consider the goals properly for maximization or minimization problems, whereas RMCGP treats these goals as the decision maker's choices. Generally, RMCGP tries to improve the values of objective functions from lower bounds to upper bounds in connection with the interval goals, but CSF approach tries to improve the values of objective functions from lower bounds to the efficient frontiers. So, RMCGP does not guarantee an efficient solution, whereas CSF provides a more satisfactory solution in "more is better" or "less is better" for an optimization problem. In addition to this, an important aspect about the usage of RMCGP or CSF in practice is that how can the values of the parameters α_i in RMCGP for $i = 1, 2, \ldots, n$ or β in CSF be determined practically. The values of the weights w_i for $i = 1, 2, \ldots, n$, are assigned by the decision maker and these weights are obtained by a Analytic Hierarchy Process, a Analytic Network Process, a simple additive weighting method, etc. In our discussion, we take $\alpha_i = w_i$ and w_i=1/(total variation of the i-th goal), for $i = 1, 2, \ldots, n$. After that, the value of the parameter β will be interactively determined by considering the decision maker's preferences. Considering different values of β in CSF, the decision maker can get an efficient solution according to the desired utility value of the objective functions. The decision maker may want to change the initial range of interval goals and the initial values of parameters due to a certain effect of learning after an efficient solution is obtained.

4.5 Conclusion

In this chapter, we have considered a multi-choice multi-objective transportation problem, where the cost, demand and supply parameters are of multi-choice types. Another important notion of this study is to incorporate the goal preferences of the decision maker. We have proposed this concept for the first time to solve multi-choice multi-objective transportation problem by employing the Conic Scalarization approach with less number of variables and with minimum computational burden. The MCMTP is given a new direction to handle the real-life multi-objective transportation problem when the transportation parameters are multi-choices in nature. Two numerical examples are presented in this chapter to explore the applicability and suitability of our approach for solving MOTP and MCMTP with consideration of decision maker preferences. In addition, the proposed method can be used as a decision making aid for multi-choice multi-objective decision making problems from real-life situations like economical, agricultural, industrial management, etc.

The notion of multi-choice parameters can also be used in real-world supply chain management problems. In that context, the number of variables increases in GP or RMCGP but in Conic Scalarization approach, the proposed method allows for a better solution satisfying all the goals; consequently, the decision maker can take a proper decision under a multi-choice environment of multi-objective transportation problem.

Chapter 5

Cost and Time Minimizing Transportation Problem*

Time is an important factor in real-life MOTPs. Considering this fact, in this chapter, we introduce the study of minimizing cost and time through single objective function in the light of multi-choice environment with interval analysis. The parameters of transportation problem follow multi-choice interval valued type so this form of TP is called Multi-Choice Interval Transportation Problem (MCITP). A procedure is shown for converting from MCITP to deterministic TP and then we solve it. Finally, a case study is presented to illustrate the usefulness of the discussion.

5.1 Introduction

Transportation problem was mainly developed to reduce the transportation cost in earlier days. Nowadays decision making problems like fixing of cost of goods, profit for sellers, taking decisions for real-life multiple objectives etc. are guided by TP and the classical TP has been taken into account in different mathematical models. In this study, the classical TP has been designed under the environment of multi-choice and interval programming. Minimizing the transportation cost is not only the main issue in this chapter but also time minimizing during transportation (for delivering the goods) is another impor-

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tant issue. At present, in the competitive market scenario, transportation cost and time both need to minimize in business economy, mostly it is needed in case of transporting perishable goods. The basic difference between the cost and the time minimizing transportation problem is that the transportation cost is highly related with how much amount of goods delivered whereas in the transportation time, this is not so.

The classical transportation problem can be described to a special case of linear programming problem and its model is applied to determine how many units of commodity of goods to be shipped from each origin to various destinations, satisfying supply and demand constraints, while it optimizes total cost of transportation. The transportation cost, amount of goods available at the supply points and the amounts required at the demand points are the parameters in the transportation problem. In earlier days, transportation problem was developed with the assumption that the supply, demand and cost parameters are exactly known. But in real-life applications, all the parameters of the transportation problem are not generally defined precisely. There may have some situations where several routes are available for transporting the goods of TP. In several routes, different costs may exist for transporting the goods. In this consideration, the transportation cost becomes multi-choice type. Again, when the transportation cost are tagged with the cost of perishable goods then it may not be said precisely as a fixed value to treat in simply TP. It may have a value lies in an interval [a,b]. Similar consideration may be taken for supply and demand parameters in TP of this chapter. Keeping this point of view, this chapter is designed with these parameters of transportation problem as multi-choice interval valued type.

In presence of multi-choice interval valued cost, demand and supply; transportation problem becomes a Multi-Choice Interval Transportation Problem (MCITP). An MCITP cannot be solved directly unless it converts into deterministic form. To reduce MCITP into a Multi-Choice Transportation Problem (MCTP), at first, the parameters are introduced in a multi-choice interval TP such that, these multi-choice intervals reduce to simply multi-choice trans-

portation problem. Again, to convert this multi-choice numbers, we use a general transformation technique with the help of binary variables in such a way that it changes into a deterministic TP.
A special emphasis of this chapter is that, the reduction of interval cost to a real number is made by a parameter which is a function of transportation time. We consider the parameter λ which depends on time as follows:

$$\lambda = \begin{cases} 0, & \text{if } t < t_0 \\ \frac{t-t_0}{t}, & \text{if } t \geq t_0 \end{cases}$$

where t_0 is the range period of transportation time as per decision maker's choice and t is the actual time of transportation. Clearly, λ is an increasing function of time. If the delivery of goods made within the range period then the minimum transportation cost is a which belongs to an interval cost $[a, b]$ otherwise the transportation cost (C_{ij}) becomes $C_{ij} = a(1 - \lambda) + \lambda b$.
It is cleared from the relation i.e., $C_{ij} = a(1 - \lambda) + \lambda b$ that C_{ij} takes the minimum value when λ tends to zero. As λ is an increasing function in time, so it tends to the minimum value when time of transportation goes to minimum time. Thus, if we wish to minimize the value of C_{ij} then value of λ will also minimize and vice-versa.
Different techniques have been proposed to solve TP with single or multi-objective environment by several researchers. But, up-to-date, no specific method is available to determine optimal time and cost for transportation problem through single objective function. So, the main aim of this chapter is as follows:

- We propose a new way in which time and cost both are minimized through single objective transportation problem.

- The study of proposed TP which involves the multi-choice interval valued cost, supply and demand parameters.

5.2 Mathematical model

A general transportation problem is a typical problem where the main objective is to minimize the transportation cost and is defined as follows:

Model 5.1

$$\text{minimize} \quad Z = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}x_{ij} \tag{5.1}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \leq a_i \quad (i = 1, 2, \ldots, m), \tag{5.2}$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j \quad (j = 1, 2, \ldots, n), \tag{5.3}$$

$$x_{ij} \geq 0, \quad \forall \quad i \text{ and } \quad j \tag{5.4}$$

where C_{ij} $(i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$ is the transportation cost per unit commodity from the i^{th} origin to the j^{th} destination. Here a_i $(i = 1, 2, \ldots, m)$ and b_j $(j = 1, 2, \ldots, n)$ are availability and demand in the i^{th} origin and the j^{th} destination respectively and $\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} b_j$ is the feasibility condition. Due to some unstable situations of market or for the cause of special discount to the customers in business ground, there may exist some cases where the cost parameter per unit commodity in transportation problem is not a crisp value but it may lie in an interval. When the discount is offered to the customers, then it may not be guessed the actual price for the goods to be sold in the market and in that case, if we wish to fix the cost of goods then, it may again lie in an interval. Again, due to multiple routes of the transportation, the cost parameters consider multi-choice types. Also, the supply in the origin and demand in the destination may not be fixed always. Due to weather condition, variation in share market or unpredictable expectation in the market etc., both the purchaser and the supplier predict the amount of buying and selling goods, so it becomes interval-valued type.

So, on the basis of these phenomenons, we consider the parameters of TP as multi-choice interval valued type and then the corresponding mathematical model is defined as follows:

Model 5.2

$$\text{minimize} \quad Z = \sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{C}_{ij}^{1} \text{ or } \tilde{C}_{ij}^{2} \text{ or} \ldots \text{or } \tilde{C}_{ij}^{k})x_{ij} \quad (k = 1, 2, \ldots, K)$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \leq (\tilde{a}_i^1 \text{ or } \tilde{a}_i^2 \text{ or } \ldots \text{ or } \tilde{a}_i^p) \ (i = 1, 2, \ldots, m), \tag{5.5}$$

$$\sum_{i=1}^{m} x_{ij} \geq (\tilde{b}_j^1 \text{or } \tilde{b}_j^2 \text{ or } \ldots \text{ or } \tilde{b}_j^q) \ (j = 1, 2, \ldots, n), \tag{5.6}$$

$$x_{ij} \geq 0, \quad \forall \ i \text{ and } j. \tag{5.7}$$

Here the multi-choice parameters $\tilde{C}_{ij}^{k}, \tilde{a}_i^p, \tilde{b}_j^q$ are interval numbers and these are defined as $\tilde{C}_{ij}^{k} = [C_{ij}^{k^l}, C_{ij}^{k^u}]$, $\tilde{a}_i^p = [a_i^{p^l}, a_i^{p^u}]$, $\tilde{b}_j^q = [b_j^{q^l}, b_j^{q^u}]$ and then the feasibility condition is changed into
$\sum_{i=1}^{m} \max_{p^u}(a_i^{1^u}, a_i^{2^u}, \cdots, a_i^{p^u}) \geq \sum_{j=1}^{n} \min_{q^l}(b_j^{1^l}, b_j^{2^l}, \cdots, b_j^{q^l})$.
The feasibility condition may be defined in different ways according to decision maker's choice. Here, we provide the largest possible feasible region in our proposed model.
In TP, time of transportation, especially for transporting the perishable goods, it is an important factor. Again, due to multiple routes of transportation, the transportation time is also multi-choice which is available to the DM. Keeping this point of view, we construct an another objective function to minimize the transportation time as follows:

$$\text{minimize} \quad T = \sum_{i=1}^{m}\sum_{j=1}^{n}(T_{ij}^{1} \text{ or } T_{ij}^{2} \text{ or } \ldots \text{ or } T_{ij}^{k})\chi_{ij},$$

$$\text{where} \quad \chi_{ij} = \begin{cases} 0, & \text{if } x_{ij} = 0 \\ 1, & \text{if } x_{ij} \neq 0 \end{cases}$$

$$\text{subject to} \quad \text{the constraints } (5.5) - (5.7).$$

Here, T_{ij}^{k} is the time of transporting the goods from i-th node to j-th destination corresponding to k-th route of transportation problem described in Model 5.2.
Hence, in our proposed model, we introduce bi-objective function and both

are to be minimized and is defined as follows (see Model 5.2A):

Model 5.2A

$$\text{minimize } Z = \sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{C}_{ij}^{1} \text{ or } \tilde{C}_{ij}^{2} \text{ or } \ldots \text{or } \tilde{C}_{ij}^{k})x_{ij} \tag{5.8}$$

$$\text{minimize } T = \sum_{i=1}^{m}\sum_{j=1}^{n}(T_{ij}^{1} \text{ or } T_{ij}^{2} \text{ or } \ldots \text{ or } T_{ij}^{k})\chi_{ij} \tag{5.9}$$

$$\text{subject to} \quad \text{the constraints } (5.5) - (5.7).$$

Most of the researchers solved the bi-objective problem and they obtained the compromise solutions by solving both the objective functions or applying methodologies on both objective functions. Here, we propose a function to reduce the interval in such a way that the solution of the first objective function Z provides the solution of itself and also the solution of the second objective function T. The procedure to find compromise solution for the objective functions Z and T by solving the objective function Z has been discussed in details in the next section.

5.3 Solution procedure

The mathematical model of transportation problem has been described in this chapter (see Model 5.2) cannot be solved directly due to present of multi-choice interval valued parameters. So, at first we reduce the problem into a deterministic TP. We develop a function which depends on time and it values lie in the interval $[0, 1]$ and this has been used to reduce the interval valued cost parameter to real valued parameter. The interval valued supply and demand parameters are also converted to real numbers by using parameters which is not necessarily depend on time. This procedure is depicted in subsection 5.3.1 in this chapter. After that multi-choice cost, demand and supply parameters without interval valued are handled to select a particular choice with the help of a suitable transformation which is discussed in the second subsection. In the last subsection, an algorithm is presented to solve the proposed TP.

5.3.1 Reduction of interval into real number using parameter

In our proposed Model 5.2, the transportation costs are multi-choice types and again each choice is of interval type i.e., $\tilde{C}_{ij}^{k} = [C_{ij}^{k^l}, C_{ij}^{k^u}]$, it means that there are some reasons for which the cost may take any value in the prescribed interval. Let us consider a parameter which depends on time. Let t_0 be the time assigned by the decision maker as minimum range period. If the delivery occurred within the minimum range period then the minimum transporting cost $C_{ij}^{k^l}$ have to be paid. Due to delay of delivery the product, the cost becomes $\tilde{C}_{ij}^{k} = C_{ij}^{k^l}(1-\lambda^{C_{ij}^k}) + C_{ij}^{k^u}\lambda^{C_{ij}^k}$. Here $\lambda^{C_{ij}^k}$ is a parameter for each k such that

$$\lambda^{C_{ij}^k} = \begin{cases} 0, & \text{if } T_{ij}^k < t_0 \\ \frac{T_{ij}^k - t_0}{T_{ij}^k}, & \text{if } T_{ij}^k \geq t_0 \end{cases} \tag{5.10}$$

where T_{ij}^k is delivery time, then $\lambda^{C_{ij}^k}$ is an increasing function of time.

Proposition 5.1:

The cost component $\tilde{C}_{ij}^{k}$ attains minimum value when transportation time in k-th route for i-th origin to j-th destination tends to the minimum value and conversely.

Proof: The interval valued cost $[C_{ij}^{k^l}, C_{ij}^{k^u}]$ in k-th route for i-th origin to j-th destination has been made to a real valued cost $\tilde{C}_{ij}^{k}$ by the following way:
$\tilde{C}_{ij}^{k} = C_{ij}^{k^l}(1-\lambda^{C_{ij}^k}) + C_{ij}^{k^u}\lambda^{C_{ij}^k}$. The value of $\tilde{C}_{ij}^{k}$ tends to minimum value as $\lambda^{C_{ij}^k}$ tends to zero. Here $\lambda^{C_{ij}^k}$ is a function of time T_{ij}^k as stated in equation (5.10). From equation (5.10) it is clear that $\lambda^{C_{ij}^k}$ tends to the minimum value as the transportation time T_{ij}^k tends to the fixed time t_0 which is assigned by DM. Therefore, $\tilde{C}_{ij}^{k}$ tends to its minimum value as the transportation time T_{ij}^k tends to t_0; i.e., the cost component $\tilde{C}_{ij}^{k}$ attains minimum value when transportation time in k-th route for i-th origin to j-th destination tends to the minimum value.

Conversely, when the transportation time in k-th route for i-th origin to j-th destination tends to the minimum value then the equation (5.10) suggests that $\lambda^{C_{ij}^k}$ tends to zero and then $\tilde{C}_{ij}^{k}$ tends to the minimum value $C_{ij}^{k^l}$. This completes the proof of the proposition.

Decision maker can also choose the function according to his choice, but it should be time dependent as per our consideration in this chapter.

Again, the supply $\tilde{a}_i^p(=[a_i^{p^l}, a_i^{p^u}])$ and demand $\tilde{b}_j^q(=[b_j^{q^l}, b_j^{q^u}])$ parameters are also multi-choice types and the choices are considered as interval numbers. To reduce the interval numbers into real numbers, we have introduced the parameters within these and as a result, these reduce to $\tilde{a}_i^p = a_i^{p^l}(1-\lambda^{a_i^p})+a_i^{p^u}\lambda^{a_i^p}$ and $\tilde{b}_j^q = b_j^{q^l}(1-\lambda^{b_j^q})+b_j^{q^u}\lambda^{b_j^q}$. Here $\lambda^{a_i^p}$ and $\lambda^{b_j^q}$ are the parameters not necessarily related to time, may be linear or stochastic or fuzzy depends upon the choice of decision maker.

Transformation of multi-choice parameters like cost, supply and demand to the equivalent form

When there are multiple choice of parameters such as cost, supply and demand, we should select a single choice satisfying supply and demand restrictions. The selection of choices should be done in such a way that the whole problem to be optimized. Introduction of binary variables is an important concept to select the choice from the problem. Using the general transformation technique described in subsection 2.3.1 of Chapter 2, we get the following results:

$$\text{Let,}\tilde{C}_{ij}^{'} = \sum_{g=1}^{t}(term)^g\left[C_{ij}^{g^l}(1-\lambda^{C_{ij}^g})+C_{ij}^{g^u}\lambda^{C_{ij}^g}\alpha_c\right] \quad \forall\ i,j, \tag{5.11}$$

where $(term)^g$ (for $g=1,2,\ldots,t$) are the t number of terms in the functions of the binary variables mentioned in above. Similarly,

$$\tilde{a}_i^{'} = \sum_{g=1}^{p}(term)^g\left[a_i^{g^l}(1-\lambda^{a_i^g})+a_i^{g^u}\lambda^{a_i^g}\right] \quad (i=1,2,\ldots,m), \tag{5.12}$$

$$\text{and} \quad \tilde{b}_j^{'} = \sum_{g=1}^{q}(term)^g\left[b_j^{g^l}(1-\lambda^{b_j^g})+b_j^{g^u}\lambda^{b_j^g}\right] \quad (j=1,2,\ldots,n). \tag{5.13}$$

5.3.2 Algorithm for solving TP with minimizing time and cost

Model 5.2A consists of two objective functions, namely transportation cost (Z) and transportation time (T) and both of them are to be minimized by optimizing the objective function Z only. To find the solution of bi-objective optimization problem (see Model 5.2A), we consider an algorithm whose steps are as follows:

Algorithm

Step 1: At first, we include the time of transportation into the interval-valued transportation cost parameter of the objective function Z in such a way that when objective function Z is minimized then both the objective functions Z and T are optimized. To do this, we change the interval-valued multi-choice cost parameters of the objective function Z of TP from Model 5.2A to simply multi-choice cost parameters of TP using the procedure described in subsection 5.3.1.

Step 2: Thereafter, we select the particular choice of cost parameters from the multi-choice costs using the procedure described in subsection 5.3.1 and the same procedure can also be applied for multi-choice supply and demand constraints of the proposed problem. Finally, we design the mathematical model without considering the objective function T (see Model 5.3) as follows:

Model 5.3

$$\begin{aligned} \text{minimize} \quad & Z = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{C}'_{ij} x_{ij} \\ \text{subject to} \quad & \sum_{j=1}^{n} x_{ij} \le \tilde{a}'_i \quad (i = 1, 2, \ldots, m), \\ & \sum_{i=1}^{m} x_{ij} \ge \tilde{b}'_j \quad (j = 1, 2, \ldots, n), \\ & x_{ij} \ge 0, \quad \forall \;\; i \text{ and } \; j. \end{aligned}$$

Here $\tilde{C}'_{ij}, \tilde{a}'_i$ and $\tilde{b}'_j$ are obtained as given in equations (5.11)-(5.13).

Step 3: Solve Model 5.3 (which is a non-linear TP) and report the optimum solution which is denoted as X^*, and the optimal value of the proposed problem with the selection of single choice of cost parameter from multi-choice cost parameters.

Step 4: The optimal solution X^* is obtained through the choice of $\tilde{C}^k_{ij}$, so the minimum transportation time T is calculated by

$$\text{minimize} \quad T = \sum_{i=1}^{m}\sum_{j=1}^{n} T_{ij}^{k}\chi_{ij}, \quad \text{where} \quad \chi_{ij} = \begin{cases} 0, & \text{if } x_{ij} = 0 \text{ in } X^* \\ 1, & \text{if } x_{ij} \neq 0 \text{ in } X^* \end{cases}$$

Here, T_{ij}^{k} is the time of transportation from i-th node to j-th destination corresponding to k-th path.

- Remark 5.1: Model 5.3 is designed based on our proposed study, whose optimal solution produces the optimum cost and time of the bi-objective transportation problem. The proposed Model contains with $m{\cdot}n$ decision variables (x_{ij}) along with the reduction parameters and binary variables. In our proposed bi-objective optimization problem, the time of transportation for each of the routes are considered as crisp valued, so the multi-choice interval-valued cost parameters reduces simply real-valued multi-choice parameters without any additional variables. Therefore, the objective function Z of Model 5.3 involves decision variables x_{ij} and binary variables z_{ij}^{k} (it is required to reduce multi-choice cost parameters to real-valued parameters) only. The number of binary variables for each cell of allocation depends on number of multi-choice parameters which are available in the respective cell (it is described in the first paragraph of subsection 5.3.2). The constraints of Model 5.3 are presented in reduced form involving reduction variables ($\lambda^*,\ * = C_{ij}^{g},\ a_i^{g},\ b_j^{g}$) and binary variables. As a whole, Model 5.3 is a non-linear constrained optimization problem which can be solved by any non-linear optimization technique. But, in a real-life TP, the number of variables increases significantly in our proposed ground. So, one can consider softwares like LINGO, MATLAB, MAPLE, etc. to solve the proposed Model 5.3. However, we consider LINGO to solve it.

- Remark 5.2: Sometimes, it is required to reduce the number of variables in a problem to make a small equivalent problem which is very easy to solve it. But, in this study, if we wish to reduce the number of variables, then we might loose some data from the set of interval-valued multi-choice cost, supply and demand parameters. As a result, we deviate the main motivation of our formulated model in connection with the real-life TP. Again, if it is done arbitrarily, then DM may not obtain a better optimal solution all time, that we describe in detail to the section "Result and discussion", according to our case study.

Proposition 5.2:
The optimal solution of mathematical model (see Model 5.3) proposes a compromise solution of the objective functions Z and T of Model 5.2A.
Proof: Introduction of time in TP proposes that, the time taken for the transportation creates a situation of less profit and this is happen for the case of transportation of perishable goods. Here, in the proposed model, we state that, if the transportation involves in time then the cost $\tilde{C}'_{ij}$ takes the minimum cost C_{ij}^{kl} corresponding the k-th route for i-th origin to j-th destination. The delay time generates a damage of perishable goods which produces less profit. And in the mean time, the function $\lambda^{C_{ij}^k}$ which reduced the interval cost to real valued cost for increasing the transportation cost to a cost $\tilde{C}'_{ij}$ (see Proposition 5.1). Also, both the objective functions Z and T have the same feasible region.

Clearly, the objective function Z gets the minimum value by Model 5.3 when $\tilde{C}'_{ij} = \min_k C_{ij}^{kl}$ and it is happen when $\lambda^{C_{ij}^k} = 0 \ \ \forall \ i, j$ and denoting this solution be X_1. Again, the objective function T can be solved separately form Z and produces the optimal solution and denoting this solution be X_2. Most of the cases, it happens that neither at X_1 nor at X_2, the optimum value of the objective functions Z and T exist. So, DM needs to find compromise solution and the solution depends on the weight preferences for the objective functions. In this situation, we propose to increase the transportation cost (as it is given by an interval valued) according to the increase of time (see Proposition 5.1). For minimum value of Z, the reduced cost $\tilde{C}'_{ij}$ is to be minimized for all nodes (i, j). By Proposition 5.1, it is clear that the objective function T (see Step 4 of Algorithm in subsection 5.3.2) produces a minimum value. Hence, the optimal solution of Model 5.3 proposes a compromise solution of the objective functions Z and T. This shows the proof of the proposition.

- Remark 5.3: Using the reduction procedure and utilizing the Propositions 5.1 and 5.2, we incorporate time in the objective function of Model 5.3. So, we conclude that the transportation time is minimized in Step 4 of Algorithm in subsection 5.3.2, through the optimum solution of Model 5.3.

5.4 Case study

The transportation of perishable goods like vegetables, fruits, fishes, etc. is very related with time. When we transport these perishable goods, it is not so easy to fix the cost as well as the way of transportation when there exist several routes. In order to show the applicability of this chapter, let us include the following case study.

A store keeper has three stores in different places namely S_1, S_2 and S_3 of vegetables items. He supplies the vegetables from four popular markets in different places namely M_1, M_2, M_3 and M_4. There are several routes to deliver the goods to the markets and the transportation costs (in \$) in each route are interval valued multi-choice types (these are considered due to increasing the fuel price, road tax etc.) which are presented in Table 5.1. To transport the goods, times (in minutes) are also prescribed for each route which is provided in Table 5.1 adjacent to the right of interval valued multi-choice cost.

Table 5.1: Transportation cost (in \$) and time (in minute) for transporting the goods.

	M_1	M_2	M_3	M_4
S_1	[10,20] 230, [12,18] 180	[20,25] 240, [22,24] 220	[18,22] 200, [12,20] 280	[23,28] 220, [20,24] 220, [22,26] 240
S_2	[12,20] 220, [11,17] 230, [20,24] 200	[22,25] 220, [20,24] 230	[14,23] 210, [14,20] 300, [15,20] 200	[20,28] 220, [22,24] 210, [21,26] 230,[14,20] 480, [16,20] 210
S_3	[15,20] 210, [16,18] 180	[20,25] 240, [20,24] 250	[22,24] 170, [18,20] 210, [12,20] 280	[22,28] 230,[24,26] 250

Again, the store keeper may have more options for collecting vegetables from the dealers of different locations and each of the dealers has some capacity of supplying vegetables. In that case, he has the several options for storing the goods in the origins and the supply becomes interval valued. Also, due to weather conditions, fluctuation in the market, the demands are also taken as multi-choice interval valued. The interval valued multi-choice capacities in the stores S_i $(i = 1, 2, 3)$ are $a_1 = [50, 70]$, $a_2 = ([45, 70]$ or $[50, 60])$, $a_3 = ([40, 50]$ or $[50, 60]$ or $[45, 65]$ or $[55, 60])$ and same for the demand parameters to the markets M_j $(j = 1, 2, 3, 4)$ are $b_1 = ([60, 65]$ or $[50, 60])$, $b_2 = ([40, 45]$ or $[50, 55]$ or $[48, 60])$, $b_3 = [60, 80]$, $b_4 = ([55, 75]$ or $[60, 70])$. He expects that the goods will not be deteriorated within 200 minutes. He wishes to minimize the transportation cost with less deterioration with the consideration of total transportation time. The mathematical model is formulated corresponding to available data as follows:

$$\begin{aligned}\text{minimize}\quad & z = ([10,20] \text{ or } [12,18])x_{11} + ([20,25] \text{ or } [22,24])x_{12} + ([18,22]\\ & \text{or } [12,20])x_{13} + ([23,28] \text{ or } [20,24] \text{ or } [22,26])x_{14} + ([12,20]\\ & \text{or } [11,17] \text{ or } [20,24])x_{21} + ([22,25]\text{or}[20,24])x_{22} + ([14,23]\\ & \text{or } [14,20] \text{ or } [15,20])x_{23} + ([20,28] \text{ or } [22,24] \text{ or } [21,26] \text{ or } [14,20]\\ & \text{or } [16,20])x_{24} + ([15,20] \text{ or } [16,18])x_{31} + ([20,25] \text{ or } [20,24])x_{32}\\ & +([22,24] \text{ or } [18,20] \text{ or } [12,20])x_{33} + ([22,28] \text{ or } [24,26])x_{34} \quad (5.14)\end{aligned}$$

$$\text{subject to}\quad \sum_{j=1}^{4} x_{1j} \leq [50,70], \tag{5.15}$$

$$\sum_{j=1}^{4} x_{2j} \leq ([45,70] \text{ or } [50,60]), \tag{5.16}$$

$$\sum_{j=1}^{4} x_{3j} \leq ([40,50] \text{ or } [50,60] \text{ or } [45,65] \text{ or } [55,60]), \tag{5.17}$$

$$\sum_{i=1}^{3} x_{i1} \geq ([60,65] \text{ or } [50,60]), \tag{5.18}$$

$$\sum_{i=1}^{3} x_{i2} \geq ([40,45] \text{ or } [50,55] \text{ or } [48,60]), \tag{5.19}$$

$$\sum_{i=1}^{3} x_{i3} \geq [60, 80], \tag{5.20}$$

$$\sum_{i=1}^{3} x_{i4} \geq ([55, 75] \text{ or } [60, 70]), \tag{5.21}$$

$$x_{ij} \geq 0, \quad i = 1, 2, 3 \text{ and } \quad j = 1, 2, 3, 4.$$

Using the procedure described in subsection 5.3.3, the objective function (5.14) is reduced as follows:

Model 5.4

minimize

$$z = C_{11}x_{11} + C_{12}x_{12} + C_{13}x_{13} + C_{14}x_{14} + C_{21}x_{21} + C_{22}x_{22}$$
$$+ C_{23}x_{23} + C_{24}x_{24} + C_{31}x_{31} + C_{32}x_{32} + C_{33}x_{33} + C_{34}x_{34},$$

where

$$C_{11} = [10(1 - \lambda_{11}^1) + 20\lambda_{11}^1]z_{11}^1 + [12(1 - \lambda_{11}^2) + 18\lambda_{11}^2](1 - z_{11}^1),$$

$$C_{12} = [20(1 - \lambda_{12}^1) + 25\lambda_{12}^1]z_{12}^1 + [22(1 - \lambda_{12}^2) + 24\lambda_{12}^2](1 - z_{12}^1),$$

$$C_{13} = [18(1 - \lambda_{13}^1) + 22\lambda_{13}^1]z_{13}^1 + [12(1 - \lambda_{13}^2) + 20\lambda_{13}^2](1 - z_{13}^1),$$

$$C_{14} = [23(1 - \lambda_{14}^1) + 28\lambda_{14}^1]z_{14}^1 z_{14}^2 + [20(1 - \lambda_{14}^2) + 24\lambda_{14}^2]$$
$$(1 - z_{14}^1)z_{14}^2 + [22(1 - \lambda_{14}^3) + 26\lambda_{14}^3](1 - z_{14}^2)z_{14}^1,$$

$$C_{21} = [12(1 - \lambda_{21}^1) + 20\lambda_{21}^1]z_{21}^1 z_{21}^2 + [11(1 - \lambda_{21}^2) + 17\lambda_{21}^2]$$
$$(1 - z_{21}^1)z_{21}^2 + [20(1 - \lambda_{21}^3) + 24\lambda_{21}^3](1 - z_{21}^2)z_{21}^1,$$

$$C_{22} = [22(1 - \lambda_{22}^1) + 25\lambda_{22}^1]z_{22}^1 + [20(1 - \lambda_{22}^2) + 24\lambda_{22}^2](1 - z_{22}^1),$$

$$C_{23} = [14(1 - \lambda_{23}^1) + 23\lambda_{23}^1]z_{23}^1 z_{23}^2 + [14(1 - \lambda_{23}^2) + 20\lambda_{23}^2]$$
$$(1 - z_{23}^1)z_{23}^2 + [15(1 - \lambda_{23}^3) + 20\lambda_{23}^3](1 - z_{23}^2)z_{23}^1,$$

$$C_{24} = [20(1 - \lambda_{24}^1) + 28\lambda_{24}^1]z_{24}^1 z_{24}^2 z_{24}^3 + [22(1 - \lambda_{24}^2) + 24\lambda_{24}^2](1 - z_{24}^1)$$
$$z_{24}^2 z_{24}^3 + [21(1 - \lambda_{24}^3) + 26\lambda_{24}^3](1 - z_{24}^2)z_{24}^1 z_{24}^3 + [14(1 - \lambda_{24}^4) + 20\lambda_{24}^4]$$
$$(1 - z_{24}^3)z_{24}^1 z_{24}^2 + [16(1 - \lambda_{24}^5) + 20\lambda_{24}^5](1 - z_{24}^1)(1 - z_{24}^2)z_{24}^3,$$

$$C_{31} = [15(1 - \lambda_{31}^1) + 20\lambda_{31}^1]z_{31}^1 + [16(1 - \lambda_{31}^2) + 18\lambda_{31}^2](1 - z_{31}^1),$$

$$C_{32} = [20(1 - \lambda_{32}^1) + 25\lambda_{32}^1]z_{32}^1 + [20(1 - \lambda_{32}^2) + 24\lambda_{32}^2](1 - z_{32}^1),$$

$$C_{33} = [22(1-\lambda_{33}^1) + 24\lambda_{33}^1]z_{33}^1 z_{33}^2 + [18(1-\lambda_{33}^2) + 20\lambda_{33}^2]$$
$$(1-z_{33}^1)z_{33}^2 + [12(1-\lambda_{33}^3) + 20\lambda_{33}^3](1-z_{33}^2)z_{33}^1,$$
$$C_{34} = [22(1-\lambda_{34}^1) + 28\lambda_{34}^1]z_{34}^1 + [24(1-\lambda_{34}^2) + 26\lambda_{34}^2](1-z_{34}^1),$$
$$1 \le z_{14}^1 + z_{14}^2 \le 2,\ 1 \le z_{21}^1 + z_{21}^2 \le 2,$$
$$1 \le z_{23}^1 + z_{23}^2 \le 2,\ 1 \le z_{24}^1 + z_{24}^2 + z_{24}^3 \le 3,$$
$$1 \le z_{24}^1 + z_{21}^3,\ 1 \le z_{24}^2 + z_{24}^3,$$
$$1 \le z_{33}^1 + z_{33}^2 \le 2,$$
$$z_{ij}^k = 0 \text{ or } 1,\quad i = 1,2,3;\ j = 1,2,3,4;\ k = 1,2,3.$$

$$\left.\begin{array}{r}
\lambda_{11}^1 = \frac{3}{23}, \lambda_{11}^2 = 0, \lambda_{12}^1 = \frac{4}{24}, \lambda_{12}^2 = \frac{2}{22},\\
\lambda_{13}^1 = 0, \lambda_{13}^2 = \frac{8}{28}, \lambda_{14}^1 = \frac{2}{22}, \lambda_{14}^2 = \frac{2}{22},\\
\lambda_{14}^3 = \frac{4}{24}, \lambda_{21}^1 = \frac{2}{22}, \lambda_{21}^2 = \frac{3}{23}, \lambda_{21}^3 = 0,\\
\lambda_{22}^1 = \frac{2}{22}, \lambda_{22}^2 = \frac{3}{23}, \lambda_{23}^1 = \frac{1}{21},\\
\lambda_{23}^2 = \frac{10}{30}, \lambda_{23}^3 = 0, \lambda_{24}^1 = \frac{2}{22}, \lambda_{24}^2 = \frac{1}{21},\\
\lambda_{24}^3 = \frac{3}{23}, \lambda_{24}^4 = \frac{28}{48}, \lambda_{24}^5 = \frac{5}{25}, \lambda_{31}^1 = \frac{1}{21},\\
\lambda_{31}^2 = 0, \lambda_{32}^1 = \frac{4}{24}, \lambda_{32}^2 = \frac{5}{25}, \lambda_{33}^1 = 0,\\
\lambda_{33}^2 = \frac{1}{21}, \lambda_{33}^3 = \frac{8}{28}, \lambda_{34}^1 = \frac{3}{23}, \lambda_{34}^2 = \frac{5}{25}
\end{array}\right\}$$

The equations (5.15)-(5.21) are also reduced to deterministic form in the same way as before and are listed as follows:

$$\sum_{j=1}^{4} x_{1j} \le [50(1-\lambda_1^1) + 70\lambda_1^1],$$
$$\sum_{j=1}^{4} x_{2j} \le [45(1-\lambda_2^1) + 70\lambda_2^1]z_2^1 + [50(1-\lambda_2^2) + 60\lambda_2^2],$$
$$\sum_{j=1}^{4} x_{3j} \le [40(1-\lambda_3^1) + 50\lambda_3^1]z_3^1 z_3^2 + [50(1-\lambda_3^2) + 60\lambda_3^2](1-z_3^1)z_3^2 + [45$$
$$(1-\lambda_3^3) + 65\lambda_3^3](1-z_3^2)z_3^1 + [55(1-\lambda_3^4) + 60\lambda_3^4](1-z_3^2)(1-z_3^1),$$

$$\sum_{i=1}^{3} x_{i1} \geq [60(1-\lambda^1)+65\lambda^1]z^1 + [50(1-\lambda^2)+60\lambda^2](1-z^1),$$

$$\sum_{i=1}^{3} x_{i2} \geq [40(1-\lambda^3)+45\lambda^3]z^2z^3 + [50(1-\lambda^4)+55\lambda^4](1-z^2)z^3$$

$$+[48(1-\lambda^5)+60\lambda^5](1-z^3)z^2,$$

$$\sum_{i=1}^{3} x_{i3} \geq [60(1-\lambda^6)+80\lambda^6],$$

$$\sum_{i=1}^{3} x_{i4} \geq [55(1-\lambda^7)+75\lambda^7]z^4 + [60(1-\lambda^8)+70\lambda^8](1-z^4),$$

$$0 \leq z_3^1 + z_3^2 \leq 2,\ 1 \leq z^2 + z^3 \leq 2,$$

$$z_j^t = 0 \ \ or \ \ 1, \quad j=1,2,3; \quad t=1,2,3,$$

$$z^s = 0 \ \ or \ \ 1, s=1,2,3,4,$$

$$0 \leq \lambda_j^t \leq 1, \quad j=1,2,3; \ t=1,2,3,$$

$$0 \leq \lambda^s \leq 1, \ s=1,2,\ldots,8,$$

$$x_{ij} \geq 0, \quad i=1,2,3; \ j=1,2,3,4.$$

Here, we solve the problem by LINGO software to compute the compromise solution which is presented in the next section including a discussion.

5.5 Result and discussion

Solving Model 5.4 using LINGO software, we have described the optimal solution in Table 5.2. The minimum cost Z is \$3077.223.

Table 5.2: Optimum value of decision variables.

Variables	x_{11}	x_{12}	x_{13}	x_{14}	x_{21}	x_{22}	x_{23}	x_{24}	x_{31}	x_{32}	x_{33}	x_{34}
Values of C_{ij}^k	50	0	20	0	0	15	0	55	0	25	40	0

For the minimum value of the transportation cost, the selected cost coefficients among the multi-choice cost coefficients corresponding to the decision variables are obtained and is shown in Table 5.3.

Table 5.3: Selected cost for transporting the goods.

	x_{11}	x_{12}	x_{13}	x_{14}	x_{21}	x_{22}	x_{23}	x_{24}	x_{31}	x_{32}	x_{33}	x_{34}
Values of C_{ij}^k	11.3	20.83	14.29	20.36	11.78	20.52	14.43	16.19	15.23	20.79	14.28	22.78

The allocations are made corresponding to the following routes for x_{11} along [10,20] 230, x_{13} along [12,20] 280, x_{22} along [20,24] 230, x_{24} along [16,20] 210, x_{32} along [20,24] 250, x_{33} along [12,20] 280, where the numbers followed by the intervals denote the transportation time. The time of transportation for the required items is $T = (230+280+230+210+250+280)$ minutes=1480 minutes. The best choices from interval valued supply and demands for the origins and destinations respectively are as follows: $a_1 = [50,70], a_2 = [45,70], a_3 = [45,60]$, $b_1 = [50,60], b_2 = [40,45], b_3 = [60,80], b_4 = [55,75]$.

The obtained solution shows that allocation is made in the cell $(2,4)$ and the path selected for the allocation is $[16,20]210$ and the corresponding cost is \$16.19. There is an another route in which the assigned cost is $[14,20]480$. If the time of transportation is not considered in the problem then the DM would select this route ($[14,20]480$) for transporting the goods as the minimum transportation cost in this route is \$14. Because of transportation time, the transportation cost is increased to \$17.5 and this would have increased the total transportation cost. So, a confusing case occurs to take the decision by the DM for selecting routes of transportation. In this chapter, the proposed model removes the complexities for selecting the routes with the minimum transportation cost and time.

Our proposed methodology is not comparable directly with any other existing techniques for solving MOTP like fuzzy programming, weighting method, goal programming, etc. One can wish to compare our technique with existing methodology for solving MOTP like fuzzy programming, weighting method, goal programming, etc. He/She should have to consider some assumptions to formulate the mathematical model for given bi-objective TP. To justify the effectiveness of our proposed methodology, assuming that minimum transportation cost is to be paid if the selected multi-choice route takes the maximum time among the routes corresponding the node. If the route is chosen which takes

minimum time of transportation among the routes then it produces maximum transportation cost regarding that route which indicates the upper bound of interval cost. Under this assumption, we have solved the bi-objective problem then we have obtained the following results:
The minimum transportation cost is \$2790.00 and time of transportation for this is 1710 minutes. Again considering the time minimization for the objective function then minimum time of transportation is 1250 minutes, corresponding cost of transportation is \$4265.00.
Now to justify that obtained solution of our proposed methodology is a better solution, let us introduce a utility function in the form
$f(g) = w_1\frac{\bar{Z}-Z}{\bar{Z}-\underline{Z}} + w_2\frac{\bar{T}-T}{\bar{T}-\underline{T}}$, where $\bar{Z}$ =Value of Z when transportation time is minimum, $\bar{T}$ =Value of T when transportation cost is minimum, $\underline{Z}$= Minimum transportation cost, $\underline{T}$= Minimum transportation time, Z and T are the values of the cost and time to the objective functions by our proposed methodology, w_1 and w_2 are weights for the objective functions. The value of function f lies between 0 and 1. The bigger value of f proposes a better compromise solution of the bi-objective TP. Considering the equal weights 0.5 for the objective functions, we get $f(g) = 0.66$. This value suggests that the obtained solution is provided a better compromise solution.

5.6 Conclusion

The main aim of this chapter is to minimize the transportation cost as well as minimize the transportation time for transporting the perishable goods through single objective TP under the environment of multi-choice interval valued programming. Until now, researchers have used the methodology of multi-objective transportation problem to optimize the transportation cost and time but in this chapter, we have optimized the transportation cost and time without using multi-objective TP. Here we have considered the single objective transportation problem in which the parameters are interval valued multi-choice types. To solve this type of transportation problem, our methodology is provided a correct direction for getting the fruitful optimal solution.

Again, when there are several routes with varying costs, the decision maker can use the proposed methodology to solve this type of transportation problem which is presented in this chapter. To show the reality and feasibility of the proposed study, a case study is considered to analyze the situation. The proposed methodology may be used to solve time-cost trade off transportation problem in multi-objective environment when the parameters incorporate the uncertain type of data.

Chapter 6

Transportation Problem Under Fuzzy Decision Variable*

In usual way, fuzzy programming in OP is used to solve multi-objective optimization problems. In this chapter, we introduce the concept of fuzzy decision variable into the fuzzy transportation problem to solve some complicated real-life decision making problems. This chapter consists of two parts. In the first part, the study on transportation problem under fuzzy decision variable is presented and in the last, the study of TP under fuzzy decision variable is extended into multi-objective environment.

6.1 TP under fuzzy decision variable

In this part, a new formulation of mathematical model of Fuzzy Transportation Problem (FTP) with fuzzy goal to the objective function is designed. After that, the solution technique of the proposed model is included through multi-choice goal programming approach. The proposed approach is not only improved the applicability of goal programming in real world situations but is also provided useful insight about the solution of a new class of the TP. Finally, a real-life example is incorporated to analyze the feasibility and usefulness of this study.

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6.1.1 Introduction

At the beginning, fuzzy set theory is considered as a tool to solve optimization problems [Zadeh (166)]. Basically, the concept of fuzzy numbers is introduced in the transportation parameters (cost, supply and demand) to make it a fuzzy TP. To the best of our knowledge, the notion of decision variable in TP as a fuzzy variable has not available in the literature. We consider that the expectations in the destinations of TP are fuzzy numbers and they are taken as fuzzy goals. In the destinations, there are multi-choice fuzzy expectations. In this situation, decision maker would have to take a decision for supplying goods in such a way that the profit would maximize, with the best possible fulfillment of requirement at the destinations. Again, it is not true that in a TP at each node there is an allocation, it depends on the best fit of the problem. In this case, when there is no need of allocation in a cell, then we assign a crisp goal '0' with high priority value. In our proposed TP, the requirement in the allocated cells are one among a multi-choice fuzzy numbers along with '0'. With this assumption, we design a TP whose decision variables are fuzzy. This situation of decision making problem is solved using the multi-choice goal programming approach.

But many situations occur where goal is taken in favor of DM for the outcome of objective function to find the best fitted goal in decision making problem. Here, we describe a situation where the decision maker prefers the goals for allocation as well as the best outcome according to his/her choice.

The main aim of the first part of this chapter is to formulate the Fuzzy Transportation Problem (FTP) where the decision variables are multi-choice fuzzy goals and the objective function has also a fuzzy goal. The methodology for solving the formulated model and the way of selection of optimum goal corresponding to the objective function are introduced with the help of multi-choice goal programming approach.

6.1.2 Problem environment

In real-world situations, uncertainty is a common phenomena especially in the ground of optimization. TP is a class of optimization problem which considers in the field of uncertain optimization problems. A number of studies [cf., Ebrahimnejad (31), Kaur and Kumar (68), Kaur and Kumar (69) and many others] has been developed to accommodate several ambiguous situations in real-life transportation problem. Many cases, the studies are incorporated by considering transportation parameters like cost, supply, and demand as uncertain parameters such as stochastic, fuzzy, interval-valued etc. Nevertheless, in most of the cases [cf., Waiel (157), Ebrahimnejad (31), Kaur and Kumar (68), Kaur and Kumar (69)], the concept of uncertainty is incorporated in proposed models by theoretical point of view which is not generally considered as practically in connection with TP. In this proposed study, we present a new class of TP under fuzziness and it is denoted as Fuzzy Transportation Problem (FTP). In traditional way of FTP, the transportation parameters fully or partially considered as fuzzy numbers. Whereas, in our newly designed FTP, we consider the situation where the parameters are not treated as fuzzy numbers but we choose the decision variable as fuzzy. Mainly, in classical sense of FTP, the optimal solution is obtained by transporting the crisp amount of goods through the fuzzy transportation penalty along with supply and demand restrictions. Here, DM only optimizes the value of the objective function through the optimal value of decision variables using different approaches. In many existing studies, the optimum solution is followed by minimum cost for delivering the amount of goods to the purchaser, i.e., cost is the only factor which belongs to the specified fuzzy data. But, we consider here the FTP, not only optimizing the cost, also optimizing the expected quantity required by the customer. Because of that we choose fuzzy decision variable in the proposed FTP. We consider the amount of goods required by the purchaser with specific priority level, which refers to fuzzy expectations in the allocation cells. So, we consider multi-choice fuzzy expected amount of goods with respective priority level in the allocation cells. A graphical network is provided in Figure 6.1 where TP contains the allocation cells. In a classical TP network, there is no idea of

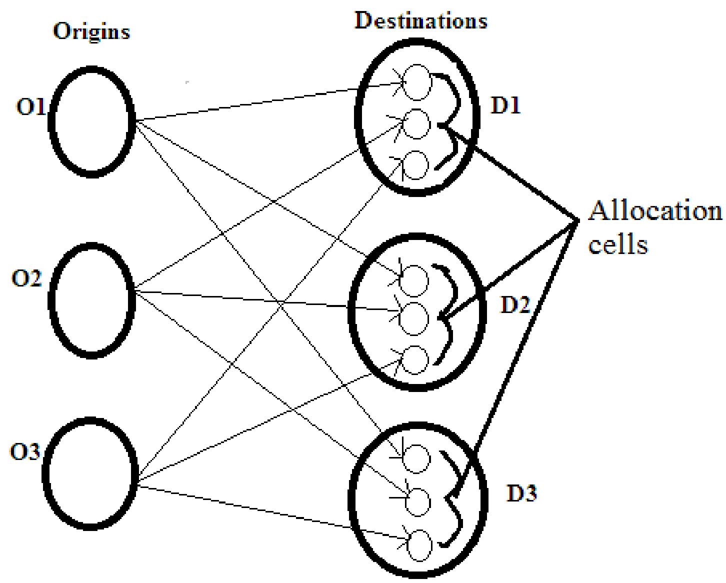

Figure 6.1: Graphical network of TP with allocation cells.

allocation cells under the management of the demand nodes (D1, D2 and D3). In our proposed study, we optimize the priority level of expected amount of goods in allocation cells satisfying the restricted demands in the corresponding demand nodes. Many cases where the demands are assumed as multi-choice numbers in the destinations, but our study includes a new direction which allows the selection of fuzzy allocations among the multi-choice fuzzy numbers to the allocation cells and seeks the optimal solution of the objective function. Due to this fact, here decision variable is taken as *fuzzy decision variable* in our proposed TP in the chapter.

6.1.3 Mathematical model

The first subsection considers a brief introduction of multi-choice goal programming and the mathematical model of TP with goal under fuzzy decision variable is formulated in later subsection.

Multi-choice goal programming

In the literature of goal programming, Chang (17) described MCGP approach which allows the DM to set multi-choice aspiration levels (MCALs) for each goal (i.e., one goal is mapping with multiple aspiration levels). The mathematical model of goal programming is considered as follows:

GP

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{K} w_i |Z^i(X) - g_i| \\ \text{subject to} \quad & x \in F \quad (F \ \text{ is the feasible set}). \end{aligned}$$

where w_i $(i = 1, 2, \ldots, K)$ are the weights attached to the deviation of the achievement function $Z^i(x)$ and g_i is the goal corresponding to the i-th objective function. $|Z^i(x) - g_i|$ represents the deviation of the i-th goal. Later on, a modification of GP is provided which noted as Weighted Goal Programming (WGP).

Sometimes, it may not be possible to assign crisp goals corresponding to each of the objective functions and then we consider fuzzy goals. Again, the fuzzy goals may be multiple-choice corresponding to some objective functions. Under the environment of Fuzzy Multi-Choice Goal Programming (FMGP), the formulation of GP reduces to the following form:

FMGP

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{K} w_i |Z^i(X) - \tilde{g}_i^1 \ or \ \tilde{g}_i^2 \ or \ \ldots \ \ or \ \tilde{g}_i^t| \\ \text{subject to} \quad & x \in F, \end{aligned}$$

where w_i $(i = 1, 2, \ldots, K)$ are the weights relative to the importance of the objective functions and the aspiration levels $\tilde{g}_i^t \ \forall \ i, t$ are assumed to be triangular fuzzy numbers with membership functions $\tilde{\mu}_i^t \ \forall \ i, t$.

TP under multi-choice goal programming

The mathematical model of the transportation problem is defined as follows:
Model 6.1

$$\text{minimize} \quad Z = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}x_{ij} \tag{6.1}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \leq a_i \ (i = 1, 2, \ldots, m), \tag{6.2}$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j \ (j = 1, 2, \ldots, n), \tag{6.3}$$

$$x_{ij} \geq 0 \ \ \forall \ \ i \text{ and } \ j, \tag{6.4}$$

where x_{ij} is the decision variable and C_{ij} $(i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$ is the transportation cost per unit commodity from the i^{th} origin to the j^{th} destination. Here a_i $(i = 1, 2, \ldots, m)$ and b_j $(j = 1, 2, \ldots, n)$ are availability and demand in the i^{th} origin and the j^{th} destination respectively and $\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} b_j$ is the feasibility condition.
In real-life practical problems, some situations may occur where we need to optimize the objective function according to the decision maker's preferences. In this case, the decision variables (x_{ij}) of TP are taken as real variables and the crisp solutions are obtained. In our daily life, there may occur some cases where the expectations in the allocation cells of TP are described as the fuzzy goals, again these are multi-choices. Here, we consider that if the allocation made at any cell then it is one of the goal value among the set of values assigned by the DM. According to the concept of the TP, it is not necessary that at each of cells, there will be some allocations. So, in each cell, we assign a goal value '0' with a small deviation in such a way that if allocation is not made in the cell then aspiration level produces a high value. So, in the TP, the decision variables are not behaved as in classical TP and they are taken as fuzzy variable $(\tilde{x}_{ij})$. To the best of our knowledge, until now no work has been done for solving this typical TP whose decision variables are fuzzy multi-choices. The mathematical model of this fuzzy transportation problem can be

written as follows:

Model 6.2

$$\text{minimize} \quad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} \tilde{x}_{ij} \tag{6.5}$$

$$\text{subject to} \quad \sum_{j=1}^{n} \tilde{x}_{ij} \leq a_i \ (i = 1, 2, \ldots, m), \tag{6.6}$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} \geq b_j \ (j = 1, 2, \ldots, n), \tag{6.7}$$

$$\tilde{x}_{ij} \geq 0 \ \ \forall \ \ i \text{ and } \ j. \tag{6.8}$$

The mathematical formulation of Model 6.2 is supposed to be very simple but it is not so. The solution procedure of FTP is described in next subsection.

6.1.4 Solution procedure

This subsection contains two folded in which the first fold describes an algorithm for solving the FTP and later one provides a reduction procedure to convert the FTP to a deterministic model. Model 6.2 consists of fuzzy decision variables which is not easy to solve by existing approaches for solving the TP. So, we design an algorithm to solve the FTP.

Algorithm:

- **Step 1:** First, we fix the multi-choice fuzzy goals corresponding to the objective function and allocation cells.

- **Step 2:** We formulate fuzzy membership functions corresponding to the objective function and allocation cells using the multi-choice fuzzy goals. Again, we use binary variables to select a value among the multi-choice membership values of the objective function and each of the allocation cells. The procedure is shown in detail in the subsection 6.1.5.

- **Step 3:** Thereafter, the objective function is formulated by adding the variables of membership functions multiplied by the proper weights, corresponding to the objective function and expected allocation goals to the cells. This procedure is also shown in subsection 6.1.5.

- **Step 4:** The objective function of Step 3 is formulated as maximization type, the optimum solution of this objective function is required to satisfy the multi-choice goals of the allocation cells, for which we need to introduce two auxiliary constraints corresponding to each of the cells.

- **Step 5:** Furthermore, solving the objective function of Step 3 satisfying the constraints proposed in Step 4, we obtain optimum goal of objective function along with the optimum expected amount of goods in the allocation cells.

- **Step 6:** Stop.

6.1.5 Reduction procedure of the FTP to deterministic model

In our proposed mathematical Model 6.2, we consider the FTP under fuzzy multi-choice goal environment. The objective function of the FTP has a specific fuzzy goal $\tilde{g}$. Assume that $\{\tilde{g}_{ij}^t : t = 1, 2, \ldots, p\} \bigcup \{0\}$ be the set of all possible allocation at the node $(i,\ j)$ in the FTP. Here, p is the number of multi-choice fuzzy goals at the $(i,\ j)$-th node and it may change for different node $(i,\ j)$. Without loss of generality, we consider the allocation goals $\tilde{g}_{ij}^t$ as triangular fuzzy number. Again, we assume that the allocation should be through the selection of fuzzy goals from the set $\{\tilde{g}_{ij}^t : t = 1, 2, \ldots, p\} \bigcup \{0\}$ at each of the node $(i,\ j)$ in the FTP.

To solve Model 6.2, our aim goes to make the allocation in such a way that the aspiration value for each node and objective functions become high. Due to this situation, assigning the weights corresponding to each node and the objective function is very much important to improve a better solution of Model 6.2. To do this, we construct a crisp model of TP which is a maximizing type problem whatever the nature of the objective function of the TP.

Again for maximizing the value of an objective function, the number of fuzzy allocation goals $\tilde{g}_{ij}^t$ may not be equal for all nodes. If there be only one fuzzy allocation goal $\tilde{g}_{ij}^1$ (and for no allocation goal value is '0') for each node, then corresponding mathematical model i.e., Model 6.3 is derived from Model 6.2 as follows:

Model 6.3

$$\text{maximize} \quad Z = \sum_{i=1}^{m}\sum_{j=1}^{n} w_{ij}\mu_{ij} + w\mu \tag{6.9}$$

$$\text{subject to} \quad \mu_{ij} \leq 1 - [\frac{y_{ij} - \tilde{g}_{ij}^{1}}{d_{ij}^{1-}} z_{ij}^{1} + \frac{y_{ij} - 0}{\epsilon}(1 - z_{ij}^{1})], \tag{6.10}$$

$$\mu_{ij} \leq 1 - [\frac{\tilde{g}_{ij}^{1} - y_{ij}}{d_{ij}^{1+}} z_{ij}^{1} + \frac{0 - y_{ij}}{\epsilon}(1 - z_{ij}^{1})], \tag{6.11}$$

$$z = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij} y_{ij}, \tag{6.12}$$

$$\mu \leq 1 - \frac{z - \tilde{g}}{d^{-}}, \tag{6.13}$$

$$\mu \leq 1 - \frac{\tilde{g} - z}{d^{+}}, \tag{6.14}$$

$$\mu_{ij} \geq 0, \tag{6.15}$$

$$\mu \geq 0, \tag{6.16}$$

$$\sum_{j=1}^{n} y_{ij} \leq a_i \quad (i = 1, 2, \ldots, m), \tag{6.17}$$

$$\sum_{i=1}^{m} y_{ij} \geq b_j \quad (j = 1, 2, \ldots, n), \tag{6.18}$$

$$y_{ij} \geq 0, \quad z_{ij}^{1} = 0 \text{ or } 1 \ \forall \ i, \ j. \tag{6.19}$$

Here, d_{ij}^{1-} and d_{ij}^{1+} are the maximum allowable negative and positive deviations for $\tilde{g}_{ij}^{1}$. d^{+} and d^{-} are the positive and negative deviations respectively corresponding to the objective function Z. A very small positive number ϵ is used to assign a high aspiration value '1' if the allocation be not made in a cell. This situation is created because it is not necessary that in each cell the allocation has to be made.

Now, assume that if each node has two fuzzy aspiration levels for corresponding goal, then fuzzy goal programming is selected any one of these goals in such a way that it provides the optimal solution. Based on the model of Chang (17), the equations [(6.10) and (6.11)] reduce as follows:

$$\mu_{ij} \leq 1-[\frac{y_{ij}-\tilde{g}^1_{ij}}{d^{1-}_{ij}} z^1_{ij} z^2_{ij} + \frac{y_{ij}-\tilde{g}^2_{ij}}{d^{2-}_{ij}} z^1_{ij}(1-z^2_{ij}) + \frac{y_{ij}-0}{\epsilon}(1-z^1_{ij})z^2_{ij}],$$
$$\mu_{ij} \leq 1-[\frac{\tilde{g}^1_{ij}-y_{ij}}{d^{1+}_{ij}} z^1_{ij} z^2_{ij} + \frac{\tilde{g}^2_{ij}-y_{ij}}{d^{2+}_{ij}} z^1_{ij}(1-z^2_{ij}) + \frac{0-y_{ij}}{\epsilon}(1-z^1_{ij})z^2_{ij}],$$
$$z^1_{ij}+z^2_{ij} \geq 1,$$
$$z^1_{ij}, z^2_{ij} = 0 \text{ or } 1 \quad (i=1,2,\ldots,m; j=1,2,\ldots,n).$$

Here, d^{t-}_{ij} and d^{t+}_{ij} are the maximum allowable negative and positive deviations from $\tilde{g}^t_{ij}$ for $t=1,2$.

Again, if each node has three fuzzy aspiration levels for corresponding goal, then fuzzy goal programming is selected any one of these goals in such a way that it provides the optimal solution. Based on the model of Chang (17), the equations [(6.10) and (6.11)] reduce in the following form as:

$$\mu_{ij} \leq 1-[\frac{y_{ij}-\tilde{g}^1_{ij}}{d^{1-}_{ij}} z^1_{ij} z^2_{ij} + \frac{y_{ij}-\tilde{g}^2_{ij}}{d^{2-}_{ij}} z^1_{ij}(1-z^2_{ij}) + \frac{y_{ij}-\tilde{g}^3_{ij}}{d^{3-}_{ij}}(1-z^1_{ij})z^2_{ij}$$
$$+\frac{y_{ij}-0}{\epsilon}(1-z^1_{ij})(1-z^2_{ij})],$$
$$\mu_{ij} \leq 1-[\frac{\tilde{g}^1_{ij}-y_{ij}}{d^{1+}_{ij}} z^1_{ij} z^2_{ij} + \frac{\tilde{g}^2_{ij}-y_{ij}}{d^{2+}_{ij}} z^1_{ij}(1-z^2_{ij}) + \frac{\tilde{g}^3_{ij}-y_{ij}}{d^{3+}_{ij}}(1-z^1_{ij})z^2_{ij}$$
$$+\frac{0-y_{ij}}{\epsilon}(1-z^1_{ij})(1-z^2_{ij})],$$
$$z^1_{ij}, z^2_{ij} = 0 \text{ or } 1 \quad (i=1,2,\ldots,m; j=1,2,\ldots,n).$$

Here, d^{t-}_{ij} and d^{t+}_{ij} are considered as the maximum allowable negative and positive deviations respectively for $\tilde{g}^t_{ij}$ for $t=1,2,3$. If we consider s number of multi-choice goals corresponding to the cell $(i,\ j)$, then the linear membership function μ_{ij} for the fuzzy goals of $(i,\ j)$-th node can be defined as follows:

$$\mu_{ij} = \begin{cases} 0, & \text{if } y_{ij} \geq \tilde{g}^t_{ij} + d^{t+}_{ij}, \\ 1-\sum_{t=1}^{s} \frac{y_{ij}-\tilde{g}^t_{ij}}{d^{t+}_{ij}} F_{ij}(B), & \text{if } \tilde{g}^t_{ij} \leq y_{ij} \leq \tilde{g}^t_{ij} + d^{t+}_{ij}, \\ 1, & \text{if } y_{ij} = \tilde{g}^t_{ij}, \\ 1-\sum_{t=1}^{s} \frac{\tilde{g}^t_{ij}-y_{ij}}{d^{t-}_{ij}} F_{ij}(B), & \text{if } \tilde{g}^t_{ij} - d^{t-}_{ij} \leq y_{ij} \leq \tilde{g}^t_{ij} \end{cases}$$

for all $i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$. Here, $F_{ij}(B)$ represents a function of binary serial numbers that ensure only one fuzzy goal among the set of multi-choice fuzzy goals at each node $(i,\ j)$ satisfying the optimality of the objective function (For detailed discussion, someone can follow Roy et al. (135)) d_{ij}^{t+} and d_{ij}^{t-} are the maximum allowable positive and negative deviations corresponding to the t^{th} goal among the multi-choice fuzzy goals in the (i, j)-th node, respectively.

It may be noted that, it is not necessary that the allocation cells have the same number of multi-choice goals. Model 6.3 is developed according to the number of fuzzy goals in each of the allowable cells and then solve it to find the optimal solution of the FTP.

Remark:

In construction of objection function of Model 6.2, we use the normalized weights w_{ij} corresponding the (i, j)-th cell and the weight w for the objective function which produces a better optimal solution of the FTP.

6.1.6 Numerical experiment

Three investors namely F1, F2 and F3 made a plan to invest their money in business in such a way that they will earn the maximum profits. At the beginning of the business, they decided to keep the amounts $\$1000, \$1200, \$1100$ in hand. Then, they want to invest their amounts in two locations A and B. For the business purpose, the mentioned locations are required to minimum investment \$1600 and \$1650 at A and B respectively. Table 6.1 presents the required amounts [in dollar (\$)] at the destinations which are fuzzy multi-choice numbers. The deviations are presented within brackets adjacent to the required amounts (these amounts are fuzzy numbers) which shown in Table 6.1.

Table 6.1: Required amount in $ and deviations in locations.

	A	B
F1	$\widetilde{700}(25), \widetilde{850}(50), \widetilde{1050}(70), \widetilde{600}(25)$	$\widetilde{980}(40)$
F2	$\widetilde{800}(50), \widetilde{650}(40)$	$\widetilde{500}(25), \widetilde{400}(30), \widetilde{600}(30)$
F3	$\widetilde{850}(60), \widetilde{700}(50)$	$\widetilde{550}(25), \widetilde{450}(30), \widetilde{1050}(100)$

Without loss of generality, we consider the positive and negative deviations are same in our proposed problem. It is also to be noted that if the allocations are not made at any node, then there may be a crisp allocation '0'. So, in each node, there is a crisp choice which is not shown in Table 6.1. The expected profit from their investment policy per $100 from the destinations are depicted in Table 6.2

Table 6.2: Profit earned from investment (per $100).

	A	B
F1	4.5	4.0
F2	5.0	6.0
F3	5.5	5.0

The investors are connected to each other and they make investment in such a way that they made a total profit maximum $170 with not less by $150. According to the business policy presented here, we can say that the allocations in the locations are fuzzy multi-choice numbers. So, to solve this type of problem, the presented procedure is useful to produce a better optimal solution. We consider equal weight '0.1' for each cells and the weight '0.4' to the objective function, then the mathematical model for the proposed problem becomes as follows:

Model 6.4

$$\begin{aligned}
\text{maximize} \quad & Z = 0.1(\mu_{11} + \mu_{12} + \mu_{21} + \mu_{22} + \mu_{31} + \mu_{32}) + 0.4\mu \\
\text{subject to} \quad & \mu_{11} \leq 1 - [\frac{y_{11} - 700}{25} z_{11}^1 z_{11}^2 z_{11}^3 + \frac{y_{11} - 850}{50}(1 - z_{11}^1) z_{11}^2 z_{11}^3 \\
& + \frac{y_{11} - 600}{25}(1 - z_{11}^2) z_{11}^1 z_{11}^3 + \frac{y_{11} - 1050}{70}(1 - z_{11}^3) z_{11}^1 z_{11}^2],
\end{aligned}$$

$$\mu_{11} \leq 1 - [\frac{700 - y_{11}}{25} z_{11}^1 z_{11}^2 + \frac{850 - y_{11}}{50} z_{11}^1 (1 - z_{11}^2)$$
$$+ \frac{600 - y_{11}}{25} (1 - z_{11}^2) z_{11}^1 z_{11}^3 + \frac{1050 - y_{11}}{70} (1 - z_{11}^3) z_{11}^1 z_{11}^2],$$
$$\mu_{12} \leq 1 - \frac{y_{12} - 980}{40} z_{12}^1,$$
$$\mu_{12} \leq 1 - \frac{980 - y_{12}}{40} z_{12}^1,$$
$$\mu_{21} \leq 1 - [\frac{y_{21} - 800}{50} z_{21}^1 z_{21}^2 + \frac{y_{21} - 650}{40} z_{21}^1 (1 - z_{21}^2)],$$
$$\mu_{21} \leq 1 - [\frac{800 - y_{21}}{50} z_{21}^1 z_{21}^2 + \frac{650 - y_{21}}{40} z_{21}^1 (1 - z_{21}^2)],$$
$$\mu_{22} \leq 1 - [\frac{y_{22} - 500}{25} z_{22}^1 z_{22}^2 + \frac{y_{22} - 400}{30} z_{22}^1 (1 - z_{22}^2)$$
$$+ \frac{y_{22} - 600}{30} z_{22}^2 (1 - z_{22}^1)],$$
$$\mu_{22} \leq 1 - [\frac{500 - y_{22}}{25} z_{22}^1 z_{22}^2 + \frac{400 - y_{22}}{30} z_{22}^1 (1 - z_{22}^2)$$
$$+ \frac{600 - y_{22}}{30} z_{22}^2 (1 - z_{22}^1)],$$
$$\mu_{31} \leq 1 - [\frac{y_{31} - 850}{60} z_{31}^1 z_{31}^2 + \frac{y_{31} - 700}{50} z_{31}^1 (1 - z_{31}^2)],$$
$$\mu_{31} \leq 1 - [\frac{850 - y_{31}}{60} z_{31}^1 z_{31}^2 + \frac{700 - y_{31}}{50} z_{31}^1 (1 - z_{31}^2)],$$
$$\mu_{32} \leq 1 - [\frac{y_{32} - 550}{25} z_{32}^1 z_{32}^2 + \frac{y_{32} - 450}{30} z_{32}^1 (1 - z_{32}^2),$$
$$+ \frac{y_{32} - 1050}{100} z_{32}^2 (1 - z_{32}^1)],$$
$$\mu_{32} \leq 1 - [\frac{550 - y_{32}}{25} z_{32}^1 z_{32}^2 + \frac{450 - y_{32}}{30} z_{32}^1 (1 - z_{32}^2)$$
$$+ \frac{1050 - y_{32}}{100} z_{32}^2 (1 - z_{32}^1)],$$
$$t = [4.5y_{11} + 4.0y_{12} + 5.0y_{21} + 6.0y_{22} + 5.5y_{31} + 5.0y_{32}] \frac{1}{100},$$
$$\mu \leq 1 - \frac{170 - t}{20},$$
$$y_{11} + y_{12} \leq 1000,$$
$$y_{21} + y_{22} \leq 1200,$$
$$y_{31} + y_{32} \leq 1100,$$
$$y_{11} + y_{21} + y_{31} \geq 1600,$$

$$y_{12} + y_{22} + y_{32} \geq 1650,$$
$$0 \leq \mu_{ij} \leq 1; 0 \leq \mu \leq 1; z_{ij}^k = 0 \text{ or } 1 \quad \forall i, j, k.$$

Model 6.3 is solved by Lingo software and the solution is listed as follows: Optimum value of $Z = 0.6633$. The optimal solution (i.e., t=maximum benefit from the business) of the problem is \$165.90. The finance allocations are shown in Table 6.3.

Table 6.3: Finance allocation of invest amount (in \$) in crisp form.

Decision Variable	y_{11}	y_{12}	y_{21}	y_{22}	y_{31}	y_{32}
	1000	0	610	590	0	1100

The selection of fuzzy numbers $\tilde{x}_{ij}$ from the fuzzy multi-choice numbers to get the optimum solution is given in Table 6.4.

Table 6.4: Allocation of invest amount in the form of fuzzy number.

Decision Variable	$\tilde{x}_{11}$	$\tilde{x}_{12}$	$\tilde{x}_{21}$	$\tilde{x}_{22}$	$\tilde{x}_{31}$	$\tilde{x}_{32}$
	$\widetilde{1050}$	0	$\widetilde{650}$	$\widetilde{600}$	0	$\widetilde{1050}$

6.1.7 Result and discussion

The first part of this chapter, a study on the TP with fuzzy decision variables i.e., FTP is presented. The numerical example presents the applicability of proposed methodology for solving FTP with fuzzy decision variables. The concept of this chapter is provided a technique whose decision variables are fuzzy numbers, so it cannot be compared to any other model provided in the literature. If someone wishes to solve the problem by considering a general TP with goal corresponding the objective function, then he/she may use GP technique. To give a comparison of our study, we formulate a mathematical model according to the procedure of GP and solving it we derive the following solution:
Optimal value of objective function is \$175.25 and

$x_{11} = 550;\ x_{12} = 450;\ x_{21} = 0;\ x_{22} = 1200;\ x_{31} = 1100;\ x_{32} = 0$; In this case, the value of the objective function is more than the obtained optimal solution from Model 6.3, but the allocation made in cells do not fulfil the requirements as expected in the allocations cells. In this context, one may propose that TP which can optimize the objective function but cannot optimize the goals of the requirements in the cells of allocation. In this regard, our proposed methodology is better than the technique like GP to solve TP. Also, to the best of our knowledge, there is no specific methodology available in the literature to formulate and solve FTP with respect to the restrictions that we have presented in our proposed model.

Comparision with the existing studies

To justify the effectiveness of our proposed FTP in compare to existing studies, here we consider three important research works on FTP (cf., Ebrahimnejad (31), Kaur and Kumar (68), Kaur and Kumar (69)).

Ebrahimnejad (31) established a study on FTP by employing the uncertain situation in real-life TP. In this proposed research work, transportation cost, supply and demand parameters are considered as triangular fuzzy numbers. Thereafter, a two-step method is proposed for solving the FTP. In the first step, fuzzy arithmetic is introduced to convert FTP into a linear programming problem with fuzzy costs and crisp constraints. The second step is presented a new decomposition procedure to convert the resulting problem into crisp valued bounded transportation problems and solving these, they obtained the optimum solution. Again the study of Ebrahimnejad (31) is a way for solving TP with fuzzy parameters. In usual TP, it is always seen that the amount of goods transported to the destinations (i.e., value of x_{ij}) is also tended to a minimum value which produces the minimum transportation cost. In this study, this technique is also considered, but in our proposed TP, we do not follow this rule and consider the way to find the value of x_{ij} which is more satisfactory at the allocation cell (i.e., the most likely amount of goods expected by the purchaser) preserving the optimality criteria of the objective function. Furthermore, we search the optimum value of x_{ij} from the set of multi-choice fuzzy expected values at the cells along with the choice of fuzzy allocation at the cells through our proposed optimization technique. As a whole, we have treated the decision variables as fuzzy numbers in our proposed FTP, which is totally different from the study of FTP by Ebrahimnejad (31).

Again, Kaur and Kumar (68) entitled a study incorporating generalized trapezoidal fuzzy numbers in a FTP. In Kaur and Kumar (68), a new method is presented for solving FTPs by assuming that a decision maker is uncertain about the precise values of the transportation cost, availability and demand of the product. The solution of the proposed problem Kaur and Kumar (68) is a fuzzy optimum solution. We observe that, the fuzzy optimum solution in the problem (68) can be obtained by considering the three simple TPs with transportation cost a, b and c of the triangular fuzzy number (a, b, c) separately. So, this is not provided enough information about on fuzzy decision variable, whereas our study introduces the fuzziness in the decision variable not considering the fuzziness in transportation parameters. As a whole, our study is significantly different form the study of (68) though both the studies build on the ground of FTP.

The TP proposed by Kaur and Kumar (69), is considered using generalized fuzzy numbers for the transportation cost, supply and demand parameters. In this study, there is an algorithm to solve TP under fuzziness which is also called as FTP. According to our best knowledge, the proposed algorithm of (69) entitled a solution for the best possible values of real-decision variable x_{ij} satisfying the supply and demand restrictions. Again the values of x_{ij} will tend to minimum value as the optimization problem is so turned. So, the study does not take care about the restrictions of requirements at the allocation cells. In this chapter, we have improved the concept of fuzzy decision variables and followed the restrictions of routes which are known as the allocation cells through the optimization of the objective function.

Also, there are several research works in the literature in which real-life FTPs involving different types of fuzziness are solved via algorithms, but to the best of our knowledge, until now no one has used fuzzy decision variable in FTP under multi-choice fuzzy goals to the allocation cells and the objective function. In this chapter, a new algorithm is proposed for solving the type of FTP by assuming fuzzy decision variable, which is totally different from the existing algorithms for solving FTP.

6.2 MOTP under fuzzy decision variable

The study under fuzzy decision variable in a TP is extended in multi-objective environment, which has been presented in this part of the chapter. The utility of the study is illustrated by a numerical example.

6.2.1 Mathematical model

TP with single objective function is not enough to formulate all real-life decision making problems. To overcome this difficulty, we have incorporated the multiple objective function with TP. The mathematical model of MOTP can be written as follows:

Model 6.5

$$\begin{aligned} \text{minimize} \quad & Z^t = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^t x_{ij} \quad (t = 1, 2, \ldots, K) \\ \text{subject to} \quad & \text{the constraints } (6.2) - (6.4). \end{aligned}$$

If in a real-life MOTP the allocation cells have multi-choice options for allocating then the mathematical model of MOFTP can be written as follows:

Model 6.6

$$\begin{aligned} \text{minimize} \quad & Z^t = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^t \tilde{x}_{ij} \\ \text{subject to} \quad & \text{the constraints } (6.6) - (6.8). \end{aligned}$$

The multi-objective mathematical formulation of Model 6.6 is seemed to be very simple but it is not so. There are several methods like GP, RMCGP, Fuzzy programming approach are used to solve the MOTP.

Assuming that $\{\tilde{g}_{ij}^t : t = 1, 2, \ldots, p\} \bigcup \{0\}$ be the set of all possible allocation at the node (i, j). To make a clear understanding of MOTP with fuzzy decision variable, here we have considered the supply (a_i) and demand (b_j) as constant values. If someone considers these parameters as fuzzy then also it can be taken into consideration in further improvement of the mathematical models. Without loss of generality, we consider the allocation goals $\tilde{g}_{ij}^t$ as triangular fuzzy number.

Again for maximizing the value of an objective function, the number of fuzzy allocation goals $\tilde{g}_{ij}^t$ may not be equal for all nodes. If there be only one fuzzy allocation goal $\tilde{g}_{ij}^1$ (and for no allocation goal value is 0) for each node, then the corresponding mathematical model (see Model 6.7) is to derive from Model 6.6 as follows:

Model 6.7

$$\text{maximize} \quad Z = \sum_{i=1}^{m}\sum_{j=1}^{n} w_{ij}\mu_{ij} + \sum_{k=1}^{K} w_k\mu_k \tag{6.20}$$

$$\text{subject to} \quad \mu_{ij} \leq 1 - [\frac{y_{ij} - \tilde{g}_{ij}^1}{d_{ij}^{1-}} z_{ij}^1 + \frac{y_{ij} - 0}{\epsilon}(1 - z_{ij}^1)], \tag{6.21}$$

$$\mu_{ij} \leq 1 - [\frac{\tilde{g}_{ij}^1 - y_{ij}}{d_{ij}^{1+}} z_{ij}^1 + \frac{0 - y_{ij}}{\epsilon}(1 - z_{ij}^1)] \tag{6.22}$$

$$(i = 1, 2, \ldots, m; j = 1, 2, \ldots, n),$$

$$Z^k = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^k y_{ij} \quad \forall\ k, \tag{6.23}$$

$$\mu_k \leq 1 - \frac{Z^k - \tilde{g}_k}{d_k^-} \quad \forall\ k, \tag{6.24}$$

$$\mu_k \leq 1 - \frac{\tilde{g}_k - Z^k}{d_k^+} \quad \forall\ k, \tag{6.25}$$

$$\mu_{ij} \geq 0 \quad (i = 1, 2, \ldots, m; j = 1, 2, \ldots, n), \tag{6.26}$$

$$\mu_k \geq 0 \quad (k = 1, 2, \ldots, p), \tag{6.27}$$

$$\sum_{j=1}^{n} y_{ij} \leq a_i \quad (i = 1, 2, \ldots, m), \tag{6.28}$$

$$\sum_{i=1}^{m} y_{ij} \geq b_j \quad (j = 1, 2, \ldots, n), \tag{6.29}$$

$$y_{ij} \geq 0, \quad z_{ij}^1 = 0 \text{ or } 1 \quad \forall\ i \text{ and } j. \tag{6.30}$$

Here, d_{ij}^{1-} and d_{ij}^{1+} are the maximum allowable negative and positive deviations respectively for $\tilde{g}_{ij}^{1}$. d_k^+ and d_k^- are the positive and negative deviations respectively corresponding to objective functions Z^k.

A very small positive number ϵ is used to assign a high aspiration value '1' if the allocation be not made in a cell. This situation has been created because it is not necessary that in each cell the allocation has to be made.

If the number of allocation be more than one, then we apply the procedure introduced in subsection 6.1.5. The Model 6.7 can be developed according to the number of fuzzy goals in each of the allowable cells and then solve it to find the compromise solution of the MOFTP.

6.2.2 Numerical example

A store keeper collects vegetables from three markets S1, S2 and S3. The maximum capacity of supply vegetables in three sources S1, S2 and S3 are 150 Kg, 220 Kg and 200 Kg respectively. The store keeper supplies the vegetables into another two markets A and B. The minimum capacity of vegetables in destinations are 200 Kg and 250 Kg respectively. The collection of vegetables from a market may not be always a crisp value. Table 6.5 presents the possibilities of collecting vegetables, which are multi-choices and fuzzy numbers also. Hence, we may consider the positive and negative deviations as same and they are presented here by brackets adjacent to the required amounts (these amounts are fuzzy numbers) in the Table 6.5.

Table 6.5: Required vegetables in kg and deviations in locations.

	A	B
F1	$\widetilde{50}(10), \widetilde{95}(10), \widetilde{120}(10), \widetilde{70}(5)$	$\widetilde{90}(30)$
F2	$\widetilde{120}(20), \widetilde{60}(10)$	$\widetilde{50}(5), \widetilde{80}(10), \widetilde{60}(5)$
F3	$\widetilde{80}(5), \widetilde{60}(10)$	$\widetilde{65}(5), \widetilde{45}(5), \widetilde{150}(10)$

It is also to be noted that if the allocations are not made at any node, then there may be a crisp allocation '0'. So, in each node there is a crisp choice which is not shown in Table 6.5. The expected profits of per Kg of vegetables are shown in Table 6.6:

Table 6.6: Profit in ($) per Kg.

	A	B
F1	8.5	7.0
F2	7.0	6.5
F3	5.5	6.0

The transportation cost per Kg of vegetables from supplying sources to destinations are shown in Table 6.7:

Table 6.7: Transportation cost per Kg in $.

	A	B
F1	14.5	24.0
F2	10.0	18.0
F3	15.5	12.0

In the proposed problem, obviously the store keeper would like to maximize his profit by maximizing profit goal and minimizing the cost of transportation. He expects maximum profit of $3200 and not less by $3000. Also he wishes to minimize the transportation cost with a value minimum $6500 and not more than $6700. According to the possibilities of collecting vegetables presented here, we can say that the allocations in the locations are multi-fuzzy numbers. So, to solve this type of problem, the presented methodology must be helpful to produce the best solution.

Here, we have considered equal weights '0.05' for each cells and weights '0.4' and '0.3' for the objective functions of profit and transportation cost respectively, then the mathematical model for the proposed problem becomes as follows:

Model 6.8

$$\begin{aligned}
\text{maximize} \quad & Z = 0.05(\mu_{11} + \mu_{12} + \mu_{21} + \mu_{22} + \mu_{31} + \mu_{32}) + 0.4\mu_1 + 0.3\mu_2 \\
\text{subject to} \quad & \mu_{11} \leq 1 - [\frac{y_{11} - 50}{10} z_{11}^1 z_{11}^2 z_{11}^3 + \frac{y_{11} - 95}{10}(1 - z_{11}^1) z_{11}^2 z_{11}^3 \\
& + \frac{y_{11} - 120}{10}(1 - z_{11}^2) z_{11}^1 z_{11}^3 + \frac{y_{11} - 70}{5}(1 - z_{11}^3) z_{11}^1 z_{11}^2],
\end{aligned}$$

$$\mu_{11} \leq 1 - [\frac{50 - y_{11}}{10} z_{11}^1 z_{11}^2 + \frac{95 - y_{11}}{10} z_{11}^1 (1 - z_{11}^2)$$
$$+ \frac{120 - y_{11}}{10} (1 - z_{11}^2) z_{11}^1 z_{11}^3 + \frac{70 - y_{11}}{5} (1 - z_{11}^3) z_{11}^1 z_{11}^2],$$
$$\mu_{12} \leq 1 - \frac{y_{12} - 90}{30} z_{12}^1,$$
$$\mu_{12} \leq 1 - \frac{90 - y_{12}}{30} z_{12}^1,$$
$$\mu_{21} \leq 1 - [\frac{y_{21} - 120}{20} z_{21}^1 z_{21}^2 + \frac{y_{21} - 60}{10} z_{21}^1 (1 - z_{21}^2)],$$
$$\mu_{21} \leq 1 - [\frac{120 - y_{21}}{20} z_{21}^1 z_{21}^2 + \frac{60 - y_{21}}{10} z_{21}^1 (1 - z_{21}^2)],$$
$$\mu_{22} \leq 1 - [\frac{y_{22} - 50}{5} z_{22}^1 z_{22}^2 + \frac{y_{22} - 80}{10} z_{22}^1 (1 - z_{22}^2)$$
$$+ \frac{y_{22} - 60}{5} z_{22}^2 (1 - z_{22}^1)],$$
$$\mu_{22} \leq 1 - [\frac{50 - y_{22}}{5} z_{22}^1 z_{22}^2 + \frac{80 - y_{22}}{10} z_{22}^1 (1 - z_{22}^2)$$
$$+ \frac{60 - y_{22}}{5} z_{22}^2 (1 - z_{22}^1)],$$
$$\mu_{31} \leq 1 - [\frac{y_{31} - 80}{5} z_{31}^1 z_{31}^2 + \frac{y_{31} - 60}{10} z_{31}^1 (1 - z_{31}^2)],$$
$$\mu_{31} \leq 1 - [\frac{80 - y_{31}}{5} z_{31}^1 z_{31}^2 + \frac{60 - y_{31}}{10} z_{31}^1 (1 - z_{31}^2)],$$
$$\mu_{32} \leq 1 - [\frac{y_{32} - 65}{5} z_{32}^1 z_{32}^2 + \frac{y_{32} - 45}{5} z_{32}^1 (1 - z_{32}^2)$$
$$+ \frac{y_{32} - 150}{10} z_{32}^2 (1 - z_{32}^1)],$$
$$\mu_{32} \leq 1 - [\frac{65 - y_{32}}{5} z_{32}^1 z_{32}^2 + \frac{45 - y_{32}}{5} z_{32}^1 (1 - z_{32}^2)$$
$$+ \frac{150 - y_{32}}{10} z_{32}^2 (1 - z_{32}^1)],$$
$$Z_1 = 8.5 y_{11} + 7.0 y_{12} + 7.0 y_{21} + 6.5 y_{22} + 5.5 y_{31} + 6.0 y_{32},$$
$$Z_2 = 14.5 y_{11} + 24 y_{12} + 10 y_{21} + 18 y_{22} + 15.5 y_{31} + 12 y_{32},$$
$$\mu_1 \leq 1 - \frac{3200 - Z_1}{200},$$
$$\mu_2 \leq 1 - \frac{Z_2 - 6500}{200},$$
$$y_{11} + y_{12} \leq 150,$$
$$y_{21} + y_{22} \leq 220,$$

$$y_{31} + y_{32} \leq 200,$$
$$y_{11} + y_{21} + y_{31} \geq 200,$$
$$y_{12} + y_{22} + y_{32} \geq 250,$$
$$z_{11}^1 + z_{11}^2 \geq 1,$$
$$z_{11}^1 + z_{11}^3 \geq 1,$$
$$z_{31}^1 + z_{31}^2 \geq 1,$$
$$z_{21}^1 + z_{21}^2 \geq 1,$$
$$0 \leq \mu_{ij} \leq 1; 0 \leq \mu_p \leq 1; z_{ij}^k = 0 \text{ or } 1 \ \forall\, i, j, k, p.$$

Model 6.8 is solved by Lingo software and the solutions are obtained as follows: Optimum value of $Z = 0.88$; The optimal solution of the problem is $Z^1 =$ \$3181.50, $Z^2 =$ \$6500.00, i.e., the maximum profit is \$3181.5 and the minimum transportation cost is \$6500.0. The allocations are as follows:
$y_{11} = 70.0; y_{12} = 60.0; y_{21} = 134.5; y_{22} = 50.0; y_{31} = 0.0; y_{32} = 150.0$; The selection of fuzzy decision variables (i.e., the solution of MOFTP in terms of fuzzy variables) for getting the optimum solution of y_{ij} is made in the following way: $x_{11} = \widetilde{70}; x_{12} = \widetilde{90}; x_{21} = \widetilde{120}; x_{22} = \widetilde{60}; x_{31} = \widetilde{0}; x_{32} = \widetilde{150}$;

6.2.3 Result and discussion

In this chapter, we present a MOTP with fuzzy decision variables. The numerical example presents the applicability of proposed methodology for solving MOTP with fuzzy decision variables. The concept of this chapter is provided a MOTP whose decision variables are fuzzy numbers so it cannot compare to any other model provided in the literature. If someone wishes to solve the problem by considering a general MOTP, then he/she may use GP or RMCGP technique. To give a comparison of our study, we formulate a mathematical model acording to the procedure of RMCGP and solving it we have declared the following solution:
Optimal value of objective functions are $Z_1 =$ \$3200, $Z_2 =$ \$6500 and $x_{11} = 0.0; x_{12} = 98.0; x_{21} = 220.0; x_{22} = 0; x_{31} = 0; x_{32} = 162.33$; The optimal value of the objective function by RMCGP of the MOTP is more than the obtained

optimal solution in Model 6.8 but, the allocation made in cell does not fulfill the requirements as expected in the allocation cells. In this context, we may propose that MOTP can optimize the objective functions and find a compromise solution but cannot optimize the goals of the requirements in the cells of allocation. In this regard, our proposed methodology is better than the RMCGP technique to solve MOTP. Also, to the best of our knowledge, there is no specific methodology to formulate and solve MOTP with respect to the restrictions that we have presented in our proposed model.

6.3 Conclusion

In this chapter, we have considered a FTP in which the expected allocations in the destinations are multi-choice fuzzy numbers. There may occur some problems of FTP whose mathematical model and solution procedure are not described in the literature but in our chapter, we have formulated a mathematical model and on solving, we have obtained better solution of it. Again, realizing the facts of real-world decision making problems, we have extended our study of FTP into multi-objective ground and considered it as MOFTP. Finally, numerical examples are presented to justify the feasibility of our proposed models.

Chapter 7

MOTP under Cost Reliability*

Realizing the fact that transportation time is an important factor in TP, in this chapter we introduce the concept of cost reliability into the multi-objective transportation problem with multi-choice fuzzy goals to the objective functions under uncertain environment. The mathematical model of MOTP using cost reliability is presented and the solution procedure through fuzzy multi-choice goal programming is enlisted in the core part of this chapter. The numerical example is presented to justify the significance of the proposed study in the chapter.

7.1 Introduction

The parameters of the TP such as the supply, the demand and the cost parameters are not precise due to the complex real world problems. To analyze the TP in the complex ground, here we treat the supply and the demand parameters as uncertain variables, and the transportation cost is analyzed through reliability. Due to damage of transporting goods for delay of transportation within the schedule time or loss of wealth due to transportation before schedule time, as a result customers are affected. To remove this factor, we introduce the reliability in the cost parameters of the TP. Again, the study of MOTP

*A part of this chapter is accepted to publish in ***International Journal of Computational Intelligence Systems (IJCIS)***, Atlantis Press and Taylor & Francis, SCIE, IF: 0.391, Vol. **9**, No. 5, pp. 839-849 in which the objective functions are conflicting type, the reliability in cost penalty may cause to increase the cost of transportation or increase the values of objective functions which are minimization type and to decrease the profit of DM or decrease the value of objective functions which are maximization type. When the cost reliability factor is introduced as the rate of damage property which creates a diverse relation for both type of objective functions, but how much amount of loss caused by the factor is totally independent to the objective functions. So, it is clear that the reliability in cost parameter preserves the conflicting nature to the objective functions of the MOTP. Major studies in the ground of the MOTP, where the uncertainties are taken into consideration through fuzzy, stochastic, interval etc. for the transportation cost, the demand, and the supply parameters in the TP but here for the first time in the MOTP, we analyze the uncertainty into the cost parameter of MOTP using reliability.

Considering the situations of real-life decision making problems, we design this chapter on the TP in multi-objective ground where the objective functions are conflicting. Many situations occurred where the solution of a MOTP is found as compromise solution, but the solution often depends on the weights of objective functions proposed by the DM. Then in the MOTP the compromise solution satisfying the goals of the objective functions which plays an effective role for solving it. In this case, we propose goals to each objective functions of the MOTP. It is not always true that the solution of the MOTP will be specified as single choice of goals of the DM, because in case, it works in favor of DM but it is not so favored for customers. So, the DM needs to consider a multiple choice of goals corresponding to the objective functions and to select the best goals through the solution. Again, we consider the goals to the objective functions as multi-choice fuzzy numbers to accommodate the real life decision making situations.

The main aim of this chapter is to study the MOTP in which the objective functions are conflicting and non-commensurable to each other and each objective function has multi-choice fuzzy goals. Also, considering the present market scenario, we incorporate the concept of cost reliability into the transportation cost; and the demand and the supply parameters are treated as the uncertain variables. The methodologies are presented to tackle the situations and to predict the optimal goals of the objective functions as well as the optimal solution of the MOTP.

7.2 Problem background

In the TP, the completion time of transportation of amount of goods should be finished within the specified time, otherwise there may be created a damage of the items or storing problem and/or the customer may reject the ordered item. In that situation, the transportation cost or the profit may not be considered as crisp value. Then the selection of goals for the objective functions or the solution of the MOTP cannot be made in a usual way. To overcome this difficulty, by selecting the proper goals to the objective functions, here, we incorporate the concept of reliability for the cost parameters in the TP. In that situation, we introduce a new term "cost reliability" for the transportation cost in the proposed study.

Generally, Reliability refers the probability of a machine operating its intended purpose adequately for the period of time desired under the operating conditions encountered. More precisely, reliability is the probability with which the devices will not fail to perform a required operation for a certain period of time.

Definition 7.2.1 *(Cost Reliability): Cost reliability is the probability that the transportation of goods will not fail to complete the transporting of goods in the schedule time, which creates a probabilistic cost in the transportation problem. The probabilistic cost in the transportation problem increases the original value of transportation of goods and simultaneously makes a difference in profit margin*

We assume that failure rate λ, by the ratio of due time or over time, $\delta\tau$ to complete the transportation of goods; and the total estimated time T of transportation i.e., $\lambda = \frac{\delta\tau}{T}$, where τ represents the amount of late or early transportation time corresponding to the schedule time of transportation. Here, we define the cost reliability $R(\tau)$, which is a function of time τ as follows:

$$R(\tau) = \frac{Amount\ of\ goods\ remains\ in\ good\ condition\ due\ to\ loss\ of\ time}{Total\ amount\ of\ \ goods}$$

The probability of failure, $Q(\tau)$ can be expressed as follows:

$$Q(\tau) = \frac{Amount\ of\ goods\ damaged\ due\ or\ over\ time}{Total\ amount\ of\ goods}$$

Clearly, $R(\tau) + Q(\tau) = 1$ so, $Q(\tau) = 1 - R(\tau)$.
Therefore, $R(\tau) = e^{-\int_0^\tau \lambda d\tau}$. Assuming that the failure rate λ, is a constant in respect to time τ, then we have $R(\tau) = e^{-\lambda\tau}$. And finally, $Q(\tau)$ can be found as $Q(\tau) = 1 - R(\tau) = 1 - e^{-\lambda\tau}$.
To analyze the proposed study with a better impact in reality, we consider the parameter λ, is the ratio of a function of decision variable x_{ij}, and a_i is the constant supply of the i^{th} source, i.e., $\lambda = \frac{x_{ij}}{a_i}$. When the value of λ increases the value of reliability $R(\tau)$ decreases which means that if the amount of transported goods becomes larger, then the amount of items may be defective in bigger rate. Again the value of $R(\tau) = e^{-\lambda\tau}$ depends on the time τ, then we would like to consider the time τ as the expected loss of time to complete the work. If the transportation made in time, then $\tau = 0$, so reliability value is maximized, i.e., $R(\tau) = 1$. Again it is true that in the TP, some variables may take value zero. It means no item is transported in that route. So the reliability is again equal to 1 for this path in the proposed model which does not create any complexity to take the decision for the DM.
Taking advantage of the reliability function in the real-life decision making problem, we formulate the MOTP where the objective functions are conflicting and connected with some multi-choice goals. The way of formulating the mathematical model of MOTP using cost reliability is included in detail in the next section.

7.3 Mathematical model

According to the challenging competitive market scenario, there are several objective functions related to a transportation problem like minimizing the transportation cost, maximizing the transportation profit, minimizing the toll tax etc. Again, there is no connection between the cost parameters in the different objective functions of the TP, so, they are considered as conflicting and non-commensurable to each other. In multi-objective environment of the TP, i.e., the MOTP can be defined as follows:

Model 7.1

$$\text{minimize/maximize} \quad Z^t = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}^t x_{ij} \quad (t = 1, 2, \ldots, K) \tag{7.1}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \leq a_i \quad (i = 1, 2, \ldots, m), \tag{7.2}$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j \quad (j = 1, 2, \ldots, n), \tag{7.3}$$

$$x_{ij} \geq 0 \ \ \forall \ \ i, \ j. \tag{7.4}$$

Here, C_{ij}^t, a_i and b_j are the transportation cost, the supply and the demand parameters respectively for the t-th objective function in the MOTP.

In Model 7.1, the quantities C_{ij}^t, a_i and b_j are all assumed to be crisp numbers. However, if sometime, the transportation plan is made in advance, so the quantities are not generally fixed but approximate amounts of these are obtained from practical experience or expert knowledge. In this case, we may assume the quantities are uncertain variables. Then the transportation Model 7.1 is only a conceptual model rather than a mathematical model because there does not exist a natural order-ship in the complex world. A large number of decision making problems has been solved by several researchers in which uncertain situation is directly introduced in the parameters of the MOTP, but here, we introduce the uncertainty through reliability by considering due or early transportation time of delivering the goods. Due to the late or early reach of the transporting goods, the customer or the store keeper fails to manage it. So, the DM should consider in his mind the matter and as a whole the optimum value of the objective functions are affected. Considering this situation, we introduce time in the cost parameter which reduces the cost parameter of the MOTP with reliability. When the cost reliability is considered for all the objective functions then time is taken as independent to each other and the conflicting nature of the objective functions preserved in the MOTP. Again, the supply and demand quantities are not crisp due to weather condition, seasonal effect, market situation etc. So, the uncertain measure (here denoted as $\mathbb{M}$) from an uncertainty distribution for supply and demand constraints are considered. Then Model 7.1 reduces to the following mathematical model (Model 7.2) as follows:

Model 7.2

$$\text{minimize/maximize} \quad Z^t = \sum_{i=1}^{m}\sum_{j=1}^{n} Re(C_{ij}^t)x_{ij} \ (t = 1, 2, \ldots, K) \tag{7.5}$$

$$\text{subject to} \quad \mathbb{M}(\sum_{j=1}^{n} x_{ij} \leq a_i) \geq \gamma_i \ \ (i = 1, 2, \ldots, m), \tag{7.6}$$

$$\mathbb{M}(\sum_{i=1}^{m} x_{ij} \geq b_j) \geq \delta_j \ \ (j = 1, 2, \ldots, n), \tag{7.7}$$

$$x_{ij} \geq 0 \ \ \forall \ \ i, \ j. \tag{7.8}$$

It is assumed that $a_i > 0, \ b_j > 0$ and the specified stochastic levels or predetermined confidence levels are defined as $0 \leq \gamma_i \leq 1 \ \ \forall \ i$ and $0 \leq \delta_j \leq 1 \ \ \forall \ j$. In Model 7.2, $R(C_{ij}^t)$ is considered as cost parameter under reliability. The delay of supply of items causes to damage the items, in this case the value of profit function (maximization type) decreases. Again, the penalty cost due to loss of time is considered when the objective function is of minimization type (like transportation cost) which increases the value of the respective objective function. Then $R(C_{ij}^t)$ takes the following form of cost parameter as:

$R(C_{ij}^t) = C_{ij}^t + C_{ij}^t(1 - R_{ij}^t)$, for the objective function is of minimization type and

$R(C_{ij}^t) = C_{ij}^t - C_{ij}^t(1 - R_{ij}^t)$, for the objective function is of maximization type. Here R_{ij}^t is the reliability of the t^{th} objective function for the (i, j)-th node. which depends on fixed time (τ), decision variable (x_{ij}) and demand (a_i). For consistency of reliability in each node, the DM measures the time (τ) in a unit scale, otherwise there may occur large deviations in the cost values and produces an optimal solution, which is not significantly a good result.

Many real-life MOTPs, the DM wishes to solve the objective functions by considering certain goals to the objective functions, but how the goals to be chosen which is a complex task to the DM. To remove this difficulty, we consider the fuzzy multi-choice goals with the respective deviations corresponding to each objective function. In that situation, the DM wishes to fix the goals in such a way that the compromise solution becomes a better solution corresponding to the chosen goals. Let us consider the fuzzy multi-choice goals $\tilde{g}_t^1, \tilde{g}_t^2, \ldots, \tilde{g}_t^p$ $(t = 1, 2, \ldots, K)$ for the t^{th} objective function. Then each of the objective functions in the mathematical Model 7.2, there are multi-choice fuzzy goals, so, we present the procedure to select the optimum goals and get the optimal solution in the next section as a solution procedure.

7.4 Solution procedure

In this section, we present the solution procedure using FMCGP.
According to the real-life decision making problem, the goals of the objective functions cannot be predicted as crisp values always, they may be considered as fuzzy goals. So the goals are fuzzy multi-choice goals corresponding to the objective functions. Under the environment of FMCGP, the formulation of GP reduces to the following form:

FMGP

$$\begin{aligned} \text{minimize} \quad & \sum_{t=1}^{K} w_t |Z^t(X) - \hat{g}_t^1 \text{ or } \hat{g}_t^2 \text{ or } \ldots \text{ or } \hat{g}_t^p| \\ \text{subject to} \quad & X \in F. \end{aligned}$$

Here, the aspiration levels $\hat{g}_t^p$, $\forall\ i$ and t are assumed to be triangular fuzzy numbers with membership functions $\mu_t^p\ \ \forall\ p,\ t$.

7.4.1 Reduction of uncertainty in the MOTP

Here, we introduce the concept of uncertainty distribution in order to describe the uncertain variable. Due to insufficient information of demand and supply in the MOTP, we incorporate the uncertainty in the constraints. However, in our proposed study, we consider the demands and the supply parameters

as Normal distribution. There are several ways to tackle the uncertain constraints, here we use the uncertain measure proposed by Liu (86). Let us introduce some useful definitions; and theorem about on the uncertain variable.
According to our assumption that the supply and demand parameters in Model 7.2 are taken as uncertain measure (here denoted as $\mathbb{M}$) from an uncertainty distribution for supply and demand, so we define uncertain distribution function as follows:

Definition 7.4.1 *(Liu (86)) Let ξ be an uncertain variable. Then the uncertainty distribution denoted as Φ of ξ is defined by $\Phi(x) = \mathbb{M}\{\xi \leq x\}$ for any real number x.*

Without loss of generality, we may consider the Normal distribution for the supply and demand parameters in Model 7.2. A Normal uncertain distribution function and its inverse function are considered as follows:

Definition 7.4.2 *An uncertain variable ξ is called Normal if it has a Normal uncertain distribution $\Phi(x) = [1 + exp(\frac{\pi(e-x)}{\sqrt{3}\sigma})]^{-1}$ for any real number x, which is denoted by $N(e, \sigma)$ where e and σ are real numbers with $\sigma > 0$.*

Definition 7.4.3 *(Liu (86)) Let ξ be an uncertain variable with regular uncertainty distribution $\Phi(x)$. Then the inverse function $\Phi^{-1}(\alpha)$ is called the inverse uncertainty distribution of ξ, for any real number α.*

As the supply and demand parameters are uncertain variable, so, uncertain measure is introduced in the supply and demand constraints of Model 7.2. These constraints are reduced into the equivalent crisp forms by using the following measure inversion theorem.

Theorem 7.1: (Measure Inversion Theorem Liu (86) Let ξ be an uncertain variable corresponding to uncertain distribution $\Phi(x)$. Then for any real number x, we have, $\Phi(x) = \mathbb{M}\{\xi \leq x\}$ then $\mathbb{M}\{\xi \geq x\} = 1 - \Phi(x)$.

The inverse uncertainty distribution of Normal uncertain variable $N(e,\sigma)$ is defined as follows:
$\Phi^{-1}(x) = e + \frac{\sqrt{3}\sigma}{\pi} ln \frac{\alpha}{1-\alpha}$, where ln denotes natural logarithm.
Model 7.2 is not a deterministic form due to presence of uncertain variable in the constraints. Assuming that a_i $(i = 1, 2, \ldots, m)$ and b_j $(j = 1, 2, \ldots, n)$ are independent uncertain variables with uncertainty distributions θ_i $(i = 1, 2, \ldots, m)$ and ψ_j $(j = 1, 2, \ldots, n)$ respectively. Then the measure inversion theorem is provided the following results:
$\mathbb{M}(\sum_{j=1}^{n} x_{ij} \leq a_i) \geq \gamma_i$ is equivalent to $\sum_{j=1}^{n} x_{ij} \leq \theta_i^{-1}(1-\gamma_i)$,
and $\mathbb{M}(\sum_{j=1}^{n} x_{ij} \geq b_j) \geq \delta_j$ is reduced to
$\sum_{j=1}^{n} x_{ij} \geq \psi_j^{-1}\delta_j$, for $i = 1, 2, \leq, m; j = 1, 2, \leq, n$.
Using above results, Model 7.2 is equivalent to the following model:

Model 7.3

$$\text{minimize/maximize} \qquad Z^t = \sum_{i=1}^{m}\sum_{j=1}^{n}[C_{ij}^t \pm C_{ij}^t(1-R_{ij}^t)]x_{ij} \tag{7.9}$$

$$\text{subject to} \qquad \sum_{j=1}^{n} x_{ij} \leq \theta_i^{-1}(1-\gamma_i) \quad (i = 1, 2, \ldots, m), \tag{7.10}$$

$$\sum_{i=1}^{m} x_{ij} \geq \psi_j^{-1}\delta_j \quad (j = 1, 2, \ldots, n), \tag{7.11}$$

$$x_{ij} \geq 0 \;\; \forall \;\; i, \; j. \tag{7.12}$$

Here, we consider the uncertain MOTP under fuzzy multi-choice goal environment i.e., each objective function of the MOTP has some specific fuzzy goals, $\hat{g}_t^k$ for $(t = 1, 2, \ldots, K)$ for $k = 1, 2, \ldots, p$. According to the problem, the DM can assign the weights to each objective function in such way that it produces a better compromise solution. To do this, we construct a crisp model of the MOTP which is of maximizing type problem whatever the nature of the objective functions of the MOTP.
Again for optimizing the values of the objective functions, the number of fuzzy allocation goals $\hat{g}_t^k$ may not be equal for all the objective functions. If there be only one fuzzy goal $\hat{g}_t^1$ for each objective function, then the corresponding mathematical model (i.e., Model 7.4) is derived from Model 7.3 as follows:

Model 7.4

$$\text{maximize} \quad z = \sum_{t=1}^{K} w_t \mu_t \tag{7.13}$$

$$\text{subject to} \quad Z^t = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}^t \pm [C_{ij}^t (1 - R_{ij}^t)] x_{ij}, \tag{7.14}$$

$$\mu_t \leq 1 - \frac{Z^t - \hat{g}_t^1}{d_t^{1-}} \quad \forall t, \tag{7.15}$$

$$\mu_t \leq 1 - \frac{\hat{g}_t^1 - Z^t}{d_t^{1+}} \quad \forall t, \tag{7.16}$$

$$\mu_t \geq 0 \quad \forall t, \tag{7.17}$$

$$\sum_{j=1}^{n} x_{ij} \leq \theta_i^{-1}(1 - \gamma_i) \quad (i = 1, 2, \ldots, m), \tag{7.18}$$

$$\sum_{i=1}^{m} x_{ij} \geq \psi_j^{-1} \delta_j \quad (j = 1, 2, \ldots, n), \tag{7.19}$$

$$x_{ij} \geq 0 \quad \forall \ i, \ j. \tag{7.20}$$

where d_t^{1-} and d_t^{1+} are the negative and positive deviations corresponding to the goals $\hat{g}_t^1$ of the objective function Z^t respectively. Now, assuming that if each objective function has two fuzzy aspiration levels, then FMCGP selects any one of these goals in such a way that it provides a better optimal solution. Based on the model of Chang (17), the equations; i.e., (7.15) and (7.16) reduce to the following form as:

$$\mu_t \leq 1 - \left[\frac{Z^t - \hat{g}_t^1}{d_t^{1-}} z_t^1 + \frac{Z^t - \hat{g}_t^2}{d_t^{2-}} (1 - z_t^1) \right] \quad \forall t,$$

$$\mu_t \leq 1 - \left[\frac{\hat{g}_t^1 - Z^t}{d_t^{1+}} z_t^1 + \frac{\hat{g}_t^2 - Z^t}{d_t^{2+}} (1 - z_t^1) \right] \quad \forall t,$$

$$z_t^1 = 0 \text{ or } 1 \quad \forall \ t.$$

Here, d_t^{k-} and d_t^{k+} are the maximum allowable negative and positive deviations respectively for $\hat{g}_t^k$ for $k = 1, 2$.

According to the real-life phenomenon, the objective function may have more than two fuzzy aspiration levels, then we design the corresponding mathematical model in the following way.

So, when each objective function has three fuzzy aspiration levels, FMCGP takes any one of these goals in such a way that it produces to a better optimal solution. Therefore, based on the model of Chang (17), the equations (7.15) and (7.16) reduce as follows:

$$\begin{aligned}
&\mu_t \leq 1 - [\frac{Z^t - \hat{g}_t^1}{d_t^{1-}} z_t^1 z_t^2 + \frac{Z^t - \hat{g}_t^2}{d_t^{2-}}(1 - z_t^1) z_t^2 + \frac{Z^t - \hat{g}_t^3}{d_t^{3-}} z_t^1 (1 - z_t^2)] \ \ \forall t,\\
&\mu_t \leq 1 - [\frac{\hat{g}_t^1 - Z^t}{d_t^{1+}} z_t^1 z_t^2 + \frac{\hat{g}_t^2 - Z^t}{d_t^{2+}}(1 - z_t^1) z_t^2 + \frac{\hat{g}_t^3 - Z^t}{d_t^{3+}} z_t^1 (1 - z_t^2)] \ \ \forall t,\\
&z_t^1 + z_t^2 \geq 1 \ \forall \ t,\\
&z_t^k = 0 \text{ or } 1 \ \ \forall \ \ t, \ k.
\end{aligned}$$

Similarly, d_t^{k-} and d_t^{k+} are the maximum allowable negative and positive deviations respectively for $\hat{g}_t^k$ for $k = 1, 2, 3$.

If $\hat{g}_t^k$ for $(t = 1, 2, \ldots, K)$ denotes the fuzzy multi-choice goals for the objective functions Z^t $(t = 1, 2, \ldots, r)$, then the linear membership function μ_t for t-th objective function can be defined as follows:

$$\mu_t = \begin{cases} 0, & \text{if } \ Z^t \geq \tilde{g}_t^k + d_t^{k+},\\ 1 - \sum_{k=1}^{p} \frac{Z^t - \tilde{g}_t^k}{d_t^{k+}} F_k(B), & \text{if } \ \tilde{g}_t^k \leq Z_t \leq \tilde{g}_t^k + d_t^{k+},\\ 1, & \text{if } \ Z_t = \tilde{g}_t^k,\\ 1 - \sum_{k=1}^{p} \frac{\tilde{g}_t - Z^t}{d_t^{k-}} F_k(B), & \text{if } \ \tilde{g}_t^k - d_t^{k-} \leq Z_t \leq \tilde{g}_t^k \end{cases}$$

Here, $F_k(B)$ indicates a function of binary serial numbers that ensures only one aspiration level must be chosen from each goal (cf., Roy et al. (135)). In general, d_t^{k+} and d_t^{k-} are the maximum allowable positive and negative deviations respectively from the k-th aspiration level of the t-th objective function respectively.

It may be noted that, it is not necessary that each objective function has the same number of multi-choice goals. Then Model 7.4 is developed according to the number of fuzzy goals for the objective functions and then solving, we obtain the aspiration level of each objective function as well as the optimal solution of the MOTP.

7.5 Numerical example

To test the efficiency of our proposed study, we consider a coal transportation problem, which mainly refers to the MOTP with fuzzy multi-choice goals to the objective functions. The MOTP is designed based on uncertain supply and demand along with the concept of reliability to the cost parameter of the TP.

The DM plans to distribute the coals from three mines M1, M2 and M3 to four Thermal Plants which are situated in the cities, C1, C2, C3 and C4. During the planning, he wishes to optimize the following objective functions as:

- minimize the transportation cost (Z^1),
- minimize the toll tax (Z^2),
- maximize the profit (Z^3).

According to the market scenario, the DM cannot predict the optimal goals for the objective functions Z^1, Z^2 and Z^3 in a crisp way. According to the experience of the DM in the work field, the DM has the ideas about the nature of the objective functions and using this he considers a number of fuzzy goals corresponding to the objective functions. Again, from his experience he guesses the following events.

- The cost C_{ij}^1 for transporting one unit of goods from the resource i to the destination j and the approximate loss of time of delivery from the sources to the destinations are also known to the DM, for $i = 1, 2, 3$ and $j = 1, 2, 3, 4$ respectively.
- The toll tax cost C_{ij}^2 for transporting one unit of goods from the resource i to the destination j and it is fixed value for $i = 1, 2, 3$ and $j = 1, 2, 3, 4$ respectively.
- The profit C_{ij}^3 for transporting one unit of goods from the resource i to the destination j and the DM made a prediction of approximate loss of

time of delivery from the sources to the destination, for $i = 1, 2, 3$ and $j = 1, 2, 3, 4$ respectively.

The data for the costs C_{ij}^t for $t = 1, 2, 3$ are represented in Tables 7.1, 7.2 and 7.3.

Table 7.1: Transportation cost C_{ij}^1 (in \$) and loss of time (in week).

	C1	C2	C3	C4
M1	(20, 0.1)	(18, 0.1)	(22, 0.1)	(24, 0.1)
M2	(10, 0)	(12, 0.2)	(15, 0)	(13, 0)
M3	(22, 0)	(20, 0.1)	(24, 1)	(23, 0.15)

Table 7.2: Toll tax cost C_{ij}^2 (in \$) for transportation.

	C1	C2	C3	C4
M1	5	6	4	3
M2	6	5	5	4
M3	9	8	8	10

Table 7.3: Profit C_{ij}^3 (in \$) and loss of time (in week).

	C1	C2	C3	C4
M1	(3, 0.1)	(3.5, 0.1)	(2.5, 0.1)	(5, 0.1)
M2	(3, 0)	(6, 0.2)	(4, 0)	(4, 0)
M3	(4, 0)	(3, 0.1)	(4, 1)	(5, 0.15)

The supply parameters $a_1,\ a_2$ and a_3 of mines M1, M2 and M3 and the demand parameters $b_1,\ b_2,\ b_3$ and b_4 of cities C1, C2, C3 and C4 follow Normal distribution $N(e_i^1, \sigma_i^1)$, for $i = 1, 2, 3$; and $N(e_j^2, \sigma_j^2)$, for $j = 1, 2, 3, 4$ respectively. The data for supply a_i and demand b_j $\forall\, i, j$ are presented in Tables 7.4 and 7.5.

Table 7.4: Supply parameter a_i follows Normal distribution $N(e_i^1, \sigma_i^1)$.

M1	M2	M3
(55, 4)	(60, 5)	(70, 4)

Table 7.5: Demand parameter b_j follows Normal distribution $N(e_i^2, \sigma_i^2)$.

C1	C2	C3	C4
(40, 3)	(36, 4)	(35, 5)	(40, 3)

Again, in the proposed problem, obviously the DM would like to minimize the transportation cost and toll tax cost and maximize the profit. In this situation, the DM expects that the possible expenditure for transportation cost may be any one of fuzzy interval values 2900(100), 4000(100) and 3400(100), here the number within first bracket denotes the deviations (positive and negative deviations are same here).

Thereafter, DM expects the expenditure due to toll tax cost either 950(50) or 1250(40). He also predicts the profit goal may be chosen among the fuzzy values 480(30), 1050(20), 650(50) and 900(50). He wishes to consider the equal weights '0.3' to the objective functions Z^1 and Z^2 and an weight '0.4' for the last objective function Z^3 respectively. Then the proposed problem takes the mathematical form as follows:

Model 7.5

$$Z^1 = \sum_{i=1}^{3}\sum_{j=1}^{4} R[C_{ij}^1]x_{ij},$$

$$Z^1 \text{ has the goals } [2900(100),\ 4000(100),\ 3400(100)],$$

$$Z^2 = \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}^2 x_{ij},$$

$$Z^2 \text{ has the goals } [950(50),\ 1250(40)],$$

$$Z^3 = \sum_{i=1}^{3}\sum_{j=1}^{4} R[C_{ij}^3]x_{ij},$$

$$Z^3 \text{ has the goals } [480(30),\ 1050(20),\ 650(50),\ 900(50)],$$

$$\text{subject to} \quad \mathbb{M}(\sum_{j=1}^{4} x_{ij} \leq a_i) \geq \gamma_i \quad (i = 1, 2, 3),$$

$$\mathbb{M}(\sum_{i=1}^{3} x_{ij} \geq b_j) \geq \delta_j \quad (j = 1, 2, 3, 4),$$

$$x_{ij} \geq 0 \quad \forall\ i, j.$$

The uncertain parameters are taken as Normal variable $N(e, \sigma)$ where e is the expectation and σ is the standard deviation. The inverse uncertain distribution is defined as $\phi^{-1}(x) = e + \frac{\sqrt{3}\sigma}{\pi} ln\frac{1-x}{x}$.

Then Model 7.5 reduces to the following form as:

Model 7.6

$$
\begin{aligned}
& Z^1 = \sum_{i=1}^{3}\sum_{j=1}^{4}[C_{ij}^1 + C_{ij}^1(1 - R_{ij}^1)]x_{ij}, \\
& Z^1 \text{ has the goals } [2900(100),\ 4000(100),\ 3400(100)], \\
& Z^2 = \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}^2 x_{ij}, \\
& Z^2 \text{ has the goals } [950(50),\ 1250(40)], \\
& Z^3 = \sum_{i=1}^{3}\sum_{j=1}^{4}[C_{ij}^3 - C_{ij}^3(1 - R_{ij}^3)]x_{ij}, \\
& Z^3 \text{ has the goals } [480(30),\ 1050(20),\ 650(50),\ 900(50)], \\
\text{subject to} \quad & \sum_{j=1}^{4} x_{ij} \le e_i^1 + \frac{\sqrt{3}\sigma_i^1}{\pi} ln\frac{1-\gamma_i}{\gamma_i} \quad (i = 1,2,3), \\
& \sum_{i=1}^{3} x_{ij} \ge e_j^2 + \frac{\sqrt{3}\sigma_j^2}{\pi} ln\frac{1-\delta_j}{\delta_j} \quad (j = 1,2,3,4), \\
& x_{ij} \ge 0 \ \ \forall\ i,j.
\end{aligned}
$$

Here, R_{ij}^1 and R_{ij}^3 are the reliability of completing the job of transportation in time, They are taken as function of the decision variables and the expected loss time for completion the work. The source capacity (a_i) is uncertain in nature, but we use this value for maximum priority in R_{ij}^1 and R_{ij}^3. Let us assume the confidence levels be $\gamma_i = 0.85$ and $\delta_j = 0.9$, $\forall\ i = 1,2,3$ and $j = 1,2,3,4$. Finally, the proposed MOTP is converted to a single objective problem using the goals as prescribed by the DM and we obtain the following model as:

Model 7.7

$$
\begin{aligned}
\text{minimize} \quad & z = 0.3\mu_1 + 0.3\mu_2 + 0.4\mu_3 \\
\text{subject to} \quad & Z^1 = (40 - 20e^{\frac{-0.1}{55}x_{11}})x_{11} + (36 - 18e^{\frac{-0.1}{55}x_{12}})x_{12} \\
& +(44 - 22e^{\frac{-0.1}{55}x_{13}})x_{13} + (26 - 13e^{\frac{-0.1}{55}x_{14}})x_{14} \\
& +(20 - 10e^{\frac{-0}{60}x_{21}})x_{21} + (24 - 12e^{\frac{-0.2}{60}x_{22}})x_{22} \\
& +(30 - 15e^{\frac{-0}{60}x_{23}})x_{23} + (26 - 13e^{\frac{-0}{60}x_{24}})x_{24} \\
& +(44 - 22e^{\frac{-0}{70}x_{31}})x_{31} + (40 - 20e^{\frac{-0.1}{70}x_{32}})x_{32}
\end{aligned}
$$

$+(48-24e^{\frac{-1}{70}x_{33}})x_{33}+(46-23e^{\frac{-0.15}{70}x_{34}})x_{34},$

$Z^2=5x_{11}+6x_{12}+4x_{13}+3x_{14}+6x_{21}+5x_{22}$

$+5x_{23}+4x_{24}+9x_{31}+8x_{32}+8x_{33}+10x_{34},$

$Z^3=3e^{-\frac{0.1}{55}x_{11}}x_{11}+3.5e^{-\frac{0.1}{55}x_{12}}x_{12}+2.5e^{-\frac{0.1}{55}x_{13}}x_{13}+$

$5e^{-\frac{0.1}{55}x_{14}}x_{14}+3e^{-\frac{0}{60}x_{21}}x_{21}+6e^{-\frac{0.2}{60}x_{22}}x_{22}+$

$4e^{-\frac{0}{60}x_{23}}x_{23}+4e^{-\frac{0}{60}x_{24}}x_{24}+4e^{-\frac{0}{70}x_{31}}x_{31}+$

$3e^{-\frac{0.1}{70}x_{32}}x_{32}+4e^{-\frac{1}{70}x_{33}}x_{33}+5e^{-\frac{0.15}{70}x_{34}}x_{34},$

$\mu_1\leq 1-[\frac{Z^1-2900}{100}z_1^1z_1^2+\frac{Z^1-4000}{100}(1-z_1^1)z_1^2+\frac{Z^1-3400}{100}z_1^1(1-z_1^2)],$

$\mu_1\leq 1-[\frac{2900-Z^1}{100}z_1^1z_1^2+\frac{4000-Z^1}{100}(1-z_1^1)z_1^2+\frac{3400-Z^1}{100}z_1^1(1-z_1^2)],$

$\mu_2\leq 1-[\frac{Z^2-950}{50}z_2^1+\frac{Z^2-1250}{40}(1-z_2^1)],$

$\mu_2\leq 1-[\frac{950-Z^2}{50}z_2^1+\frac{1250-Z^2}{40}(1-z_2^1)],$

$\mu_3\leq 1-[\frac{Z^3-480}{30}z_3^1z_3^2+\frac{Z^3-1050}{20}(1-z_3^1)z_3^2+\frac{Z^3-650}{50}z_3^1(1-z_3^2)$

$+\frac{Z^3-900}{50}(1-z_3^1)(1-z_3^2)],$

$\mu_3\leq 1-[\frac{480-Z^3}{30}z_3^1z_3^2+\frac{1050-Z^3}{20}(1-z_3^1)z_3^2+\frac{650-Z^3}{50}z_3^1(1-z_3^2)$

$+\frac{900-Z^3}{50}(1-z_3^1)(1-z_3^2)],$

$z_1^1+z_1^2\geq 1,$

$z_k^t=0 \text{ or } 1\ \forall\ k,t,$

$0\leq\mu_k\leq 1\ \ (k=1,2,3),$

$x_{11}+x_{12}+x_{13}+x_{14}\leq 51.175,$

$x_{21}+x_{22}+x_{23}+x_{24}\leq 55.218,$

$x_{31}+x_{32}+x_{33}+x_{34}\leq 66.175,$

$x_{11}+x_{21}+x_{31}\geq 43.634,$

$x_12+x_22+x_32\geq 40.486,$

$x_{13}+x_{23}+x_{33}\geq 41.057,$

$$x_{14} + x_{24} + x_{34} \geq 43.634,$$

$$x_{ij} \geq 0 \;\; \forall \; i, j.$$

7.6 Result and discussion

Solving Model 7.7 using LINGO software, we derive the following solution: Optimum value of $z = 0.82$. Hence, the optimal values of the objective functions are: $Z^1 = 3400.00$, $Z^2 = 980.13$ and $Z^3 = 650$. The allocations are made as follows:

$x_{11} = 3.12$; $x_{12} = 0.0$; $x_{13} = 18.95$; $x_{14} = 29.10$; $x_{21} = 11.26$; $x_{22} = 25.07$; $x_{23} = 4.36$; $x_{24} = 14.54$; $x_{31} = 29.26$; $x_{32} = 15.42$; $x_{33} = 17.74$; $x_{34} = 0.0$;

Selected goal for the objective functions are as follows:

3400(100) for Z^1, 950(50) for Z^2 and 650(50) for Z^3.

Here, we present a study on the MOTP with fuzzy multi-choice goals for each objective functions under uncertain environment. Especially, here, we incorporate the situation of cost reliability with the cost parameters due to delay in delivery of goods before/after schedule time. The numerical example is presented the applicability of proposed methodology for solving the MOTP with fuzzy multi-choice goals and uncertain supply and demand parameters. Usually, the MOTP having the objective functions are of either maximization type or minimization type, but here, the aim of the DM is not likely to the solution of traditional MOTP. This work presents the uncertainty under the expectation of an upcoming event and also proposes to fix-up the goals for each of the objective functions for the DM corresponding to the possible optimal values. On the other hand, time is very much important for transporting the goods to real-life transportation problems, so the decision making under time consideration and cost reliability provide a proper way of selecting the goals for the objective functions.

In the proposed problem, if the DM wishes to find the optimal solution of the objective function under traditional way like goal programming approach, then certainly he may consider minimum goals for first two objective functions Z^1 and Z^2 i.e., 2900 and 950 respectively and for the profit objective function

Z^3, the maximum goal is 1050. Considering these goals corresponding to the objective functions, we attempt to solve the proposed problem using goal programming, but we cannot obtain any feasible solution through GP. Again we have tested that, if the DM considers the goal values 4000 for the objective function Z^1 and 1250 for the objective function Z^2 then the DM calculated the maximum profit goal 1050 for the objective function Z^3 through goal programming approach. Basically, in most cases, the customers pay the transportation cost and toll tax cost, if the costs increase then the DM may loose the customer in near future which happens due to maximum profit goal of the DM. As a result the optimal solution through GP has not produced a better result in both cases. To test the utility of our proposed method, we consider multi-choice goals (2900, 4000, 3400) for Z^1; (950, 1250) for Z^2; (480, 1050, 650, 900) for Z^3 and solving the problem by revised multi-choice goal programming (RMCGP) approach, we see that the selected optimal goal values are 2900 for Z^1; 950 for Z^2; 480 for Z^3. In this case the DM achieves the solution in which selected profit goal is minimum value and it is not satisfactory to the DM. So, the proposed approach is better than GP or RMCGP technique for selecting optimal goals. Hence, in that situation, the DM cannot fix the goals of the objective functions using the existing techniques, whereas our present methodology can give a better solution as well as a better selection of goals for the MOTP through uncertain environment.

7.7 Conclusion and limitations of the study

In this chapter, we have analyzed the real-life MOTP through the concept of reliability and uncertain environments. We have proposed a new kind of uncertainty on cost parameter based on the concept reliability. Beside this, we have established the MOTP under the consideration of fuzzy multi-choice goals to the objective functions and supply and demand are taken as uncertain in nature. A solution procedure for solving the MOTP; and the selection of goals for the objective functions has been discussed by taking a real-life example. The obtained results have indicated that the proposed approach has a better impact to solve the MOTP under uncertain environment; and it has the advantages of selection of goals for real-life decision making problems under uncertain environment. This method is not only proposed the subjective preference into real-life decision-making problems, but also can realize the better selection of goals to the objective functions.

The proposed method has the following limitations for formulating the mathematical model. Firstly, the value of delay time or over time of transportation in the reliability function should have to be taken in a unit scale and in this case the time $\tau \leq 1$, otherwise, the value of reliability function may not provide for selecting the goals and optimal solution. Another important factor is that at least one multi-choice fuzzy goals for each of the objective functions (Z^i) must intersect the interval range [min Z^i, max Z^i] for all i for getting the optimal solution of the MOTP, otherwise it produces infeasible solution. Thus for a better result of the MOTP, the above two restrictions must be taken into care by the DM.

Chapter 8

Two-Stage Grey Transportation Problem *

In this chapter, we propose how to select optimum goals corresponding to the objective functions in a Two-Stage TP under grey environment. Here, we present the Two-Stage TP considering multiple objective functions under grey environment and establish the approach for selection of optimum goals along with the optimum solution of the proposed model. A numerical example is incorporated to justify the efficiency of the proposed study.

8.1 Introduction

The traditional way of a Transportation Problem (TP) is a type of decision making problem which minimizes the transportation cost for transporting the goods from origin to destinations satisfying some restrictions according to the feasibility of the problem. But, it is not exactly true that the TP only minimizes the transportation cost. In addition to this, it can have applications in a decision making problem like minimizing the purchasing cost and maximizing the profit, etc. In a TP, sometimes it is also to be considered that before transporting the goods in the destinations from the sources, goods are to be stored at warehouses from sources. So, the Decision Maker (DM) utilizes the concept for managing Two-Stage transportation in his position which maximizes

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the profit. A TP is called a Two-Stage transportation problem, if it consists of transporting the goods by two Stages, namely, One-Stage transportation problem and in Another-Stage transportation problem. The TP which is considered in two stages of collecting the goods in warehouses is called an One-Stage transportation problem; the TP considered in Another Stage of distributing the goods is referred to as an Another Stage transportation problem.
In Figure 8.1, a network is considered to illustrate a Two-Stage transportation problem. Here, the DM has three warehouses such as S1, S2 and S3

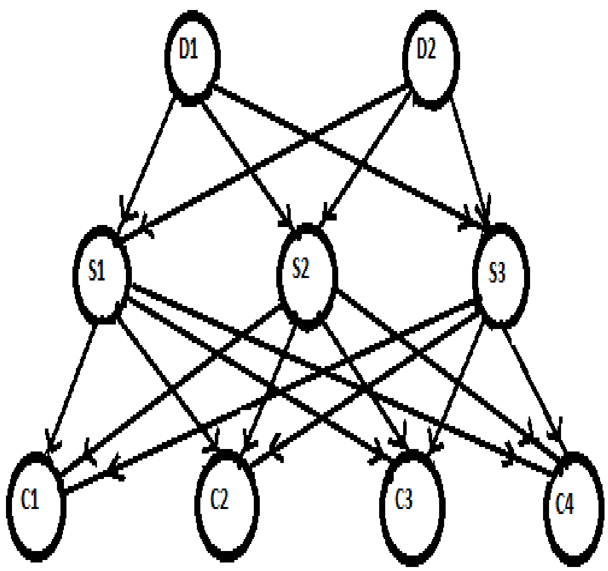

Figure 8.1: Graphical Network represents a Two-Stage transportation problem.

which collect the goods from two cities: D1 and D2 (One-Stage transportation problem). Again the goods are supplied to four cities: C1, C2, C3 and C4 (Another-Stage transportation problem) by the help of the DM. So, the goods are in and out at the warehouses S1, S2 and S3 by two-times transportation, and so it is a Two-Stage transportation problem. Again, there may be more than one objective function in a Two-Stage transportation problem which makes it a multi-objective Two-Stage transportation problem. In this situation, taking decision is not an easy task for the DM unless he or she selects the proper goals for the objective functions. So, we make a connection between the goals and the objective functions of a multi-objective Two-Stage transportation problem.

Goal Programming (GP) is a decision making tool for the MOTP. In most of the cases, a GP is used to produce a better solution of the MOTP under the assumption of certain goals. Later on, RMCGP is also introduced to improve the solution of the MOTP by a GP. In both methods, certain goals are to be chosen by the DM and the solution is obtained satisfying the optimal values of the goals. But, it is difficult for the DM to guess the possible goals of the objective functions in connection with the methodologies of GP or RMCGP.

Because of the globalization of the market, the supply and demand in a TP are not always fixed, so these are considered to be an uncertain number or, more specifically, grey numbers. A *grey number* means a number which is not known exactly. The interval grey number can be treated as continuous and discrete. It may be a discrete number within certain range or any number within a range of lower and upper limits. Again, if the number is not known precisely, but is taken from a set of numbers, then it becomes a multi-choice grey number. In a real-life TP, the capacity of supply (a) may not be a fixed number always, but there exists an upper limit ($\bar{a}$) of supply that can be utilized. Again, it is also noticed that the supply cannot be less than an amount which is the lower limit ($\underline{a}$) of supply. Then the upper and lower limits of supply describe the situation of interval grey supply ($\underline{a}, \bar{a}$) in a TP. Again, if the supply is not a fixed number and it is considered from a set of multi-choice numbers ($a^1, a^2, \ldots, a^p$), then the supply becomes multi-choice grey supply. The demands (b) at the destinations may not be fixed in advance; it also has an upper limit ($\bar{b}$) according to the market situation is concerned, Therefore, it has a lower limit ($\underline{b}$) also. In that situation, we may assign an interval grey demand ($\underline{b}, \bar{b}$) in our proposed study. Furthermore, if the possible demands are multi-choice real numbers which belong to the set of multi-choice values ($b^1, b^2, \ldots, b^q$), then it also becomes a grey demand. Here we use two types of greyness in our model, namely, interval grey number and multi-choice grey number.

Based on the above discussions, realizing the real-life situations, we allow the supply and demand constraints of a TP as grey supply and demand constraints. Again, a real-life TP has many dimensions; so to accommodate these phenomena, here we employ the multi-objective TP. Thereafter, we define the goal space for the objective functions. Then we solve a Two-Stage MOTP for optimal selection of goals from the goal space to the objective functions through a utility function for proper choice of goals and optimal solution of the MOTP. Many researchers adopted various methodologies for analyzing multi-criteria decision making problems and MOTPs. Nevertheless, to the best of our knowledge, the existing approaches are not sufficient to tackle the MOTP when the supply and demand are interval grey numbers, and to answer the question of how to select goal values corresponding to the objective functions. To tackle such a type of MOTP, we propose a new approach for solving a Two-Stage MOTP problem which will be of a great impact for the scientific community, especially, in Operational Research.

8.1.1 Problem environment

A large number of studies have been accommodated by different researchers for solving MOTP with goals through the objective functions. But, in many cases, we observe that the objective function is considered associated with the goals, and then solution procedures are developed to find solutions to the best according to decision maker's choice. In our research, we formulate a mathematical model on the MOTP which is a Two-Stage transportation problem. Here, a Two-Stage transportation problem refers to a TP in which goods are collected at some warehouses and then the goods are delivered to several destinations. So, the warehouses are the demand points for sources and supply points for destinations. Goods are transported at two Stages, i.e., for gathering and distributing, and the whole system is managed by only one decision maker. The DM wishes to maximize his benefit and to minimize the transportation costs. The data regarding the demand and the supply are not necessarily exact in TP due to the globalization of markets or other conditions like weather conditions, seasonal effects, etc. So, without loss of generality, these are taken as grey numbers in the MOTP. The grey numbers may be interval-valued numbers, multi-choice numbers, stochastic numbers, fuzzy numbers, etc. Here, we consider two types of grey numbers, namely, interval grey numbers and multi-choice grey numbers in our proposed MOTP. Another important notion for this MOTP is that if the DM wishes to maximize his profit without considering the transportation cost, then customers will be affected. In this situation, the optimal goals for the objective functions and the corresponding solutions are not specified in the literature until now. Based on all these ideas, we propose a new way to select the optimal goals for the objective functions of a regarded MOTP.

8.1.2 Theoretical background

Goal programming is a branch of Multi-Objective Optimization (MOO). It is also considered as a branch of optimization problem in multi-criteria decision analysis. It can be thought of as an extension or generalization of linear programming to handle multiple, normally conflicting objective measures. Each of these measures is given a goal or target value to be achieved. Unwanted deviations from this set of target values are then minimized through an achievement function. This can be a vector or a weighted sum, depending on the goal programming used. The mathematical model for solving a Multi-Objective Decision Making (MODM) problem using a GP can be considered in the following form:

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{K} w_i |Z^i(x) - g_i| \\ \text{subject to} \quad & x \in F, \end{aligned}$$

where F is the feasible set and w_i are the weights attached to the deviation of the achievement function. Here, $Z^i(x)$ is the i-th objective function of the i-th goal and g_i is the aspiration level of the i-th goal. Here, $|Z^i(x) - g_i|$ represents the deviation of the i-th goal.

Weighted goal programming allows for direct trade-offs among all unwanted deviational variables by placing them in a weighted, normalized single achievement function. In fact, weighted goal programming is also termed as non-preemptive goal programming in the literature. Mathematically, Weighted Goal Programming (WGP) can be displayed in the following form:

$$\begin{aligned}
\text{minimize} \quad & \sum_{i=1}^{K} w_i(d_i^+ + d_i^-) \\
\text{subject to} \quad & Z^i(x) - d_i^+ + d_i^- = g_i \quad (i = 1, 2, \ldots, K), \\
& d_i^+ \geq 0, d_i^- \geq 0 \quad (i = 1, 2, \ldots, K), \\
& x \in F,
\end{aligned}$$

where d_i^+ and d_i^- signify over and under achievements of the i-th goal, respectively.

However, the conflicts of resources and the lack of available information make it almost impossible for the DM to set the specific aspiration levels and to choose a better decision when there is a multi-choice of goals connecting with the objective functions. To tackle this situation, Chang (17) proposed the mathematical model on RMCGP to solve a MODM problem. The mathematical model of a MODM using RMCGP is prescribed as follows:

RMCGP

$$\begin{aligned}
\text{minimize} \quad & \sum_{i=1}^{K} \left[w_i(d_i^+ + d_i^-) + \alpha_i(e_i^+ + e_i^-) \right] \\
\text{subject to} \quad & Z^i(x) - d_i^+ + d_i^- = y_i \quad (i = 1, 2, \ldots, K), \\
& y_i - e_i^+ + e_i^- = g_{i,max} \text{ or } g_{i,min} \quad (i = 1, 2, \ldots, K), \\
& g_{i,min} \leq y_i \leq g_{i,max} \quad (i = 1, 2, \ldots, K), \\
& d_i^+, d_i^-, e_i^+, e_i^- \geq 0 \quad (i = 1, 2, \ldots, K), \\
& x \in F,
\end{aligned}$$

where y_i is the continuous variable associated with i-th goal which restricted between the upper ($g_{i,max}$) and lower ($g_{i,min}$) bounds, and e_i^+ and e_i^- are positive and negative deviations attached to the i-th goal of $|y_i - g_{i,max}|$ and α_i

is the weight attached to the deviation of $|y_i - g_{i,max}|$; the other variables are defined as in a WGP. To show the efficiency of the variables d_i^+; d_i^- and e_i^+; e_i^-, here we use d_i^+ and d_i^- to minimize the deviation in the objective function of RMCGP; and e_i^+ and e_i^- are introduced in the model to optimize the goals according to the type of the objective functions, namely, either being of maximization or of minimization type. Again, we formulate the model for both maximization and minimization type functions, so we consider e_i^+ and e_i^-. However, if the type of some objective function is known in advance, then one of the variables e_i^+ and e_i^- may be omitted according to the type of objective function, of maximization or minimization, respectively.

Theorem 8.1: WGP is a modification of GP for solving a MODM problem and RMCGP produces better results than GP and WGP.

Proof: The mathematical model of the GP for solving a MODM problem is considered as follows:

GP

$$\text{minimize} \quad \sum_{i=1}^{K} w_i |Z^i(x) - g_i| \tag{8.1}$$

$$\text{subject to} \quad x \in F. \tag{8.2}$$

Let us take $d_i^+ = Z^i(x) - g_i$ if $Z^i(x) \geq g_i$, and $d_i^+ = 0$ otherwise ($i = 1, 2, \ldots, K$). Also, we put $d_i^- = g_i - Z^i(x)$ if $Z^i(x) \leq g_i$, and $d_i^- = 0$ otherwise. Then, $Z^i(x) - g_i = d_i^+ - d_i^-$, which implies that $Z^i(x) - d_i^+ + d_i^- = g_i$. Furthermore, $|Z^i(x) - g_i| = d_i^+ + d_i^-$.

Thus, the GP model reduces to the model WGP as follows:

WGP

$$\begin{aligned}
\text{minimize} \quad & \sum_{i=1}^{K} w_i (d_i^+ + d_i^-) \\
\text{subject to} \quad & Z^i(x) - d_i^+ + d_i^- = g_i \quad (i = 1, 2, \ldots, K), \\
& d_i^+ \geq 0, d_i^- \geq 0 \quad (i = 1, 2, \ldots, K), \\
& x \in F.
\end{aligned}$$

According to the above discussion, we see that the mathematical model GP is a function of decision variables along with the weights and goals, whereas the objective function in the mathematical model of WGP consists of the variables goal deviations (d_i^+ and d_i^-). Both the models GP and WGP produce the same result, but the WGP model is easier to handle than GP as its objective function contains a minimum number of variables when compared to the GP model.

If the goal (g_i) for i-th objective function is not prescribed by a fixed value but it is considered as a range $[g_{i,min}, g_{i,max}]$, then for a better solution of a maximization type objective function g_i should attain the maximum value of the range, and for a minimization type objective function it should take its minimum value of the specified range.

In that case, we introduce a new variable y_i in model WGP as $Z^i(x) - d_i^+ + d_i^- = y_i$, and two deviation variables such as e_i^+ and e_i^- are similar to d_i^+ and d_i^-, respectively, along with the constraint $y_i - e_i^+ + e_i^- = g_{i,max}$ or $g_{i,min}$. Then, the objective function is changed into the following form as:

minimize $[\sum_{i=1}^{K} w_i(d_i^+ + d_i^-) + \sum_{i=1}^{K} \alpha_i(e_i^+ + e_i^-)]$,

where α_i are weights corresponding to the goal deviations. Using this objective function we formulate the model RMCGP. The model RMCGP minimizes both the deviations $(d_i^+ + d_i^-)$ and $(e_i^+ + e_i^-)$, whereas the problem GP only minimizes the deviation of the objective function value, i.e., $(d_i^+ + d_i^-)$. Thus, in the course of time of minimizing the objective function in RMCGP model, the second part of the objective function $\sum_{i=1}^{K} \alpha_i(e_i^+ + e_i^-)$ is also minimized. This implies that the value of y_i tends to $g_{i,max}$ for a maximizing type objective function, and y_i tends to $g_{i,min}$ for a minimizing type objective function. In WGP or GP, the goal deviations are only minimized which does not consider the type of objective function. Here, the additional variables e_i^+ and e_i^- tackle the situation which minimizes the deviations according to the nature of the objective function. Hence, we conclude that RMCGP produces a better result than WGP or GP models.

Again, in the *RMCGP* model, if we treat the goal deviations e_i^+ and e_i^- as 0, then, it takes the form of a WGP which is a modification of the model GP. So, it is clear that the solution of RMCGP is better than the solution of WGP and GP. Hence, the arguments evince the proof of the theorem. □

Considering real-world practical problems, we present the situation where supply and demand are taken as grey numbers. Basically, grey numbers are used to represent the situation where the supply and demand are not known exactly. Sometimes supply or demand may be a number among a set of values which indicates that supply or demand follow multi-choice values. Again, supply or demand may be considered as interval valued. So, different types of uncertain supply and demand situations may occur in a MOTP, which are considered as grey supply and grey demand in our proposed model.

Utility function

The term "*utility*" is measured (Berger (4), Blanchard and Fischer (9)) to be correlative as "Desire" or "Want". It has been argued already that desire cannot be measured directly, but only indirectly, by the outward phenomena in which the context is presented.

Definition 8.1.1 *A utility function describes a function $U : X \longrightarrow \mathbb{R}$ which assigns a real number in such a way that it captures the DM's preference over the desired goals of the objectives, where X is the set of feasible points.*

Satisfaction of goals does not always mean a better selection of goals for the objective function as well as the solution for the objective functions with respect to the goals. There may occur situations to compare the solutions corresponding to the goals. Here, to introduce a better goal, we include a utility function, whose value indicates to optimize the proposed problem through optimal selection goals.

8.2 Mathematical model

Generally, transportation problem is a type of decision making problem in which the main objective is to minimize the transportation cost and is defined subsequently:

Model 8.1

$$\text{minimize} \quad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij} \tag{8.3}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \leq a_i \quad (i = 1, 2, \ldots, m), \tag{8.4}$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j \quad (j = 1, 2, \ldots, n), \tag{8.5}$$

$$x_{ij} \geq 0 \ \ \forall \ \ i \text{ and } \ j, \tag{8.6}$$

where x_{ij} $(i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$ is the decision variable and C_{ij} $(i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n)$ is the transportation cost per unit commodity from the i-th origin to the j-th destination. Here, a_i $(i = 1, 2, \ldots, m)$ and b_j $(j = 1, 2, \ldots, n)$ are availability and demand at the i-th origin and the j-th destination, respectively, and the feasibility condition is $\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} b_j$.
The mathematical model of the MOTP can be taken as follows:

Model 8.2

$$\text{minimize/maximize} \quad Z^s = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}^s x_{ij} \quad (s = 1, 2, \ldots, K) \tag{8.7}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \leq a_i \quad (i = 1, 2, \ldots, m), \tag{8.8}$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j \quad (j = 1, 2, \ldots, n), \tag{8.9}$$

$$x_{ij} \geq 0 \quad \forall \ \ i \ \text{ and } \ j. \tag{8.10}$$

The objective functions in MOTP are conflicting to each other, so even if sometimes they are of the same type, i.e., two objective functions are of minimization type or maximization type, they cannot be summed up to a single objective function. For example, if the transportation cost and production cost are considered as two objective functions in MOTP, then both are minimization type but they cannot be combined to make a single objective function as the costs are dealt by two decision makers, namely, by the customer and

by the seller, respectively. Again, it is true that a maximization problem can be reduced easily to a minimization problem. But this is not what we are considering, because the reduction of maximization to minimization problem which causes the goals from positive to negative numbers. In real-life situations, the negative goals are not considered as they are meaningless. Based on these phenomena, we do not convert the maximization problem into the minimization in our proposed model.
According to real-world situations, considering the ambiguity that exists in the supply and demand parameters, we incorporate grey supply and grey demand in our study and we depict the MOTP in Model 8.3.

Model 8.3

$$\text{minimize/maximize} \quad Z^s = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^s x_{ij} \ (s = 1, 2, \ldots, K) \tag{8.11}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \leq \hat{a}_i \quad (i = 1, 2, \ldots, m), \tag{8.12}$$

$$\sum_{i=1}^{m} x_{ij} \geq \hat{b}_j \quad (j = 1, 2, \ldots, n), \tag{8.13}$$

$$x_{ij} \geq 0 \quad \forall \ i \ \text{ and } \ j, \tag{8.14}$$

where $\hat{a}_i$ and $\hat{b}_j$ are the grey supply and demand, respectively. In our proposed study we consider the grey supply $\hat{a}_i$ as anyone of interval grey numbers $\hat{a}_i = [\underline{a}_i, \bar{a}_i]$ or multi-choice grey numbers $\hat{a}_i = (a_i^1, a_i^2, \ldots, a_i^p)$; here, p is the number of possible multi-choice supply at i-th origin. Again, demand is also considered by interval grey numbers $\hat{b}_j = [\underline{b}_j, \bar{b_j}]$ $(j = 1, 2, \ldots, n)$ or multi-choice grey numbers $\hat{b}_j = (b_j^1, b_j^2, \ldots, b_j^q)$; here, q is the number of possible multi-choice demand at j-th destination. In fact, $\underline{a}_i$ and $\bar{a}_i$ are lower and upper bounds of grey supply at the i-th origin; $\underline{b}_j$, $\bar{b}_j$ are the bounds of grey demand at the j-th destination and $\sum_{i=1}^{m} \bar{a}_i \geq \sum_{j=1}^{n} \underline{b}_j$ is the feasibility condition.
We consider the following assumptions to formulate mathematical model (for Model 8.4) of a Two-Stage multi-objective TP as:

- Consider m sources D_i $(i = 1, 2, \ldots, m)$ with interval grey supply $[\underline{a}_i, \bar{a}_i]$ $(i = 1, 2, \ldots, m)$ and p destinations $C_{j'}$ $(j' = 1, 2, \ldots, p)$ with interval grey demand $[\underline{b}_{j'}, \bar{b_{j'}}]$ $(j' = 1, 2, \ldots, p)$ and there are no direct link between sources and destinations.

- Assume n nodes S_j $(j = 1, 2, \ldots, n)$ with equal interval grey supply and demand $[\underline{d}_j, \bar{d}_j]$ $(j = 1, 2, \ldots, n)$ to interact between sources D_i and destinations $C_{j'}$; this has the capacity of taking goods from sources D_i and delivering goods to the destinations $C_{j'}$.

- In our proposed model, we consider that there are no goods left after the completion of a particular time of transportation in the nodes S_j.

- We treat T objective functions for One-Stage transportation, i.e., transportation from sources D_i to destination S_j; y_{ij} is taken as decision variable and P^t_{ij} is the cost parameter for the t-th objective function.

- We describe K objective functions for Another-Stage transportation, i.e., transportation from sources S_j to destination $C_{j'}$; here, $x_{i'j'}$ is taken as a decision variable and $C^k_{i'j'}$ is the penalty parameter for the k-th objective function. In fact, the penalty parameter may be the transportation cost per unit item, producing cost per unit item of goods, selling cost per unit item of goods, etc.

Then, our mathematical model can be designed in the following form:

Model 8.4

$$\text{minimize/maximize} \qquad W^t = \sum_{i=1}^{m} \sum_{j=1}^{n} P^t_{ij} y_{ij} \quad (t = 1, 2, \ldots, T) \tag{8.15}$$

$$\text{minimize/maximize} \qquad Z^k = \sum_{i'=1}^{n} \sum_{j'=1}^{p} C^k_{i'j'} x_{i'j'} \quad (k = 1, 2, \ldots, K) \tag{8.16}$$

$$\text{subject to} \quad \sum_{j=1}^{n} y_{ij} \leq \hat{a}_i \quad (i = 1, 2, \ldots, m), \tag{8.17}$$

$$\sum_{i=1}^{m} y_{ij} \geq \hat{d}_j \quad (j = 1, 2, \ldots, n), \tag{8.18}$$

$$\sum_{j'=1}^{p} x_{i'j'} \leq \hat{d}_{i'} \quad (i' = 1, 2, \ldots, n), \tag{8.19}$$

$$\sum_{i'=1}^{n} x_{i'j'} \geq \hat{b}_{j'} \quad (j' = 1, 2, \ldots, p), \tag{8.20}$$

$$\sum_{i=1}^{m} y_{il} = \sum_{j'=1}^{p} x_{lj'} \quad (l = 1, 2, \ldots, n), \tag{8.21}$$

$$y_{ij} \geq 0 \ \ \forall\, i,\ j;\ x_{i'j'} \geq 0 \ \ \forall\, i',\ j'. \tag{8.22}$$

Here, $\hat{a}_i$, $\hat{d}_j$, $\hat{d}_{i'}$ and $\hat{b}_{j'}$ are the grey supply and demand parameters as defined in Model 8.3. Also, $\sum_{i=1}^{m} \bar{a}_i \geq \sum_{j'=1}^{p} \underline{b}_{j'}$ is the feasibility condition. Although this condition can be made according to the choice of DM, we consider the possibly large feasible region for the multi-objective Two-Stage TP.

In Model 8.4, constraints (8.17) and (8.18) represent availability and demand at the origins and destinations of One-Stage transportation, respectively. Again, constraints (8.19) and (8.20) represent availability and demand at the origin and destinations of Another-Stage transportation, respectively. Finally, constraints (8.21) represent that the amount of goods stored at the destination of One-Stage transportation will all be distributed to the designations of Another-Stage of transportation.

The objective functions W^t $(t = 1, 2, \ldots, T)$ are of conflicting nature which is considered at the first stage of Model 8.4; so these functions are not summed up, even if they are of same type. A similar concept is adopted for the objective functions at the subsequent stage of Model 8.4. Also, there are different agents to pay the cost at different stages of transportation. Due to this fact, we consider the objective functions separately in Model 8.4.

Usually, the DM determines the goals into the objective functions and, thereafter, the solution is obtained according to the best fit of goal. Here, our main interest is to constitute the way of assignment of goals to the objective func-

tions. When there are more than one objective function, conflicting in nature, in many cases optimal solutions of the objective functions occur at different points. So, consideration of specific goals and the corresponding solution are a complex task for the DM.
Now, some useful definitions regarding our proposed study in the goal space are presented. The following definitions are given with respect to s number of objective functions.

Definition 8.2.1 *(**Feasible goal**): Let Z^t be the t-th objective function of the MOTP. Then the feasible goal for the objective function Z^t refers to all the possible values of the objective function Z^t which is an interval $I^t = [\min\ Z^t, \max\ Z^t]$. Here,* $\min\ Z^t$ *and* $\max\ Z^t$ *are obtained under supply and demand constraints of the MOTP along with the nonnegativity conditions.*

Definition 8.2.2 *(**Feasible goal region**): Let I^t be the feasibility of goal for t-th objective function of the MOTP. Then the subset $I^1 \times I^2 \times \ldots \times I^s$ of $\mathbb{R}^s$ is the feasibility goal region for the MOTP with respect to s objective functions.*

Definition 8.2.3 *(**Optimum goal region**): In the usual way of the MOTP, each objective function is either of maximization type or of minimization type. Each can be solved with respect to the constraints separately to get its individual optimal solution. Let $X_1^*, X_2^*, \ldots, X_s^*$ be the respective ideal solutions of s number of objective functions. Evaluating all these objective functions at all the ideal solutions, we construct a pay-off matrix of format $s \times s$ as stated in Table 8.1.*

Table 8.1: *Pay-off Matrix for ideal solutions of the objective functions.*

$$\begin{bmatrix} Z^1(X_1^*) & Z^1(X_2^*) & \cdots & Z^1(X_s^*) \\ Z^2(X_1^*) & Z^2(X_2^*) & \cdots & Z^2(X_s^*) \\ Z^3(X_1^*) & Z^3(X_2^*) & \cdots & Z^3(X_s^*) \\ \vdots & \vdots & \vdots & \vdots \\ Z^s(X_1^*) & Z^s(X_2^*) & \cdots & Z^s(X_s^*) \end{bmatrix}.$$

Then, we find $I_o^i = [\min_j \; Z^i(X_j^*), \max_j \; Z^i(X_j^*)] \;\; i, j = 1, 2, \ldots, s$. *The subset* $I_o^1 \times I_o^2 \times \ldots \times I_o^n$ *of* $\mathbb{R}^s$ *is the optimum feasibility goal region for the MOTP with respect to the given objective functions.*

Again, presence of grey numbers in the constraints of Model 8.4 makes the model non-deterministic, and we use a reduction procedure to make the MOTP becomes deterministic one by the following rule.

8.2.1 Reduction of grey supply and demand parameters to real parameters

In most of the real-life cases, it is seen that, if the number of units of goods transported to some destination is of a rather small amount already, then the transportation cost may even be around a minimal value. So, the demand in the classical TP could be considered with the minimum value among all the uncertain possibilities through grey numbers for an optimum solution. However, in the situation of minimizing transportation cost and maximizing profit through transporting the goods, it is not predictable how much amount of goods is need to be transported for an optimum solution, it may be any one among the possible values of the grey demands. Again, if the supply amount of goods is of a high value, then total transportation cost reduces as the customers purchase goods from those origins in which the transportation cost per unit item is about its minimum. But, it is not often useful to provide the satisfactory level of supply at all the origins; so we make a reduction procedure in which the optimum solution is obtained through the optimum selection of grey supply, according to the goals of the objective functions.
If the interval grey supply is taken as $[\underline{a}_i, \bar{a}_i]$, then it reduces to a real number by considering the following reduction technique:
$a_i^{'} = \underline{a}_i(1-\lambda) + \lambda\bar{a}_i$, where $0 \leq \lambda \leq 1$. (8.A)
Similarly, the interval grey demand $[\underline{b}_j, \bar{b}_j]$ can also be reduced to a real number indicated in below:
$b_j^{'} = \underline{b}_j(1-\lambda) + \lambda\bar{b}_j$, where $0 \leq \lambda \leq 1$. (8.B)
Again, λ is considered as a real free parameter, whose value determines the

possible supply and demand to the DM. The value of λ is obtained from the solution according to the best goals assigned about the optimal demand and supply to the DM. Again, if the supply or demand quantity is a multi-choice grey number, then we use a reduction procedure by employing binary variables in the following way.

If there are two possible values of a_i, namely, a_i^1 and a_i^2, then using one binary variable we reduce the multi-choice grey number to a crisp number in the subsequent manner:
$a_i^{'} = a_i^1 z_1 + a_i^2(1 - z_1)$, where $z_1 = 0$ or 1.
If there are three possible values of a_i, namely, a_i^1, a_i^2 and a_i^3 then we use two binary variables to reduce the greyness in the following manner:
$a_i^{'} = a_i^1 z_1 z_2 + a_i^2(1 - z_1)z_2 + a_i^3(1 - z_2)z_1$, where $z_1, z_2 = 0$ or 1 and $z_1 + z_2 \geq 1$.
Furthermore, for four possible values a_i^1, a_i^2, a_i^3 and a_i^4, of a_i, we use the following reduction formulae:
$a_i^{'} = a_i^1 z_1 z_2 + a_i^2(1 - z_1)z_2 + a_i^3(1 - z_2)z_1 + a_i^4(1 - z_1)(1 - z_2)$, where $z_1, z_2 = 0$ or 1.
One issue to be noted is that, the number of binary variables is r, when $2^{r-1} < p \leq 2^r$, where p is the number of multi-choice numbers in grey supply $(a_i^1, a_i^2, \ldots, a_i^p)$. For more details about a general transformation technique, we refer to Roy et al. (135). In a similar way, the multi-choice grey uncertainty can be reduced for demand parameters, too.

8.2.2 Algorithm for selection of goals to the objective functions in Two-Stage transportation problem

The following steps are proposed for selection of goals to the objective functions in our Two-Stage MOTP with interval grey demand and supply:

Step 1: First, we reduce the constraints with grey numbers to deterministic constraints with real numbers by the help of procedure described in Subsection 8.2.1.

Step 2: We solve each of the objective functions corresponding to an One-Stage TP under the restriction of proposed problem. As a result, we obtain the solutions, $Y_1^*, Y_2^*, \ldots, Y_T^*$. Again, we solve each of the objective functions corresponding to an Another-Stage TP under the restriction of our proposed problem and we calculate the solutions, $X_1^*, X_2^*, \ldots, X_K^*$. These solutions are called as ideal solutions; often, these are known as Pareto optimal solutions.

Step 3: Using the ideal solutions obtained in Step 2, we formulate pay-off matrices, one is of format $T \times T$, and the other one is of format $K \times K$, according to Definition 5.3.

Step 4: Obtaining $I_o^i = [\min_j\ W^i(Y_j^*), \max_j\ W^i(Y_j^*)] \quad (i = 1, 2, \ldots, T)$, we then find the optimum feasibility goal region $I_o^1 \times I_o^2 \times \ldots \times I_o^T$ of $\mathbb{R}^T$ for the One-Stage MOTP. In a similar way, we define $G_o^i = [\min_j\ Z^i(X_j^*), \max_j\ Z^i(X_j^*)]$ $(i = 1, 2, \ldots, K)$, and derive the optimum feasibility goal region $G_o^1 \times G_o^2 \times \ldots \times G_o^K$ in $\mathbb{R}^K$ for the Another-Stage MOTP.

Step 5: We consider p_t as the goal value to each of the objective functions W^t and g_k as the goal value to each of the objective functions Z^k. Based on the goal values, we construct a utility function $E = E(p_1, p_2, \ldots, p_T; g_1, g_2, \ldots, g_K)$ and, then, we optimize the value of E within the feasible goal regions $I_o^1 \times I_o^2 \times \ldots \times I_o^T$ in $\mathbb{R}^T$ and $G_o^1 \times G_o^2 \times \ldots \times G_o^K$ in $\mathbb{R}^K$. If $p_1^*, p_2^*, \ldots, p_T^*; g_1^*, g_2^*, \ldots, g_K^*$ are the optimal solutions of E, then these are the optimal goals to the objective function of the MOTP.

Step 6: We formulate a single objective function whose solution provides the optimal goals and which is also yields the solution of the MOTP.

Step 7: Based on the results of optimal goals and possible optimized values of objective functions, we design an RMCGP model to obtain the optimal solution.

Step 8: Stop. **Construction of utility function**

The utility function regarding all of the goals can be formed according to the choice of the DM. Here, we propose the function E of the goals in such a way that the value of E approaches to zero when the function of the MOTP is minimized or maximized according to the choice of the DM. Let us define

$E = \sum_{t=1}^{T} \alpha_t u_t(p_t) + \sum_{i=1}^{K} w_i u_i(g_i).$ (8.C)

Here, α_t $(t = 1, 2, \ldots, T)$ and w_i $(i = 1, 2, \ldots, K)$ are the weights. Weights are mainly used for the preference of the respective objective functions in the MOTP. Basically, in MOTP the weights are chosen by the DM, which reflects the priority level of respective objective functions. In most of the cases, weights are taken in *normal form*, i.e., the sum of all weights is considered as 1. Here, we also present the weights in normal form by considering the percentage of preferences corresponding to the objective functions in a unit scale. Furthermore, u_t $(t = 1, 2, \ldots, T)$ is the utility function for the t-th objective function with respect to the goal p_t $(t = 1, 2, \ldots, T)$, and u_i $(i = 1, 2, \ldots, K)$ is the utility function for the i-th objective function with respect to goal g_i.

The objective function is considered by two types, namely, maximization type and minimization type. The utility function is regarded in such a way that the goal of a maximization type objective function is increased and goal value corresponding to a minimization type objective function is minimized. So the utility function u_i $(i = 1, 2, \ldots, K)$ is defined as follows:

$$u_i(g_i) = \begin{cases} \frac{g_i - g_i^l}{g_i^u - g_i^l}, & \text{if objective function is of minimization-type,} \\ \frac{g_i^u - g_i}{g_i^u - g_i^l}, & \text{if objective function is of maximization-type.} \end{cases} \quad (8.D)$$

Again, the utility function $u_t(p_t)$ $(t = 1, 2, \ldots, T)$ is defined in a very similar way as the function $u_i(g_i)$ has been introduced.

8.3 Numerical example

Now, we present a real-life Two-Stage TP to show the effectiveness of our proposed study. A reputed company wishes to select proper goals for objective functions and the corresponding solution. The company has three stores: S1, S2 and S3, and it purchases goods from two market cities: D1 and D2. Thereafter, the company supplies the goods to four cities: C1, C2, C3 and C4. During the purchasing of items from the marketed cities D1 and D2, the company mainly considers two criteria: one is transportation cost, and other is purchasing cost of goods. Here, the transportation of goods from D1 and D2 to the warehouses S1, S2 and S3 is a One-Stage TP. In this situation, the company would like to minimize both the value of the objective functions. Again, during the selling time, the company tackles the transportation of goods with the motivation that it wishes to minimize the transportation cost and to maximize the profit. Here, the transportation of goods from S1, S2 and S3 to C1, C2, C3 and C4 is considered as an Another-Stage TP. Indeed, the company is interested to proceed in such a way that the stored amount will all be delivered. Then, the capacities of supply at D1 and D2 and the intake capacity at S1, S2 and S3 are chosen according to the requirement of the cities C1, C2, C3 and C4, which are considered as interval grey numbers. The supplied data of the problems are shown in Tables 8.2 and 8.3.

Tables 8.2: Transportation cost (in $) for transporting the amount of goods per item.

	S1	S2	S3
D1	12	10	8
D2	11	9	9

Tables 8.3: Purchasing cost (in $) for amounting of goods per item.

	S1	S2	S3
D1	25	20	20
D2	30	25	22

Here, we assume that the One-Stage transportation system is maintained by the supplier and, therefore, the transportation cost is treated by the supplier, and the purchasing cost is considered by the buyer. So, we cannot add them to a single objective function, because the objective functions are of conflicting nature. Due to the globalization of the market, the intake amount of goods to the center D1 is followed by an interval grey number which is the [1500, 1600] unit, and the capacity of supply items of D2 is also assessed as an interval grey number, the [1550, 1750] unit. In the TP, there are certain demands at the destinations S1, S2 and S3. Demand is always varying through the market scenario, so it is unpredictable. We consider that the intake amount of goods will be delivered completely for the destinations S1, S2 and S3. Let the capacity of supply at S1, S2 and S3 be [900, 1000], [1100, 1150] and [1000, 1100], respectively. Due to fluctuation in the market, the demands at the cities C1, C2, C3 and C4 are considered by interval grey numbers. The transportation has been made in Two-Stage form, namely, D1 and D2 to S1, S2 and S3 and, then, S1, S2 and S3 to C1, C2, C3 and C4. Here, the decision maker tries to minimize the Two-Stage transportation cost and would like to maximize the profit for distributing goods from S1, S2 and S3 to C1, C2, C3 and C4. For this case, the DM assigns the profit for transferring goods which are given in Table 8.4, and the transportation costs from S1, S2 and S3 to C1, C2, C3 and C4 are presented in Table 8.5.

Table 8.4: Profit (in $) of selling goods per amount.

	C1	C2	C3	C4
S1	4.5	5.0	6.0	6.5
S2	4.0	5.5	5.5	4.5
S3	5.0	4.5	5.0	5.5

Table 8.5: Transportation cost (in $) per amount of goods for selling time.

	C1	C2	C3	C4
S1	8.0	9.0	6.5	7.5
S2	10.0	8.0	5.0	6.5
S3	6.0	9.5	6.0	7.0

Without loss of generality, we assume that the demands at the destinations C1, C2, C3 and C4 are grey numbers and they are taken as [750, 800], [850, 900], (600, 650, 700) and (690, 700, 750, 850). Here, we mainly concentrate on the following approach:

- We fix the optimum goals for minimizing transportation cost and maximizing the profit.
- The optimum amount of goods will run over the stores S1, S2 and S3.
- We find optimum solution for the proposed goals for the MOTP.

We formulate the mathematical model in which W^1 and W^2 are the objective functions of minimization type for One-Stage transportation (i.e., for keeping the goods at the stores S1, S2 and S3), and Z^1 and Z^2 are objective functions of maximization and minimization type, respectively, for Another-Stage transportation (i.e., for supplying the goods to the destination C1, C2, C3 and C4). Obviously, this problem is a multi-objective Two-Stage grey transportation problem. So, based on the supplied data, we formulate the following mathematical model as:

Model 8.5

$$\begin{aligned}
\text{minimize} \quad & W^1 = 12y_{11} + 11y_{12} + 10y_{13} + 9y_{21} + 8y_{22} + 9y_{23} && (8.23)\\
\text{minimize} \quad & W^2 = 25y_{11} + 30y_{12} + 20y_{13} + 25y_{21} + 20y_{22} + 22y_{23} && (8.24)\\
\text{maximize} \quad & Z^1 = 4.5x_{11} + 5.0x_{12} + 6.0x_{13} + 6.5x_{14} + 4.0x_{21} + 5.5x_{22} \\
& +5.5x_{23} + 4.5x_{24} + 5.0x_{31} + 4.5x_{32} + 5.0x_{33} + 5.5x_{34} && (8.25)\\
\text{minimize} \quad & Z^2 = 8x_{11} + 9x_{12} + 6.5x_{13} + 7.5x_{14} + 10x_{21} + 8.0x_{22} \\
& +5.0x_{23} + 6.5x_{24} + 6x_{31} + 9.5x_{32} + 6.0x_{33} + 7.0x_{34} && (8.26)\\
\text{subject to} \quad & y_{11} + y_{12} + y_{13} \leq [1500, 1600], && (8.27)\\
& y_{21} + y_{22} + y_{23} \leq [1550, 1750], && (8.28)\\
& y_{11} + y_{21} \geq [900, 1000], && (8.29)\\
& y_{12} + y_{22} \geq [1100, 1150], && (8.30)\\
& y_{13} + y_{23} \geq [1000, 1100], && (8.31)\\
& x_{11} + x_{12} + x_{13} + x_{14} \leq [900, 1000], && (8.32)\\
& x_{21} + x_{22} + x_{23} + x_{24} \leq [1100, 1150], && (8.33)\\
& x_{31} + x_{32} + x_{33} + x_{34} \leq [1000, 1100], && (8.34)\\
& x_{11} + x_{21} + x_{31} \geq [750, 800], && (8.35)\\
& x_{12} + x_{22} + x_{32} \geq [850, 900], && (8.36)\\
& x_{13} + x_{23} + x_{33} \geq (600 \text{ or } 650 \text{ or } 700), && (8.37)\\
& x_{14} + x_{24} + x_{34} \geq (690 \text{ or } 700 \text{ or } 750 \text{ or } 850), && (8.38)
\end{aligned}$$

$$y_{11} + y_{21} = x_{11} + x_{12} + x_{13} + x_{14}, \tag{8.39}$$

$$y_{12} + y_{22} = x_{21} + x_{22} + x_{23} + x_{24}, \tag{8.40}$$

$$y_{13} + y_{23} = x_{31} + x_{32} + x_{33} + x_{34}, \tag{8.41}$$

$$x_{i'j'} \geq 0,\ \ y_{ij} \geq 0\ \forall\ (i', j = 1, 2, 3; i = 1, 2; j' = 1, 2, 3, 4). \tag{8.42}$$

To solve Model 8.5, using the reduction procedure of grey numbers to real numbers presented in Subsection 8.2.1, we reduce the constraints (8.27)-(8.38), which contain grey numbers, to deterministic constraints as follows:

$$y_{11} + y_{12} + y_{13} \leq (1 - a_1)1500 + a_1 1600, \tag{8.43}$$

$$y_{21} + y_{22} + y_{23} \leq (1 - a_2)1550 + a_2 1750, \tag{8.44}$$

$$y_{11} + y_{21} \geq (1 - a_3)900 + a_3 1000, \tag{8.45}$$

$$y_{12} + y_{22} \geq (1 - a_4)1100 + a_4 1150, \tag{8.46}$$

$$y_{13} + y_{23} \geq (1 - a_5)1000 + a_5 1100, \tag{8.47}$$

$$x_{11} + x_{12} + x_{13} + x_{14} \leq (1 - b_1)900 + b_1 1000, \tag{8.48}$$

$$x_{21} + x_{22} + x_{23} + x_{24} \leq (1 - b_2)1100 + b_2 1150, \tag{8.49}$$

$$x_{31} + x_{32} + x_{33} + x_{34} \leq (1 - b_3)1000 + b_3 1100, \tag{8.50}$$

$$x_{11} + x_{21} + x_{31} \geq (1 - b_4)750 + b_4 800, \tag{8.51}$$

$$x_{12} + x_{22} + x_{32} \geq (1 - b_5)850 + b_5 900, \tag{8.52}$$

$$x_{13} + x_{23} + x_{33} \geq 600 z_1 z_2 + 650(1 - z_1) z_2 + 700(1 - z_2) z_1, \tag{8.53}$$

$$\begin{aligned} x_{14} + x_{24} + x_{34} \geq\ & 690 z_3 z_4 + 700(1 - z_3) z_4 + 750 z_3 (1 - z_4) \\ & + 850(1 - z_3)(1 - z_4), \end{aligned} \tag{8.54}$$

$$z_1 + z_2 \geq 1\ \ (z_1,\ z_2,\ z_3,\ z_4 = 0 \text{ or } 1), \tag{8.55}$$

$$0 \leq a_r \leq 1,\ 0 \leq b_s \leq 1\ \ (r = 1, 2 \ldots, 5; s = 1, 2, \ldots, 5). \tag{8.56}$$

Now, we minimize the objective functions W^1 and W^2 independently with respect to the constraints (8.39)-(8.56), and we construct the pay-off matrix (using Definition 8.2.3) which is shown in Table 8.6.

Table 8.6: Pay-off Matrix for W^1 and W^2.

	W^1	W^2
Y_1^*	28050	30700
Y_2^*	73250	68500

Table 8.7 represents the pay-off matrix (using Definition 8.2.3) which is designed by solving each objective functions Z^1 and Z^2 with respect to the same constraints (8.39)-(8.56).

Table 8.7: Pay-off Matrix for Z^1 and Z^2.

	Z^1	Z^2
X_1^*	18400	14400
X_2^*	22550	19075

Using the pay-off matrices from Tables 8.6 and 8.7 and also using equations $(8.C)$ and $(8.D)$, we formulate Model 8.6 with a utility function as follows:
Model 8.6

$$\begin{aligned}
\text{minimize} \quad & 0.25u_1 + 0.25u_2 + 0.25u_3 + 0.25u_4 \\
\text{subject to} \quad & u_1 = \frac{W^1 - 28050}{30700 - 28050}; u_2 = \frac{W^2 - 68500}{73250 - 68500}; \\
& u_3 = \frac{18400 - Z^1}{18400 - 14400}; u_4 = \frac{Z^2 - 22500}{22500 - 19075}; \\
& 0 \le u_i \le 1 \quad (i = 1, 2, 3, 4); \\
\text{and} \quad & \text{the constraints } (8.39) - (8.56).
\end{aligned}$$

Solving Model 8.6, we derive the optimal value of each objective function as follows:
$W^1 = 30700$, $W^2 = 72583.3$; $Z^1 = 16800$; $Z^2 = 20600$. These solutions provide the following optimal goal regions: [28050, 30700] for W^1, [68500, 72583.3] for W^2, [16800, 18400] for Z^1, and [19075, 20600] for Z^2. Referring to the optimal goal regions in RMCGP model, we construct the following model as:

Model 8.7

$$
\begin{aligned}
\text{minimize} \quad & 0.25(d_1^+ + d_1^-) + 0.25(d_2^+ + d_2^-) + 0.25(d_3^+ + d_3^-) + 0.25(d_4^+ + d_4^-) \\
& +0.25(e_1^+ + e_1^-) + 0.25(e_2^+ + e_2^-) + 0.25(e_3^+ + e_3^-) + 0.25(e_4^+ + e_4^-) \\
\text{subject to} \quad & W^1 - d_1^+ + d_1^- = y_1;\ y_1 - e_1^+ + e_1^- = 28050; \\
& W^2 - d_2^+ + d_2^- = y_2;\ y_2 - e_2^+ + e_2^- = 68500; \\
& Z^1 - d_3^+ + d_3^- = y_3;\ y_3 - e_3^+ + e_3^- = 18400; \\
& Z^2 - d_4^+ + d_4^- = y_4;\ y_4 - e_4^+ + e_4^- = 19075; \\
& 28050 \leq y_1 \leq 30700;\ 68500 \leq y_2 \leq 72583.3; \\
& 16800 \leq y_3 \leq 18400;\ 19075 \leq y_4 \leq 20600; \\
& d_i^+ \geq 0,\ d_i^- \geq 0,\ e_i^+ \geq 0,\ e_i^- \geq 0 \ \ (i = 1, 2, 3, 4); \\
\text{and} \quad & \text{the constraints } (8.39) - (8.56).
\end{aligned}
$$

Finally, we obtain the optimal solution by solving Model 8.7 as follows:
$W^1 = 30700;\ W^2 = 68500;\ Z^1 = 16800;\ Z^2 = 20600.$
The values of the decision variables are
$y_{11} = 500; y_{12} = 1100; y_{13} = 0; y_{21} = 400; y_{22} = 0; y_{23} = 1000;$ and
$x_{11} = 0; x_{12} = 0; x_{13} = 450; x_{14} = 450; x_{21} = 0; x_{22} = 850;$
$x_{23} = 250; x_{24} = 0; x_{31} = 750; x_{32} = 0; x_{33} = 0; x_{34} = 250.$
Again, the amount of goods are stocked or delivered in or from the warehouses S1, S2 and S3 are 900, 1100 and 1000 units, respectively. The amount of goods which are purchased from sources D1 and D2 are 1600 units and 1400 units, respectively. The amount of goods that are sold to the destinations C1, C2, C3 and C4 are 750 units, 850 units, 700 units and 700 units, respectively.

8.4 Sensitivity analysis

RMCGP is a technique to solve the MOTP having the goals related to each of the objective functions. Meanwhile the main drawback of a RMCGP model is about how the DM would select the possible goals of the objective functions. In real-life problems, if the goal values of the objective functions are not defined properly, then the RMCGP method will have failed to produce the best result. In this study, we describe the scenarios of a Two-Stage TP and select the goals to each of the objective functions. In the meantime, we accommodate our situation by removing the drawback of RMCGP for selecting the goals by proposing a new algorithm. With the proposed algorithm, we first determine the optimal goal values for the objective functions; then we use RMCGP to find a better compromise solution. Following the proposed example, we select the goal arbitrarily and if the selected goal does not lie in the optimal goal region, then we cannot get any feasible solution. As an example, if the DM needs a solution with the following interval goals: [22000, 24000] for W^1, [60450, 62700] for W^2, [18700, 20000] for Z^1, and [13710, 16050] for Z^2; then the RMCGP fails to produce a feasible solution. So, RMCGP does not provide any definite conclusion for selecting the interval goals of the choices of the DM. Under this circumstance, our proposed methodology helps to seek out the optimal goal region as well as the optimal solution of the MOTP. Again, one issue should be noticed, namely, that the solutions and optimal goal regions depend on the choice of weights (w_i) also. These weights are the preferences of the objective functions by the DM. If the weights are changed according to the choice of the DM, then the optimal goal region will also be changed.

In our consideration, the supply and demands are taken as interval grey numbers. From the obtained solution, we see that the amount of goods purchased from the sources D1 and D2 are 1600 units and 1400 units, respectively. In that case, the DM considers the upper bound of grey supply at D1 and an amount less than the minimum supply capacity at D2. If the DM wishes to prepare a model by considering the minimum capacity of supply rather than grey supply, then the purchasing cost would be increased which may cause a smaller profit. So, the choice about the amount of goods to be purchased from the sources are cleared by our proposed technique through grey supply, whereas the existing methods fail to select a better choice when greyness is involved in the supply. Here, the amount of goods transported to the destinations C1, C2, C3 and C4 are 750 units (lower limit of demand), 850 units (lower limit of demand), 700 units (maximum among multi-choice of demand)

and 700 (one value among the multi-choice of demand), respectively, for optimal solution. Hence, for a better solution of our proposed MOTP, the selling amount of goods regarding an optimal solution is produced by our technique when demands are grey numbers.

In most of the real-life MOTPs, we just directly address origins and destinations, but here we add a new concept by proposing a Two-Stage TP. In the modern-days business economy, the DM wishes to find the optimal solution by keeping the minimum transportation cost. But, here the DM gets a platform to fix the goals in some situations of gathering the goods in the warehouses and, in time, of distributing the goods from the warehouses. It suggests that the specific value of some storing items in the warehouses is produced with the optimal profit by the DM.

8.5 Conclusion

In this chapter, we have proposed a Two-Stage MOTP with two interesting characteristics, that one being about the goal preferences of the DM and another one consists in the selection of particular values of supply and demand from grey supply and grey demand. Solution of the MOTP under these goals through RMCGP has provided a technique for multi-objective decision making. But, the main drawback of RMCGP consists in how the DM would select the goals for the objective functions. The core contents of our proposed study has removed this difficulty by selecting the goals of the objective functions through an algorithm. We have constructed a utility function (choice by the DM) for selecting the optimal goal region. After this selection, we have solved the proposed problem by our RMCGP approach. Our suggested multi-objective Two-Stage TP has provided a new direction to select proper goals for real-life multi-objective transportation problems under the environment of grey supply and grey demand. A numerical example has been given to show the applicability and suitability of our way of solving MOTP and fixing the proper goals for the objective functions.

Chapter 9

Multi-Modal Transportation Problem*

Realizing the fact of real-life situations, we consider multiple modes of transportation for distributing the goods to respective destinations using supplementary origins in different stages and incorporate a new approach of multi-modal transportation problem in this chapter. The formulated MMTP is nothing but a linear programming problem and so, it is easy to solve by any simplex algorithm. To analyze the proposed method a numerical example is included and solved which reveals a better impact for analyzing the real-life decision making problems.

9.1 Introduction

In a classical TP, there are two types of node points, namely, origins and destinations. Sometimes, it is also need to take in consideration that the origins and the destinations may be multiple in nature. It means that, there may have origins/destinatiuons in different levels. Due to the factor of multiple routes or multi-mode of transportation in a TP, the TP becomes a Multi-Modal Transportation Problem (MMTP). Multi-modal transport which is also known as combined transport allows to transport the goods under a single contract, but it is performed with at least two modes of transport; the carrier is liable (in a

*A part of this chapter has communicated in an International Journal

legal sense) for the entire carriage, even though it is used by several different modes of transport such as sea, road, etc. The carrier does not have to possess all the means of transport, and in practice usually it does not valid. The carrier is often performed by sub-carriers which is referred to in legal language as "actual carriers". The carrier responsible for the entire carriage is addressed to as a Multi-Modal Transport Operator (MTO).
In this chapter, a new mathematical model is proposed for solving TP by incorporating the multi-modal transport systems. The proposed model is completely a Linear Programming Problem (LPP), so it is easy to understand and solve; and to apply on real-life transportation problems for DMs.

9.2 Problem environment

The term *multi-modal* is defined for several modes of transport in a transportation problem, and as a whole it is referred to here as multi-modal transportation problem. Generally, MMTP can be categorized as either passenger or freight-oriented. Goods can be transported via several modes and people also use different modes of transportation for their journey. Therefore several modes of transportation are considered as follows:

- Roadway/highway automobiles (including taxi), truck, motorcycle, etc.,
- Passenger rail and traditional freight train, etc.,
- Transit-light rail vehicles, commuter rail, buses, etc.,
- Air-passenger service and air-freight,
- Water-ferries, barges, transatlantic vessels, cruise ships, etc., and
- Non-motorized-walking, bicycling, etc.

Obviously, some other modes (e.g., bicycle, motorcar, etc.) are available to provide largely recreational transportation in the countries of different parts of the World. Other modes such as passenger air service and rail-road freight service are essential components of the transportation system, the economy and our daily life. All modes of transportation must be planned and systematically provided, like any other form of modern infrastructure (e.g., buildings, sanitation). Not only exist the several modes, but also the transportation professionals must also plan and provide for the safety and efficient transfer of goods and people among different modes. This transfer is generally referred to as an intermodal transfer. Recently James et al. (59) in University Transportation Center (UTC) defined intermodal transportation based on information fusion.

There are some important perspectives in the MMTP like as:

- Decreasing overall transportation costs by allowing each mode to be used for the portion of the trip in which it is best suited,
- Increasing economic efficiency and productivity, thereby enhancing the Nation's global competitiveness,
- Reducing congestion and the burden on over stressed infrastructure components,
- Generating higher returns from public and private infrastructure investments,
- Improving mobility for the elderly, disabled, isolated, and economically disadvantages, and
- Reducing energy consumption and contributing to improve air quality and environmental conditions (Fox et al. (37), Veloso (155)).

Now we present three useful definitions related to the new method of the MMTP.

Definition 9.2.1 ***(Ground Origins):*** *In a transportation problem, the sources have the capacity of supply the goods only but there are no such capacity to gather the goods. Then the sources are treated as Ground Origins.*

Definition 9.2.2 ***(Final Destinations):*** *In a transportation problem, the destinations have the capacity of gathering the goods only but there are no such capacity to supply the goods, considered as Final Destinations.*

There is not possible to supply the goods according to the expectation of the *final destinations* from the *ground origins* because of that to vehicle capacity/multiple routes of transport. In that case, there are required some destination points which have the capacity of supplying the goods and receiving the goods simultaneously. These nodes are known as *supplementary origins*.

Definition 9.2.3 ***(Supplementary Origins):*** *In a transportation problem, the destinations which have the capacity of gathering the goods as well as the capacity of delivering the goods which is noted as Supplementary Origins.*

In Figure 9.1, A1 and A2 are the *ground origins*; B1, B2 and C1 are the *supplementary origins*; D1 and D2 are referred as the *final destinations*. The TP

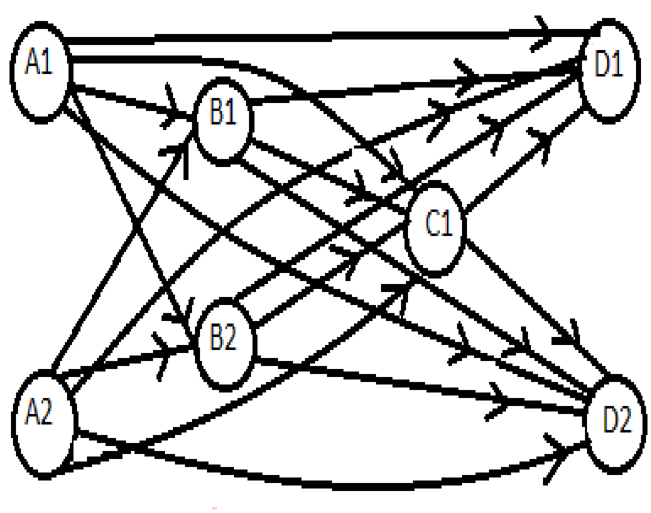

Figure 9.1: Graphical representation of the MMTP.

under the consideration of at least one *supplementary origin* is described as the MMTP. We propose to formulate the mathematical model of the MMTP and

solve it for producing a better result. To accommodate the real-life transportation problem, it is not always possible to fulfill the demand of the customers at the destinations through single transportation. Sometimes there are some restrictions for transporting the goods and so it is required to consider the multi-modes of transportation from different nodes. Then the transportation is not a simple TP, it becomes a MMTP.
The mathematical model of the proposed MMTP is shown in details in the next Section.

9.3 Mathematical model

A general transportation problem is a typical problem in which the main objective is to minimize the transportation cost which is defined as follows:
Model 9.1

$$\text{minimize} \quad Z = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}x_{ij} \tag{9.1}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \leq a_i \quad (i = 1, 2, \ldots, m), \tag{9.2}$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j \quad (j = 1, 2, \ldots, n), \tag{9.3}$$

$$x_{ij} \geq 0 \;\; \forall \;\; i \text{ and } \; j, \tag{9.4}$$

where C_{ij} $(i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$ is the transportation cost per unit commodity from the i^{th} origin to the j^{th} destination. Here a_i $(i = 1, 2, \ldots, m)$ and b_j $(j = 1, 2, \ldots, n)$ are availability and demand in the i^{th} origin and the j^{th} destination respectively and $\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} b_j$ is the feasibility condition. To formulate a mathematical model of the MMTP, we consider the following assumptions with the preferences which are decided by the Decision Maker (DM):

- Let m_1 be the number of *ground origins* and n_1 be the number of *final destinations*. The decision variables of transportation problem from the nodes are denoted as x^1_{ij1}; and C^1_{ij1} is the unit transportation cost. Also assume that vehicle has the capacities; and due to the capacities, we consider that the transported amounts of goods are multiple of α_1^1, i.e., x_{ij1}^1 is taken as multiple of α_1^1.

- Suppose there be m_2 number of *supplementary origins* in 1^{st} level with the capacity of storing amount is a_i^1, which can deliver to the final destinations and receive the items from the *ground origins*. In this case, the decision variables from *supplementary origin* in 1^{st} level to *final destinations* are x_{ij1}^2; and C_{ij1}^2 is the unit transportation cost. Due to vehicle restriction, we consider that the transported amounts of goods are multiple of α_1^2, i.e., x_{ij1}^2 is chosen as multiple of α_1^2.

- Assuming that there are m_r number of *supplementary origins* at $(r-1)^{th}$ level with the capacity of storing amount is a_i^{r-1}, which can deliver to the *final destinations* and collect the items from the *ground origins*. The decision variables in this case for transporting the goods from *supplementary origin* in $(r-1)^{th}$ level to *final destinations* are x_{ij1}^r; and C_{ij1}^r are the unit transportation cost. Also considering that x_{ij1}^r is taken as multiple of α_1^r. Thus we consider the decision variables, x_{ij1}^p and cost variables, C_{ij1}^p $(p = 1, 2, \ldots, r)$ for transportation of items which are transported to the *final destinations* along with the vehicle capacity α_1^p. Let the objective function be defined as z^1 for this transportation (from *ground origins* and all *supplementary origins* to *final destinations*).

- Considering the *supplementary origin* at $(r-1)^{th}$ level as the final destination point; and the decision variables, x_{ij2}^p and the cost variables, C_{ij2}^p $(p = 1, 2, \ldots, r-1)$ for transportation of items which are transported to the *supplementary origin* at $(r-1)^{th}$ level along with the vehicle capacity α_p^2. Let z^2 be the objective function for this case (from *ground origins* and *supplementary origins* up-to $(r-2)^{th}$ level to *supplementary origins* at $(r-1)^{th}$ level).

- Proceeding in this way, we choose the *supplementary origin* at 1^{st} level as the final destination point then the decision variables are denoted as x^p_{ijr} $(p = 1)$ and cost variables C^p_{ijr} $(p = 1)$ for transporting the items which are transported to the *supplementary origin* in 1^{st} level along with the vehicle capacity α^1_r. Let z^r be objective function for that transportation.

- If there is no other path of transportation according to our proposed multi-modal p^{th} stage $(p = 1, 2, \ldots, (r-1))$ transportation model, then we simply set '0' as the value of the transportation variable.

- In the *supplementary origin* at p^{th} stage $(p = 1, 2, \ldots, (r-1)$, we assign a capacity of storing by maximum amount a^p_i, we are not proposed any *supplementary demand* capacity at the points. We introduce an inequality restriction to each of the *supplementary origins* for amounting of goods which will be delivered from these nodes which will not exceed the amount of goods gathered there.

Based on the above discussion, we design the mathematical model of the MMTP as follows:

Model 9.2

$$
\begin{aligned}
\text{minimize} \quad & Z = z^1 + z^2 + \ldots + z^r \qquad (9.5)\\
\text{where} \quad & z^1 = \sum_{i=1}^{m_1}\sum_{j=1}^{n_1} \alpha^1_1 C^1_{ij1} x^1_{ij1} + \sum_{i=1}^{m_2}\sum_{j=1}^{n_1} \alpha^2_1 C^2_{ij1} x^2_{ij1}\\
& + \ldots + \sum_{i=1}^{m_r}\sum_{j=1}^{n_1} \alpha^r_1 C^r_{ij1} x^r_{ij1}\\
& z^2 = \sum_{i=1}^{m_1}\sum_{j=1}^{m_r} \alpha^1_2 C^1_{ij2} x^1_{ij2} + \sum_{i=1}^{m_2}\sum_{j=1}^{m_{r-1}} \alpha^2_2 C^2_{ij2} x^2_{ij2}\\
& + \ldots + \sum_{i=1}^{m_r}\sum_{j=1}^{m_r} \alpha^{r-1}_2 C^{r-1}_{ij2} x^{r-1}_{ij2}\\
& \vdots\\
& z^r = \sum_{i=1}^{m_1}\sum_{j=1}^{m_2} \alpha^1_r C^1_{ijr} x^1_{ijr}
\end{aligned}
$$

subject to

$$\sum_{j=1}^{n_1}\alpha_1^1 x_{ij1}^1+\sum_{j=1}^{m_r}\alpha_2^1 x_{ij2}^1+\ldots+\sum_{j=1}^{m_2}\alpha_r^1 x_{ijr}^1\le a_i \quad (i=1,2,\ldots,m_1), \tag{9.6}$$

$$\sum_{i=1}^{m_1}\alpha_1^1 x_{ij1}^1+\sum_{i=1}^{m_2}\alpha_1^2 x_{ij1}^2+\ldots+\sum_{i=1}^{m_{r-1}}\alpha_1^r x_{ij1}^r\ge b_j \quad (j=1,2,\ldots,n_1), \tag{9.7}$$

$$\sum_{j=1}^{n_1}\alpha_1^2 x_{ij1}^2+\sum_{j=1}^{m_r}\alpha_2^2 x_{ij2}^2+\ldots+\sum_{j=1}^{m_2}\alpha_{r-1}^2 x_{ij(r-1)}^2$$

$$\le a_i^1 \ (i=1,2,\ldots,m_2), \tag{9.8}$$

$$\sum_{j=1}^{n_1}\alpha_1^3 x_{ij1}^3+\sum_{j=1}^{m_r}\alpha_2^3 x_{ij2}^3+\ldots+\sum_{j=1}^{m_2}\alpha_{r-2}^3 x_{ij(r-2)}^3$$

$$\le a_i^2 \ (i=1,2,\ldots,m_3), \tag{9.9}$$

$$\vdots$$

$$\sum_{j=1}^{n_1}\alpha_1^r x_{ij1}^r\le a_i^r \quad (i=1,2,\ldots,m_r), \tag{9.10}$$

$$\sum_{j=1}^{n_1}\alpha_1^2 x_{tj1}^2+\sum_{j=1}^{m_r}\alpha_2^2 x_{tj2}^2+\ldots+\sum_{j=1}^{m_2}\alpha_{r-1}^2 x_{tj(r-1)}^2$$

$$\le\sum_{i=1}^{m_1}\alpha_r^1 x_{itr}^1 \quad (t=1,2,\ldots,m_2), \tag{9.11}$$

$$\sum_{j=1}^{n_1}\alpha_1^3 x_{tj1}^3+\sum_{j=1}^{m_r}\alpha_2^3 x_{tj2}^3+\ldots+\sum_{j=1}^{m_2}\alpha_{r-2}^3 x_{tj(r-2)}^3$$

$$\le\sum_{i=1}^{m_1}\alpha_{r-1}^1 x_{it(r-1)}^1+\sum_{i=1}^{m_2}\alpha_{r-1}^2 x_{it(r-1)}^2 \quad (t=1,2,\ldots,m_3), \tag{9.12}$$

$$\vdots$$

$$\sum_{j=1}^{n_1}\alpha_1^r x_{tj1}^r\le\sum_{i=1}^{m_1}\alpha_2^1 x_{it2}^1+\sum_{i=1}^{m_2}\alpha_2^2 x_{it2}^2$$

$$+\ldots+\sum_{i=1}^{m_{r-1}}\alpha_2^{r-1} x_{it2}^{r-1} \quad (t=1,2,\ldots,m_r), \tag{9.13}$$

$$x_{ijp}^{(t)}\ge 0 \ \forall \ i,\ j,\ t \text{ and } p, \tag{9.14}$$

Here, the feasibility condition of the mathematical model is $\sum_{i=1}^{m_1} a_i \ge \sum_{j=1}^{n_1} b_j$.

In Model 9.2, the maximum number of decision variables are $(m_1 \times m_2 \times \cdots \times m_r \times n_1)$. If there is no other path between the nodes in the proposed TP, then we consider the value of decision variable is 0 and remove the variable form the proposed model which reduces the number of variables.
The feasible region of the proposed model is constructed by considering the following assumptions:

- There are m_1 number of availability constraints (9.6) for the ground origins.

- There are n_1 number of demand constraints (9.7) for the final destinations.

- There are the restrictions of storing items in the supplementary origins so we introduce $(m_2 + m_3 + \ldots + m_r)$ number of inequations form (9.8) to (9.10).

- Again the delivered amount of goods from the supplementary origins do not exceed supplied amount of goods to the respective supplementary origins. To do this, we introduce $(m_2 + m_3 + \ldots + m_r)$ number of inequations from (9.11) to (9.13).

Thus, the formulated mathematical model consists of $(m_1 \times m_2 \times \cdots \times m_r \times n_1)$ number of variables and $[2(m_2+m_3+\ldots+m_r)+m_1+n_1]$ constraints along with the non-negativity conditions. Here, Model 9.2 is a completely LPP model and can be solved by any simplex algorithm like Big-M method, revised simplex method. If the number of variables are increased then one can use the software such as LINGO, MatLab, etc. for solving the Model 9.2.

9.4 Numerical example

The numerical example is presented here to justify the utility of the MMTP. Assume that the two supply centers of goods are namely, A1 and A2; and D1 and D2 are two destinations in which a homogeneous commodity of a product is to be transported. The capacity of vehicle to deliver goods is 1000 items. So it is necessarily a problem to deliver the goods when the demands at the destinations are not multiple of 1000. Again, the destinations B1 and B2 which can receive the goods from A1 and A2 and have the capacity of transport the goods to the final destinations D1 and D2. The vehicles are carrying the goods from B1 and B2 to D1 and D2 with the capacity of 100 items. So, again there is a problem to deliver goods when the amount of goods are not in multiple of 100. Also consider that there is a destination C1 which can take the goods form A1, A2, B1 and B2 and supply them to the destinations D1 and D2. The transportation from the center C1 to the destinations D1 and D2 have no such vehicle capacity i.e., any amount of goods can be transported between the nodes. The traditional approach of transportation problem cannot provide any such mathematical model to solve the proposed problem. Here to solve the problem, we formulate the mathematical model which is known as MMTP. The following notations and assumptions are considered to formulate the mathematical model of the MMTP.

- The decision variables for transporting the items are considered as follows:
  From A1 and A2 to D1 and D2 are considered as x^1_{ij1} using ship-way with vehicle capacity $\alpha^1_1 = 1000$;
  From B1 and B2 to D1 and D2 are taken as x^1_{ij2} using rail-way with vehicle capacity $\alpha^2_1 = 100$;
  From C1 to D1 and D2 are considered as x^1_{ij3} using road-way without any vehicle capacity restriction, i.e., $\alpha^3_1 = 1$.
  From A1 and A2 to C1 are considered as x^2_{ij1} with vehicle restriction $\alpha^1_2 = 500$;
  From B1 and B2 to C1 are considered as x^2_{ij2} without any vehicle restriction.
  From A1 and A2 to B1 and B2 are considered as x^3_{ij1} and there is no such any vehicle restriction.

- The feasibility of the numerical example consists of following number of constraints:
  The supply capacity at the ground origins A1 and A2 are introduced by two constraints. The demand at the final destinations D1 and D2 are considered by two constraints. Storing capacity at the supplementary origins B1, B2 and C1 provide three constraints. Amount of goods distributed from the supplementary origins B1, B2 and C1 do not exceed the amount of storing items which produces three constraints. Hence, the number of constraints in the MMTP of the numerical example is 10.

The transportation costs in different routes are represented in Tables 9.1 to 9.6.

Table 9.1: Transportation cost from A1 and A2 to D1 and D2 (in $).

	$D1$	$D2$
$A1$	15	13
$A2$	15	18

Table 9.2: Transportation cost from B1 and B2 to D1 and D2 (in $).

	$D1$	$D2$
$B1$	8	10
$B2$	9	7

Table 9.3: Transportation cost from C1 to D1 and D2 (in $).

	$D1$	$D2$
$C1$	6	5

Table 9.4: Transportation cost from A1 and A2 to C1 (in $).

	$C1$
$A1$	11
$A2$	12

Table 9.5: Transportation cost from B1 and B2 to C1(in $).

	$C1$
$A1$	8
$A2$	9

Table 9.6: Transportation cost from A1 and A2 to B1 and B2(in $).

	$B1$	$B2$
$A1$	5	4
$A2$	6	5

Again the availability of goods at each Ground Origin A1 and A2 are 1600 units. The maximum capacity of storing at the Supplementary Origins B1, B2 and C1 are 1200 units, 1300 units and 1000 units respectively. The mathematical model is designed corresponding to the available data described in Tables 9.1 to 9.6 as follows:

Model 9.3

minimize $Z = z_1 + z_2 + z_3,$

$$z_1 = 1000(15x^1_{111} + 13x^1_{121} + 15x^1_{211} + 18x^1_{221}) + 100(8x^2_{111} + 10x^2_{121}$$
$$+9x^2_{211} + 7x^2_{221}) + 6x^3_{111} + 5x^3_{121},$$
$$z_2 = 500(11x^1_{112} + 12x^1_{212}) + 8x^2_{112} + 9x^2_{212},$$
$$z_3 = 5x^1_{113} + 4x^1_{123} + 6x^1_{213} + 5x^1_{223},$$
$$1000(x^1_{111} + x^1_{121}) + 100x^1_{112} + 500(x^1_{113} + x^1_{123}) \leq 1600,$$
$$1000(x^1_{211} + x^1_{221}) + 100x^1_{212} + 500(x^1_{213} + x^1_{223}) \leq 1600,$$
$$1000(x^1_{111} + x^1_{211}) + 100(x^2_{111} + x^2_{211}) + x^3_{111} \geq 1555,$$
$$1000(x^1_{121} + x^1_{221}) + 100(x^2_{121} + x^2_{221}) + x^3_{121} \geq 1575,$$
$$x^1_{113} + x^1_{213} \leq 1200,$$
$$x^1_{123} + x^1_{223} \leq 1300,$$
$$500(x^1_{112} + x^1_{212}) + x^2_{112} + x^2_{212} \leq 1000,$$
$$x^1_{113} + x^1_{213} \leq 100(x^2_{111} + x^2_{211}) + x^2_{112} + x^2_{212},$$
$$x^1_{123} + x^1_{223} \leq 100(x^2_{121} + x^2_{221}) + x^2_{122} + x^2_{222},$$
$$500(x^1_{112} + x^1_{212}) + x^2_{112} + x^2_{212} \leq x^3_{111} + x^3_{121},$$
$$x^p_{ijk} \geq 0 \text{ (all are taken integers); } \forall\, i, j, k, p.$$

Model 9.3 is simply a LPP which can be solved through any simplex algorithm. As Model 9.3 contains the large number of variables, so we use LINGO software to obtain the solution of Model 9.3.

9.5 Result and discussion

The value of the objective function is 41700($) which is minimized. The optimal solution of Model 9.3 is presented in Tables 9.7, 9.8 and 9.10.

Table 9.7: The amounts of transported goods to final destinations D1 and D2.

Variable	x^1_{111}	x^1_{121}	x^1_{211}	x^1_{221}	x^2_{111}	x^2_{121}	x^2_{211}	x^2_{221}	x^3_{111}	x^3_{121}
Value	0	1000	0	0	1200	0	300	600	55	0

Table 9.8: The amounts of transported goods to supplementary origin C1.

Variable	x^1_{112}	x^1_{212}	x^2_{112}	x^2_{212}
Value	0	0	0	55

Table 9.9: The amounts of transported goods to supplementary origins B1 and B2.

Variable	x^1_{113}	x^1_{123}	x^1_{213}	x^1_{223}
Value	0	600	1200	355

Table 9.10: The amounts of transported goods stored at all supplementary origins.

Node	B1	B2	C1	D1	D2
Value	1200	955	55	1555	1600

In classical TP, there are only two types of nodes namely, supply node and demand node. In addition to that, there is at least one supplementary origin node which is present in the MMTP. Sometimes there are restrictions for transporting the goods between the nodes due to vehicle capacity. So, to minimize the transportation cost for delivering the goods in proper node, different types of vehicles are required.

To justify the efficiency of the proposed mathematical model of the MMTP, we describe the various possibilities in the numerical example as follows:

- Consider that the routes between the supply points, A1 and A2 to destination points D1 and D2 are sea way. So delivering the goods are made through ship. Obviously, a large amount of goods are delivered through the ship and the amount is 1000 units. In that situation, if there are no other nodes available like B1, B2 and C1, then the formulated TP is a classical TP. In this case we see that there exists a feasible solution of the proposed problem, but the transportation cost is not minimized. Because in each of the destination the minimum requirements are 1555 units and 1575 units of goods which mean at least two ships are required for delivering the goods in each node D1 and D2. So, traditional TP is not enough to give definite conclusion without considering the supplementary origins as we considered in our proposed study.

- Again, we assume that there is a connection through rail-way between B1 and B2 to D1 and D2. Then, the capacity transports in each time by the rail-way is high and we consider that at a single transport it is needed 100 amount of goods. In that situation, we solve the problem without considering the supplementary origin C1 (i.e., using the value of the variables as '0' those are taken for C1), add the total transportation cost is 42000($). The amount of transported goods to the node D1 and D2 respectively are 1600 units and 1600 units respectively. The amount of goods supplied at the supplementary origins B1 and B2 are 900 units and 1300 units respectively.

- In the similar way, if we formulate mathematical model without considering the supplementary origins B1 and C1; or B2 and C1, then the transportation cost will be increased.

According to our discussions, we analyze that introduction of multi-modal system in TP is very much essential to reduce the transportation cost for delivering the goods. But, in the classical TP, it is not so.

9.6 Conclusion

The study has been introduced the multi-modal system in the TP which has significantly analyzed through real-life decision making problems. There may occur the situations in a transportation system, due to the presence of multiple-mode of transportation in which the traditional TP fails to formulate a mathematical model and find the least-cost route of transportation. Our mathematical model, the MMTP may be applied to formulate the mathematical model under multiple-mode of transportation and its solution suggests about the selection of mode of transportation as well as optimal solution of the problem. The results of the numerical example presented in the Chapter justify the efficiency of the proposed mathematical model.

Chapter 10

Integrated Study of Transportation and Inventory *

All the previous chapters of the thesis have been described TP in several directions, especially in multi-choice environment. The concept of multi-choice study, not only improved in the area of TP, the idea of multi-choice optimization can also be extended in combined ground of inventory and transportation and build up the study of Integrated Optimization in Inventory Transportation (IOIT) problem. Formulation of IOIT model under multi-choice environment, and solution of the model are justifying the efficiency of the IOIT model in this chapter.

10.1 Introduction

Inventory is the stock of items or resources used in an organization. The study of inventory refers to know how much amount of goods have to be sold by decision maker (DM) and how much amount left after sold and how much amount need to order from suppliers to keep stock with enough product. After all, through out the system, the DM would like to optimize the profit, noting the demands of the retailers and their requirements.

Transportation problem (TP) is a well known decision making problem, mainly coming into picture by minimizing the transportation cost for transporting the

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goods from origin to destination. An IOIT problem is a problem to optimize the combination of transportation cost and inventory cost under the prerequisite assumptions.
In this chapter, main aim is to make a bridge between TP and inventory in multi-item integrated transportation problem under the multi-choice transportation cost and random supply. Then the combined form is to solve by well known optimization technique and compare the solution with the solution of basic inventory optimization. Finally, we show that the proposed methodology of our chapter has an advantage in practical importance related to cost reduction in logistic system.

10.2 Mathematical model

In the subsection, first we present the mathematical model of multi-item transportation problem. Thereafter, the mathematical model of multi-item multi-choice TP with stochastic supply is presented. We introduce the basic inventory optimization problem, and furthermore, the mathematical structure of IOIT is incorporated here.

10.2.1 Multi-item transportation problem

There may occur some situations of real-life decision making problems where the DM transports more than one item of goods which are not correlated to each other. To accommodate the situation, the TP becomes multi-item transportation problem. The mathematical model of multi-item transportation model can be expressed as follows (Model 10.1):

Model 10.1

$$\text{minimize} \quad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{t=1}^{s} C_{ijt} x_{ijt}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ijt} \leq a_{it} \quad (i = 1, 2, \ldots, m;\ t = 1, 2, \ldots, s),$$

$$\sum_{i=1}^{n} x_{ijt} \geq b_{jt}, \quad (j = 1, 2, \ldots, n;\ t = 1, 2, \ldots, s),$$

$$x_{ijt} \geq 0 \;\; \forall \, i, \; j \text{ and } t,$$

where, C_{ijt}, a_{it} and b_{jt} are the cost, supply and demand parameters in multi-item transportation problem for t-th item. The feasibility condition is $\sum_{i=1}^{m} a_{it} \geq \sum_{j=1}^{n} b_{jt}, \;\; \forall \, t$.

Transportation problem can be reduced to a multi-choice TP when at least one of the transportation parameters (parameters may be cost, demand, supply in TP) become multi-choice type. Again considering the stochastic supply instead of fixed supply in a multi-choice TP, then the multi-choice TP converts to multi-choice stochastic TP. Again, the study of multi-choice TP with stochastic supply under multi-item environment reduces to multi-item multi-choice TP with stochastic supply. The mathematical model of multi-item multi-choice TP with stochastic supply can be defined as follows:

Model 10.2

$$\text{minimize} \quad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{t=1}^{s} \left(C_{ijt}^{1} \text{ or } C_{ijt}^{2} \text{ or } \ldots \text{ or } C_{ijt}^{k} \right) x_{ijt} \quad (10.1)$$

$$\text{subject to} \quad \Pr \left(\sum_{j=1}^{n} x_{ijt} \leq a_{it} \right) \geq 1 - \gamma_{it} \quad \forall \, i, \; t, \quad (10.2)$$

$$\sum_{i=1}^{n} x_{ijt} \geq b_{jt} \quad \forall \, j, \; t, \quad (10.3)$$

$$x_{ijt} \geq 0 \quad \forall \, i, \; j \text{ and } t, \quad (10.4)$$

where γ_{it} $(0 < \gamma_{it} < 1)$ $\forall \;\; i$ and t, is a pre-specified level of probability fixed by DM.

Without loss of generality, assuming that a_{it} $(i = 1, 2, \ldots, m; \; t = 1, 2, \ldots, s)$ be specified with exponential stochastic variable and $\left(C_{ijt}^{1}, C_{ijt}^{2}, \ldots, C_{ijt}^{k} \right)$ are known as multi-choice cost parameters for $i = 1, 2, \ldots, m; \;\; j = 1, 2, \ldots, n;$ and $t = 1, 2, \ldots s$.

Many real-life problems on logistic system are designed with the combination of the inventory system and the TP. Based on the above discussion, we make a connection between the inventory system and the TP under multi-choice programming and stochastic programming. Here, we present two types of mathematical models namely, Inventory Optimization (IO) Model and Inventory Optimization in Integrated Transportation (IOIT) Model.

10.2.2 Basic Inventory Optimization (IO) model

In this chapter, we assume that there is no shortage allowed and the demand D, is continuous. The inventory is replenished every τ time, the demand quantity must meet the requirement $D\tau$ during the cycle. If Q denotes the order volume, then $Q = D\tau$. C_1 and C_2 are the unit inventory sustaining cost and the fixed order cost respectively. K is the price of goods, then the order cost is $C_2 + KD\tau$, the average order cost in τ time is $\frac{C_2}{\tau} + KD$, and the average volume of inventory is $\frac{1}{2}D\tau$ with the average inventory sustaining cost is $\frac{1}{2}C_1D\tau$. Then we define the average total inventory cost in the cycle is expressed by the following mathematical model as:

$$C(\tau) = \frac{C_2}{\tau} + KD + \frac{1}{2}C_1D\tau. \tag{10.5}$$

Differentiating equation (10.5) with respect to τ and then equate to zero, we have

$$\tau = \sqrt{\frac{2C_2}{C_1D}}.$$

Assuming that τ_0 be the optimum time period of the inventory, then,
$\tau_0 = \sqrt{\frac{2C_2}{C_1D}}$. Therefore, the order volume with respect to optimum time period τ_0 is $Q_0 = D\tau_0$.

In basic inventory model, the transportation cost is added with the inventory cost. In this case, the transportation cost is obtained by solving the simple transportation problem separately. Finally, the total logistic cost is: inventory cost+ transportation cost.

For multi-item goods, the logistic cost can be calculated by considering the inventory cost separately and then it adds with the transportation cost for multi-item goods.

10.2.3 Inventory Optimization in Integrated Transportation (IOIT) model

Under the following assumptions that all demand points are managed by DM, it does not allow any shortage and demand points are independent to each other. The supply of all the origin points is uncertain. Let us use the following notations:

m=number of the supplier,

n=number of the demander,

r_{ijk}=cost that demander j purchased goods k from supplier i,

x_{ijk}= volume of goods k transported from supplier i to demander j,

d_{jk}=requirement of goods k from demander j,

u_{jk}=unit storage cost of demander j for goods k,

s_{jk}= safe stock of demander j for goods k,

R_k= total requirement of goods k,

C_{ijk}= unit transportation cost of goods k from supplier i to demander j,

α_{jk}=initial inventory volume of demander j for goods k.

Then the mathematical model of IOIT is as follows:

$$\begin{aligned}
\text{minimize} \quad & Z = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{s} r_{ijk}x_{ijk} + \sum_{j=1}^{n}\sum_{k=1}^{s}\frac{1}{2}(d_{jk}+\alpha_{jk})u_{jk} \\
& + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{s} C_{ijk}x_{ijk} \\
\text{subject to} \quad & d_{jk}+\alpha_{jk} \geq s_{jk} \quad \forall\ j,k, \\
& \sum_{j=1}^{n} d_{jk} = R_k \quad \forall \quad k, \\
& \sum_{i=1}^{m} x_{ijk} = d_{jk} \quad \forall\ j,k, \\
& \sum_{j=1}^{n} x_{ijk} \leq a_{ik} \quad \forall\ i,k, \\
& x_{ijk} \geq 0 \quad \forall \quad j,k.
\end{aligned}$$

Here, d_{jk} and x_{ijk} are the strategy variables. The objective function of above model is the combination of ordering cost, inventory cost and transportation cost. The first constraint implies that the storage of any demand point for any goods is more than its safe stock, the second constraint describes the sum of requirement of all demand points for goods k and is equal to total demand volume, the third constraint shows that the total transportation volume of demander j for goods k and is equal to storage requirement and the last constraint implies that the transportation volume of supplier i for goods k is less than its supply capacity.

Assume that there are several routes available for transporting the goods. Due to globalization of market, the cost of carrying per unit goods for several routes are changed. That is why we consider that the cost parameters of TP are multi-choice. Again, the supply parameter of TP follows stochastic nature due to the same reason. So, the mathematical model of IOIT with multi-choice cost and stochastic supply is defined as follows:

Model 10.3

$$\begin{aligned}
\text{minimize} \quad & z = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{s} r_{ijk}x_{ijk} + \sum_{j=1}^{n}\sum_{k=1}^{s}\frac{1}{2}(d_{jk}+\alpha_{jk})u_{jk} \\
& + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{s}\left(C_{ijk}^{1} \text{ or } C_{ijk}^{2} \text{ or } \ldots \text{ or } C_{ijk}^{t}\right)x_{ijk} \\
\text{subject to} \quad & d_{jk}+\alpha_{jk} \geq s_{jk} \quad \forall\ j,k, \\
& \sum_{j=1}^{n} d_{jk} = R_k \quad \forall\ \ k, \\
& \sum_{i=1}^{m} x_{ijk} = d_{jk} \quad \forall\ j,k, \\
& \Pr\left(\sum_{j=1}^{n} x_{ijk} \leq a_{it}\right) \geq 1-\gamma_{it} \quad \forall\ i,k, \\
& x_{ijk} \geq 0 \quad \forall\ j,k.
\end{aligned}$$

10.3 Solution technique

To solve the models i.e., Model 10.2 and Model 10.3, at first, we transform the stochastic constraints to deterministic constraints which is shown in subsection 10.3.1. Then an algorithm and its flowchart are presented to solve Model 10.3 and Model 10.2.

10.3.1 Reduction of stochastic constraints to deterministic constraints

The constraints (10.2) can be represented as below when a_{it} $(i = 1, 2, \ldots, m)$ follows an exponential stochastic variable.

$$\Pr\left(\sum_{j=1}^{n} x_{ijt} \leq a_{it}\right) \geq 1 - \gamma_{it} \quad \forall \ i, \ t.$$

The above inequality can be further expressed as:

$$\Pr\left(\sum_{j=1}^{n} x_{ijt} \geq a_{it}\right) \leq \gamma_{it} \qquad \forall \ i, \ t. \tag{10.6}$$

It is assumed that a_{it} $(i = 1, 2, \ldots, m)$ are independent exponential stochastic variables with parameter θ_i which is treated as positive integers. Now the mean of a_{it}, $E(a_{it})$ is θ_{it} and variance of a_{it}, $Var(a_{it})$ is $\sigma^2_{a_{it}} = \theta^2_{it}$, which are known to us. We know that the probability density function of a_{it} $(i = 1, 2, \ldots, m)$ is given by

$$f(a_i) \ = \ \frac{1}{\theta_{it}} e^{-\frac{a_{it}}{\theta_{it}}}, \quad \text{where} \quad a_{it} > 0, \quad \theta_{it} > 0. \tag{10.7}$$

Now, inequality (10.6) can be expressed as the cumulative density function of exponential distribution:

$$\int_{0}^{\sum_{j=1}^{n} x_{ijt}} f(a_{it}) d(a_{it}) \ \leq \ \gamma_{it}. \tag{10.8}$$

Using (10.7), the above integral can be expressed as:

$$\int_{0}^{\sum_{j=1}^{n} x_{ijt}} \frac{1}{\theta_{it}} e^{-\frac{a_{it}}{\theta_{it}}} d(a_{it}) \ \leq \ \gamma_{it}. \tag{10.9}$$

$$\text{Let,} \qquad -\frac{a_{it}}{\gamma_{it}} = z.$$

The above integral can be expressed as:

$$\int_0^{-\frac{\sum_{j=1}^n x_{ijt}}{\theta_{it}}} -e^z d(z) \leq \gamma_{it}. \tag{10.10}$$

which can be integrated as:

$$[-e^z]_0^{-\frac{\sum_{j=1}^n x_{ijt}}{\theta_{it}}} \leq \gamma_{it}. \tag{10.11}$$

Taking logarithm on both sides, we have

$$-\frac{\sum_{j=1}^n x_{ijt}}{\theta_{it}} \quad \geq \quad \ln(1-\gamma_{it}) \qquad (i = 1, 2, \ldots, m). \tag{10.12}$$

Finally, the stochastic constraint (10.2) can be transformed into equivalent deterministic constraints as follows:

$$\sum_{j=1}^n x_{ijt} \quad \leq \quad -\theta_{it}\ln(1-\gamma_{it}) \quad (i = 1, 2, \ldots, m; t = 1, 2, \ldots, s). \tag{10.13}$$

Now, using the above reduction procedure of stochastic constraints to crisp constraints in Model 10.2, we obtain the multi-item multi-choice deterministic transportation problem as follows:

Model 10.4

$$\left.\begin{aligned}
&\text{minimize} \quad Z = \sum_{i=1}^m \sum_{j=1}^n \sum_{t=1}^s \left(C_{ijt}^1 \text{ or } C_{ijt}^2 \text{ or } \ldots \text{ or } C_{ijt}^k\right) x_{ijt} \\
&\text{subject to} \quad \sum_{j=1}^n x_{ijt} \leq -\theta_{it}\ln(1-\gamma_{it}) \quad \forall\ i,\ t, \\
&\qquad \sum_{i=1}^m x_{ijt} \geq b_{jt} \quad (j = 1, 2, \ldots, n;\ t = 1, 2, \ldots, s), \\
&\qquad x_{ijt} \geq 0 \quad \forall\ i,\ j \text{ and } t, \\
&\text{where} \quad \sum_{i=1}^m [-\theta_{it}\ln(1-\gamma_{it})] \geq \sum_{j=1}^n a_{it} \ \ \forall\ t, \ \ \text{(feasibility condition).}
\end{aligned}\right\} \tag{10.A}$$

Again, applying the proposed reduction procedure in Model 10.3, we obtain the following mathematical model as:

Model 10.5

$$\begin{aligned}
\text{minimize} \quad & Z = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{s} r_{ijk}x_{ijk} + \sum_{j=1}^{n}\sum_{k=1}^{s}\frac{1}{2}(d_{jk}+\alpha_{jk})u_{jk} \\
& + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{s}\left(C_{ijk}^{1} \text{ or } C_{ijk}^{2} \text{ or } \ldots \text{ or } C_{ijk}^{t}\right)x_{ijk} \\
\text{subject to} \quad & \sum_{j=1}^{n} x_{ijk} \le -\theta_{ik}\ln(1-\gamma_{ik}) \;\; \forall\, i,\; k, \\
& d_{jk}+\alpha_{jk} \ge s_{jk} \;\; \forall\, j,k, \\
& \sum_{j=1}^{n} d_{jk} = R_k \;\; \forall\, k, \\
& \sum_{i=1}^{m} x_{ijk} = d_{jk} \;\; \forall\, j,k, \\
& x_{ijk} \ge 0 \;\; \forall\, j,k, \\
\text{where} \quad & \sum_{i=1}^{m}[-\theta_{it}\ln(1-\gamma_{it})] \ge \sum_{j=1}^{n} a_{it} \;\; \forall\, i,\; t.
\end{aligned}$$

10.3.2 MATLAB approach to find optimal solution

Here, we propose the following algorithm to find the optimal solution of **Model 10.5**:

Algorithm 10.1

- **Step 1**: First, assigning the variable in multi-choice form as the inputs in following array system $c[i][j][k]$, where i and j stand for penalties of decision variables x_{ij}; and k stands for the multi-choices of respective penalties. Also calculating the total number of possibilities.

- **Step 2**: Formulating the objective function $f(c[i][j])$ for each of the choices k.

- **Step 3**: Incorporating the coefficient matrix A corresponding to the constraints.

- **Step 4**: Introducing the decision vector b.

- **Step 5**: Use MATLAB command "[x, fval, exitflag, output, lambda] = linprog(f, A, b,[ ],[ ], lb)" to find optimal solution for each k.
- **Step 6**: Storing the solutions $f(k)$ for each of the choices k.
- **Step 7**: Finally, calculating min$\{f(k)$: for each choice $k\}$ and noticed that the choice k produces to the minimum value of objective function.
- **Step 8**: Stop.

The flowchart of the above algorithm is shown in Figure 10.1. Using Algo-

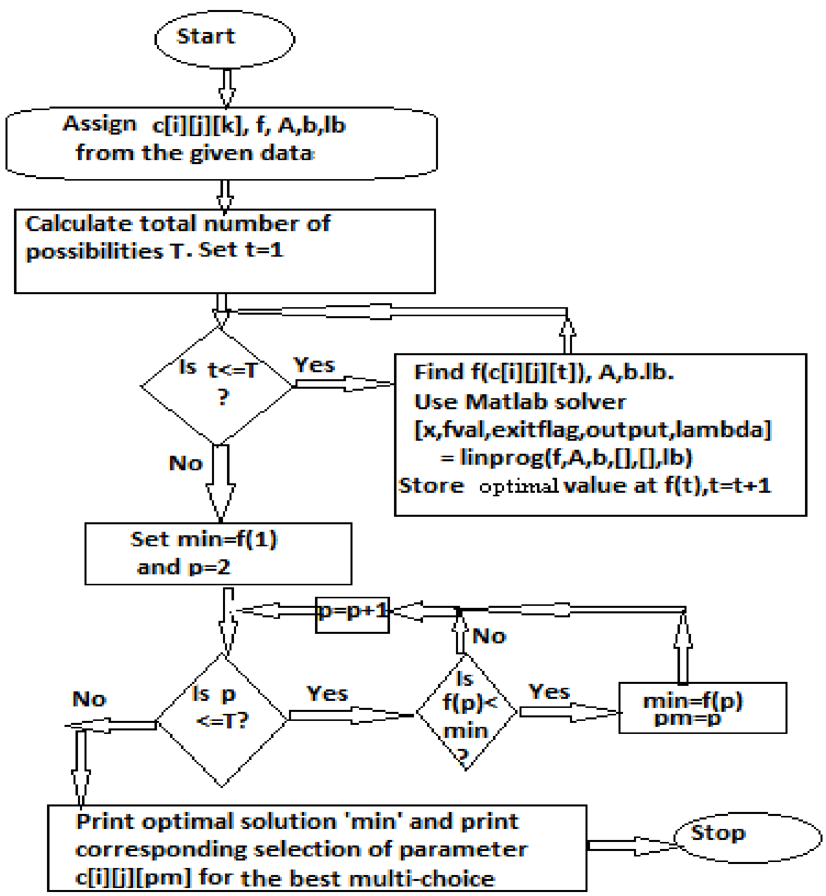

Figure 10.1: Flowchart of the algorithm for optimal solution

rithm 10.1, reader can find the solution of multi-item multi-choice IOIT with stochastic supply.

10.4 Numerical example

Here, we show that the model of multi-choice IOIT produces a better result in compare to the solution obtained from basic IO and TP.

A reputed gas agency has two distribution centers G1 and G2 and the centers deliver two type of gasses to four dealers at the different places B1, B2, B3 and B4.

The following data are given for distributing the Type 1 gas:

The demand rate D1 of B1 is 15 ton/year, the unit inventory sustaining cost C1 is $ 16 per ton/year, the fixed order cost C2 is $ 24 per time, the price of gas per unit ton is $ 100/ton, the safe stock S1 is 1.4 ton; for gas station B2, the demand rate D2 is 25 ton/year, the safe stock S2 is 2.8 ton, the others are same as gas station B1; for gas station B3, the demand rate D3 is 20 ton/year, the safe stock S3 is 2.2 ton, the others are same as gas station B1; for gas station B4, the demand rate D4 is 16 ton/year, the safe stock S4 is 1.8 ton, the others are same as gas station B1. The transportation cost (in Dollar($)) is given in Table 10.1.

Table 10.1: Transportation cost for Type 1 gas/ton.

	B1	B2	B3	B4
G1	135,132,130	128,130	105	157,150,155
G2	116,120	144	131,135,129,140	125,120,130

Assuming that the mean and variance of exponential stochastic variable with Specified Probability Level (SPL) of supplies i.e., a_i for $i = 1, 2$ are represented in Table 10.2.

Table 10.2: Table represents the data for Type 1 gas.

Mean	Variance	SPL
$E(a_1)$=θ_1= 21.5	$V(a_1) = (21.5)^2$	α_1=0.61
$E(a_2)$=θ_2= 16	$V(a_2) = (16)^2$	α_2=0.62

For the distribution of Type 2 gas, the following data are given:

The demand rate D1 of B1 is 10 ton/year, the unit inventory sustaining cost C1 is $6 per ton/year, the fixed order cost C2 is $10 per time, the price/unit ton gas is $50, the safe stock S1 is 1.0 ton; for gas station B2, the demand rate D2 is 15 ton/year, the safe stock S2 is 1.5 ton, the others are same as gas station B1; for gas station B3, the demand rate D3 is 12 ton/year, the safe stock S3 is 1.2 ton, the others are same as gas station B1; for gas station B4, the demand rate D4 is 14 ton/year, the safe stock S4 is 2.0 ton, the others are same as gas station B1. The transportation cost/ton (in Dollar($)) is given in Table 10.3.

Table 10.3: Transportation cost for Type 2 gas/ ton.

	B1	B2	B3	B4
G1	10	15	12	18
G2	15	12	11	15

Considering that the mean and variance of exponential random variable with specified probability level of supplies i.e., a_i for $i = 1, 2, 3$ are represented in Table 10.4.

Table 10.4: Table represents the data for Type 2 gas.

Mean	Variance	SPL
$E(a_1)$=θ_1= 18	$V(a_1) = (18)^2$	α_1=0.60
$E(a_2)$=θ_2= 17	$V(a_2) = (17)^2$	α_2=0.55

10.4.1 Solution by basic IO model

To find the inventory cost for the destinations B1, B2, B3 and B4, we have

For Type 1 gas:

For dealer at B1: $t_{0_1} = \sqrt{\frac{2C_2}{C_1 D}} = \sqrt{\frac{2\times 24}{16\times 15}} = 0.45$

$R_{0_1} = \sqrt{\frac{2C_2 D}{C_1}} = \sqrt{\frac{2\times 24\times 15}{16}} = 6.71$

At B2: $t_{0_2} = \sqrt{\frac{2C_2}{C_1 D}} = \sqrt{\frac{2\times 24}{16\times 25}} = 0.35$

$R_{0_2} = \sqrt{\frac{2C_2 D}{C_1}} = \sqrt{\frac{2\times 24\times 25}{16}} = 8.67$

Again at B3: $t_{0_3} = \sqrt{\frac{2C_2}{C_1 D}} = \sqrt{\frac{2\times 24}{16\times 20}} = 0.39$

$R_{0_3} = \sqrt{\frac{2C_2 D}{C_1}} = \sqrt{\frac{2\times 24\times 20}{16}} = 7.75$

Also at B4: $t_{0_4} = \sqrt{\frac{2C_2}{C_1 D}} = \sqrt{\frac{2\times 24}{16\times 16}} = 0.43$

$R_{0_4} = \sqrt{\frac{2C_2 D}{C_1}} = \sqrt{\frac{2\times 24\times 16}{16}} = 6.93$

Since the delivery cycles of four gas stations are different, So, in order to minimize the transportation costs, we need to unit their cycle, regarding 0.45 as the standard cycle, the order volume of B2 after adjustment is $8.67+\frac{0.45}{15}\times 25 = 9.42$, the order volume of B3 after adjustment is $7.75+\frac{0.45}{15}\times 20 = 8.35$, the order volume of B4 after adjustment is $6.93+\frac{0.45}{15}\times 16 = 7.41$ Then the total inventory cost for B1 is $24/.45 + 100 \times 15 + \frac{1}{2} \times 160 \times 15 \times .45 = 2093.33(\$)$. The total inventory cost for B2 is $24/0.35+100\times 25+\frac{1}{2}\times 160\times 25\times 0.35 = 3268.57(\$)$. The total inventory cost for B3 is $24/.39+100\times 20+\frac{1}{2}\times 160\times 20\times 0.39 = 2685.54(\$)$. The total inventory cost for B4 is $24/.43 + 100 \times 16 + \frac{1}{2} \times 160 \times 16 \times .43 = 2206.21(\$)$.

So, the total inventory cost for Type 1 gas is $(2093.33 + 3268.57 + 2685.54 + 2206.21)(\$) = 10253.65(\$)$. **For Type 2 gas:**

For dealer at B1: $t_{0_1} = \sqrt{\frac{2C_2}{C_1D}} = \sqrt{\frac{2\times 10}{6\times 10}} = 0.58$,

$R_{0_1} = \sqrt{\frac{2C_2D}{C_1}} = \sqrt{\frac{2\times 10\times 10}{6}} = 5.77$,

At B2: $t_{0_2} = \sqrt{\frac{2C_2}{C_1D}} = \sqrt{\frac{2\times 10}{6\times 15}} = 0.47$,

$R_{0_2} = \sqrt{\frac{2C_2D}{C_1}} = \sqrt{\frac{2\times 10\times 15}{6}} = 7.07$,

Again at B3: $t_{0_3} = \sqrt{\frac{2C_2}{C_1D}} = \sqrt{\frac{2\times 10}{6\times 12}} = 0.53$,

$R_{0_3} = \sqrt{\frac{2C_2D}{C_1}} = \sqrt{\frac{2\times 10\times 12}{6}} = 6.32$,

At B4: $t_{0_4} = \sqrt{\frac{2C_2}{C_1D}} = \sqrt{\frac{2\times 10}{6\times 14}} = 0.49$,

$R_{0_4} = \sqrt{\frac{2C_2D}{C_1}} = \sqrt{\frac{2\times 10\times 14}{6}} = 6.83$.

Since the delivery cycles of four gas stations are different, So, in order to minimize the transportation costs, we need to unit their cycle, regarding 0.58 as the standard cycle. The order volume of B2 after adjustment is $7.07+\frac{0.58}{10}\times 15 = 7.94$. the order volume of B3 after adjustment is $6.32 + \frac{0.58}{10} \times 12 = 7.02$. The order volume of B4 after adjustment is $6.83 + \frac{0.58}{10} \times 14 = 7.64$. Then the total inventory cost for B1 is $10/0.58 + 50 \times 10 + \frac{1}{2} \times 6 \times 10 \times 0.58 = 534.64(\$)$. The total inventory cost for B2 is $10/0.47 + 50 \times 15 + \frac{1}{2} \times 6 \times 15 \times 0.47 = 792.43(\$)$. The total inventory cost for B3 is $10/0.53+50\times 12+\frac{1}{2}\times 6\times 12\times 0.53 = 637.95(\$)$. The total inventory cost for B4 is $10/0.49+50\times 14+\frac{1}{2}\times 6\times 14\times 0.49 = 740.99(\$)$. So, the total inventory cost for Type 2 gas is $534.64 + 792.43 + 637.95 + 740.99 = 2706.01(\$)$ and hence the total inventory cost for two types of gas is

$(10253.65 + 2706.01)(\$) = 12959.66.(\$)$

Now, for the multi-choice multi-item transportation model according to the supplied data is described in Model 10.6 as follows:

Model 10.6

$$\begin{aligned}
\text{minimize} \quad & Z = (135, 132, 130)\, x_{111} + (128, 130)\, x_{121} + 105x_{13} \\
& + (157, 150, 155)\, x_{141} + (116, 120)\, x_{211} + 144x_{22} + \\
& (131, 135, 129, 140)\, x_{231} + (125, 120, 130)\, x_{241} + 10x_{112} + 15x_{122} \\
& +12x_{132} + 18x_{142} + 15x_{212} + 12x_{222} + 11x_{232} + 15x_{242} && (10.14) \\
\text{subject to} \quad & \Pr\left(x_{111} + x_{121} + x_{131} + x_{141} \leq 21.50\right) \geq 1 - 0.61, && (10.15) \\
& \Pr\left(x_{211} + x_{221} + x_{231} + x_{241} \leq 16.00\right) \geq 1 - 0.62, && (10.16) \\
& \Pr\left(x_{112} + x_{122} + x_{132} + x_{142} \leq 18\right) \geq 1 - 0.60, && (10.17) \\
& \Pr\left(x_{212} + x_{222} + x_{232} + x_{242} \leq 17\right) \geq 1 - 0.55, && (10.18) \\
& x_{111} + x_{211} \geq 6.71, && (10.19) \\
& x_{121} + x_{221} \geq 9.42, && (10.20) \\
& x_{131} + x_{231} \geq 8.35, && (10.21) \\
& x_{141} + x_{241} \geq 7.41, && (10.22) \\
& x_{112} + x_{212} \geq 5.77, && (10.23) \\
& x_{122} + x_{222} \geq 7.94, && (10.24) \\
& x_{132} + x_{232} \geq 7.02, && (10.25) \\
& x_{142} + x_{242} \geq 7.64, && (10.26) \\
& x_{ijt} \geq 0 \quad (i = 1, 2;\ j = 1, 2, 3, 4;\ t = 1, 2). && (10.27)
\end{aligned}$$

Using the technique described in subsection 10.3.1, the stochastic constraints (10.15) to (10.18) reduce to the following form:

$$x_{111} + x_{121} + x_{131} + x_{141} \leq 20.25, \tag{10.28}$$

$$x_{211} + x_{221} + x_{231} + x_{241} \leq 15.50, \tag{10.29}$$

$$x_{112} + x_{122} + x_{132} + x_{142} \leq 16.49, \tag{10.30}$$

$$x_{212} + x_{222} + x_{232} + x_{242} \leq 13.57. \tag{10.31}$$

We find the values of $f(=Z)$, A, b, lb from (10.14) and (10.19)-(10.31) and then we solve it by MATLAB programming.
Solving the equation (10.14) under the constraints (10.19)-(10.31) using MATLAB programming, we obtain the optimal solution as follows:
$x_{111} = 0.0, x_{121} = 9.42, x_{131} = 6.41, x_{141} = 0.0, x_{211} = 8.35, x_{221} = 0.0, x_{231} = 0.0, x_{241} = 7.41, x_{112} = 5.77, x_{122} = 7.94, x_{132} = 0.0, x_{142} = 0.0, x_{212} = 0.0, x_{222} = 6.55, x_{232} = 7.02, x_{242} = 0$; and the value of the total transportation cost for transpoting two types of gas is 4107.92(\$). The total logistic cost is the sum of inventory cost for Type 1 gas, inventory cost for Type 2 gas and the transportation cost. So, the total logistic cost is $(4107.92 + 12959.66)(\$) = 17067.58(\$)$.

10.4.2 Solution by IOIT model

Let us construct the mathematical model under IOIT technique to the following problem as:

Model 10.7

$$
\begin{aligned}
\text{minimize} \quad & Z = 100\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij1} + \frac{1}{2}(W_{11} + W_{21} + W_{31} + W_{41})16 + \\
& +50\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij2} + \frac{1}{2}(W_{12} + W_{22} + W_{32} + W_{42})6 + \\
& (135, 132, 130)\, x_{111} + (128, 130)\, x_{121} + 105x_{131} + \\
& (157, 150, 155)\, x_{141} + (116, 120)\, x_{211} + 144x_{221} + \\
& (131, 135, 129, 140)\, x_{231} + (125, 120, 130)\, x_{241} + \\
& 10x_{112} + 15x_{122} + 12x_{132} + 18x_{142} + \\
& 15x_{212} + 12x_{222} + 11x_{232} + 15x_{242} \qquad (10.32) \\
\text{subject to} \quad & \Pr(x_{111} + x_{121} + x_{131} + x_{141} \le 21.50) \ge 1 - 0.61, \qquad (10.33) \\
& \Pr(x_{211} + x_{221} + x_{231} + x_{241} \le 16.00) \ge 1 - 0.62, \qquad (10.34) \\
& \Pr(x_{112} + x_{122} + x_{132} + x_{142} \le 18) \ge 1 - 0.60, \qquad (10.35) \\
& \Pr(x_{212} + x_{222} + x_{232} + x_{242} \le 17) \ge 1 - 0.55, \qquad (10.36) \\
& W_{11} \ge 1.4, \quad W_{21} \ge 2.8, \qquad (10.37)
\end{aligned}
$$

$$W_{31} \geq 2.4, \quad W_{41} \geq 1.8, \tag{10.38}$$

$$W_{12} \geq 1.0, \quad W_{22} \geq 1.5, \tag{10.39}$$

$$W_{32} \geq 1.2, \quad W_{42} \geq 2.0, \tag{10.40}$$

$$W_{11} + W_{21} + W_{31} + W_{41} = 31.89, \tag{10.41}$$

$$W_{12} + W_{22} + W_{32} + W_{42} = 28.37, \tag{10.42}$$

$$x_{111} + x_{211} = W_{11}, \tag{10.43}$$

$$x_{121} + x_{221} = W_{21}, \tag{10.44}$$

$$x_{131} + x_{231} = W_{31}, \tag{10.45}$$

$$x_{141} + x_{241} = W_{41}, \tag{10.46}$$

$$x_{112} + x_{212} = W_{12}, \tag{10.47}$$

$$x_{122} + x_{222} = W_{22}, \tag{10.48}$$

$$x_{132} + x_{232} = W_{32}, \tag{10.49}$$

$$x_{142} + x_{242} = W_{42}, \tag{10.50}$$

$$x_{ijt} \geq 0 \quad (i = 1, 2;\ j = 1, 2, 3, 4;\ t = 1, 2). \tag{10.51}$$

We find the values of $f(= Z), A, b, lb$ from (10.32), (10.28)-(10.31) & (10.37)-(10.51) and then we solve it by MATLAB programming. Solving the equation (10.32) under the constraints (10.28)-(10.31) & (10.37)-(10.51) by using MATLAB programming, we obtain the optimal solution as follows:
$x_{111} = 0, x_{121} = 2.80, x_{131} = 17.45, x_{141} = 0.0, x_{211} = 9.84, x_{221} = 0, x_{231} = 0, x_{241} = 1.80$, $x_{112} = 14.99, x_{122} = 0.0, x_{132} = 1.5, x_{142} = 0.0, x_{212} = 0.0, x_{222} = 2.0, x_{232} = 9.88, x_{242} = 2.0$. The supplied quantities of demand nodes for Type 1 gas are $W_{11} = 9.84, W_{21} = 2.80, W_{31} = 17.45, W_{41} = 1.80$ and the supplied quantities of demand nodes for Type 2 gas are $W_{12} = 14.49, W_{22} = 3.50, W_{32} = 9.88, W_{42} = 1.80$. The selection of cost parameters for Type 1 gas are $C_{111} = 130, C_{121} = 128, C_{131} = 105, C_{141} = 150, C_{211} = 116, C_{221} = 144, C_{231} = 129, C_{241} = 120$. Also, from the solution, we obtain the total logistic cost: 8800.90($). In this logistic cost, i.e., the sum of the inventory cost and the transportation cost which is minimized together through the proposed model.

10.5 Sensitivity analysis

We solve the proposed problem using two techniques, namely IO and IOIT. In IO technique, the total logistic cost is calculated by adding the invenory cost and transportation cost separately. The solution through IOIT technique produced the logistics cost through the algorithm directly, and we see that the IOIT technique for calculating total logistics cost is significantly less than the total logistic cost through IO technique. From the obtained solution, it is observed that in IOIT model with multi-choice concept saves the cost $[\frac{17067.58-8800.90}{17067.58}] \times 100\% = 48\%$ in compare with the cost of basic IO model. The costs corresponding to IO and IOIT techniques in different aspects along with the total cost are shown in Figures 10.2 and 10.3 respectively.

According to the proposed mathematical model, the reduction of total logistic cost is made due to the fact that in IOIT, if the order quantity of goods are increased then the inventory cost becomes smaller, consequently the total logistics cost per unit item becomes smaller than the traditional IO technique. In Figures 10.2 and 10.3, it is clearly presented that the total logistic cost (\$) per unit item with respect to order volume (ton) is less in IOIT than IO technique. So, the obtained solution of proposed problem by IOIT is better than the solution by traditional IO technique.

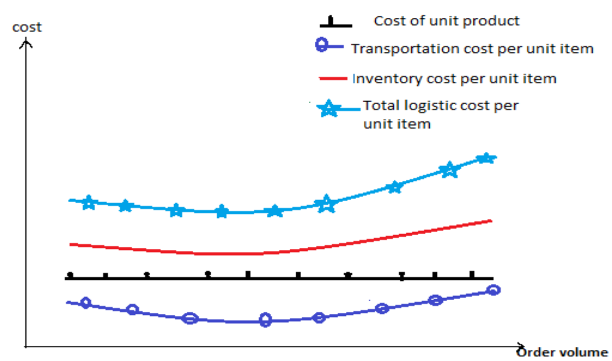

Figure 10.2: Cost in IO technique per unit item.

Here, we consider the multi-choice of transportation cost in the proposed prob-

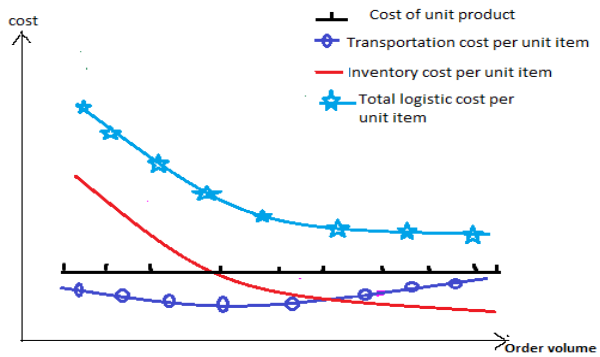

Figure 10.3: Cost in IOIT technique per unit item.

lem, and so in the algorithm, we present the situation that each combination of multi-choice costs presents an optimal solution for total transportation cost in IO technique and total logistic cost in IOIT technique.

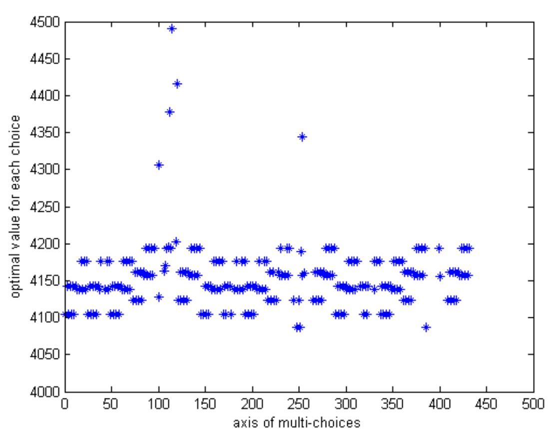

Figure 10.4: Cost of transportation for different choices in IO.

Utilizing the obtained solutions, we draw the Figures 10.4 and 10.5, which show the total transportation cost in IO technique and total logistic cost in IOIT technique respectively, for different combinations of multi-choice of parameters. The proposed problem is solved by an algorithm, and from the solution, we conclude that how much amount has to be paid for the destinations B1, B2, B3 and B4; when the demand points are managed by only one person.

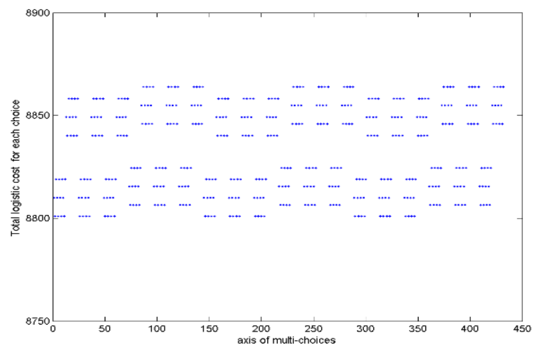

Figure 10.5: Logistic cost for different choices in IOIT.

In this regard, we present the Figures 10.6 and 10.7, which represent the amount of transportation cost, inventory cost and total logistic cost to be paid for the destination points B1, B2, B3 and B4.

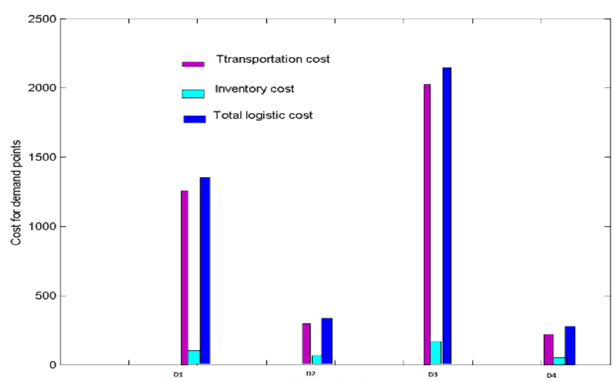

Figure 10.6: Cost at demand points for Type 1 gas.

Here, the scale of cost ($) for the Figures 10.4, 10.5, 10.6 and 10.7 are considered according to the best view of the figures to the readers, they are not correlated to each other in same scale difference, but in all Figures cost axes are in same unit $.

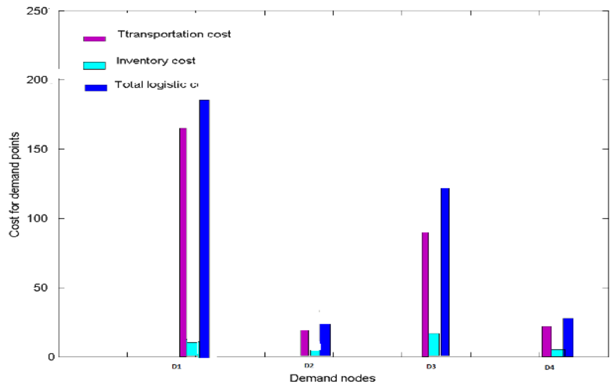

Figure 10.7: Cost at demand points for Type 2 gas.

10.6 Conclusion

In this chapter, we have focused on the study of integrated optimization in inventory transportation along with the transportation problem and combining these produce a problem named as IOIT. Here, we have considered the transportation cost in multi-choice type and the supply as random due to unstable situation of market. Again by considering the globalization situation, the decision maker is to be transported the multi-item goods instead of single item. Because of that, we have formulated the mathematical model of multi-item multi-choice IOIT with random supply. To transform the multi-choice IOIT model with random supply, we have used the stochastic programming. From the solutions of our presented numerical example, we have concluded that IOIT produces better result in compare with the solution of traditional inventory model with TP.

Chapter 11

Conclusions and Scope of Future Works

11.1 Conclusions

Transportation problem is a widely used in decision making problems under different grounds of Operations Research. In the decision making problems, the utmost important factor is decision parameters. The thesis is designed based on the study of multi-choice programming in the transportation problem. Multiple choice of parameters is a form of assessment in which DM requires to select the best possible choice out of the choices from a list which produces optimal solution. Due to presence of multiple routes of transportation, multiple type vehicles, weather condition, unpredictable market scenario, etc. the transportation parameters become multi-choice type. Keeping these points of view, we have incorporated the study of TP under multi-choice environment and established the efficiency of multi-choice study throughout the thesis.

The classical model of TP is not always produced the optimal solution according to purchasers perspectives. Introducing the concept of non-linear cost in MOTP, we have developed a non-linear MOTP under multi-choice demands. A generalized reduction procedure to convert the constraints involving multi-choice demand parameters into deterministic form is presented using binary variables. Thereafter, an algorithm is included to solve the proposed model and the proposed study in Chapter-1 and compared with related study by a numerical example and to establish the remarkable effects of the study in Chapter 2 in the proposed thesis.

The solution of the MOTP presents a set of Pareto optimal points, but a solution through the existing weighting methods is not so effective in real-life situations. In that situation, the objective functions of MOTP considers the goals, which play an important role to find a better optimal solution through goal programming. To make the solution more effective, we have introduced the utility function approach in the study of MOTP in Chapter 3. Concept of utility, in this chapter, propose a new way for extending the utilization of real-life MOTP and MCMTP and improves the skill for representing the DM's preferences in solving decision making problems.

Again, we have solved the multi-choice multi-objective transportation problem by employing the Conic Scalarization approach with less number of variables and with minimum computational burden in Chapter 4. The MCMTP is given a new direction to handle the real-life multi-objective transportation problem when the transportation parameters are multi-choices in nature. Two numerical examples are presented in this chapter to explore the applicability and suitability of our approach for solving MOTP and MCMTP with consideration of decision maker preferences.

Time is an important factor for transporting the goods in a TP. So, the cost and the time minimizing TP with multi-choice interval valued transportation parameter are considered in Chapter 5 of the proposed thesis. The main aim of this chapter is to minimize the transportation cost as well as minimize the transportation time for transporting the perishable goods through single objective TP under the environment of multi-choice interval valued programming. Till now, researchers have used the methodology of multi-objective transportation problem to optimize the transportation cost and time but in this chapter, we have optimized the transportation cost and time without using multi-objective TP. To show the reality and feasibility of the situation, a case study is considered in the chapter.

Considering the real-world situations, we have introduced the concept of fuzzy decision variable in the transportation problem in Chapter 6. A technique to solve a TP under fuzzy decision variable is furnished in this chapter in both single objective and multi-objective ground of TP. Again, in this study, the goal of purchaser as well as seller are given equally importance, whereas, most of the FTP, considered the goal of DM i.e., seller only. The concept has been extended in multi-objective ground to obtain a better result in compare to the existing method like GP.

Thereafter, we have analyzed the real-life MOTP through the concept of reliability and uncertain environments. We have proposed a new kind of uncertainty on cost parameter based on the concept of reliability in Chapter 7. Besides, we have established the MOTP under the consideration of fuzzy multi-choice goals to the objective functions and the supply and demand are taken as uncertain in nature. A solution procedure for solving the MOTP; and the selection of goals for the objective functions has been discussed by taking a real-life example. This procedure is not only proposed the subjective preference into real-life decision-making problems, but also can realize the better selection of goals to the objective functions.

Furthermore, in the proposed thesis, we have proposed a Two-Stage MOTP with the interesting characteristics that one is for the goal preferences of the DM and another one is the selection of particular values of supply and demand from the interval grey supply and demand. Solution of the MOTP under these goals through RMCGP has provided a technique for multi-objective decision making. But, the main drawback of RMCGP consists of how the DM would select the goals for the objective functions. So, we have constructed a utility function (choice by the DM) for selecting the optimal goal region. After this selection, we have solved the proposed problem by our RMCGP approach. The proposed multi-objective Two-Stage TP has provided a new direction to select proper goals for real-life multi-objective transportation problems under the environment of interval grey supply and demand.

There may have some situations in a transportation system, due to the presence of multiple-mode of transportation in which the traditional TP fails to formulate a mathematical model and find the least-cost route of transportation. We have implemented a mathematical model MMTP, which may be applied to formulate the mathematical model under multiple-mode of transportation and its solution suggests about the selection of mode of transportation as well as optimal solution of the problem.

Also, we have presented the connection between transportation and inventory optimization problem under multi-choice environment. Though, TP and inventory are two different branches of study, but here we have made a link between them under the environment of multi-choice transportation routes, and then the solution procedure is presented. Again, by considering the globalization situation, the decision maker needs to be transported the multi-item goods instead of single item. Because of that, we have formulated the mathematical model of multi-item multi-choice IOIT with random supply. To transform the multi-choice IOIT model with random supply, we have used the stochastic programming. The IOIT study reduces logistics cost significantly, which is presented in Chapter 10 of our proposed thesis.

11.2 Future works

Studying more concepts of the presented models may be a paradigm for future research. Moreover, applying this new concept is a challenging task to deal with real world decision making problems for further research.

There are many avenues of future work arising from this thesis, A few of them are appended below:

(i) The transportation problems have wide applications in many real-life problems of practical importance which reduce the cost specially in business environment. Multi-objective transportation problem with non-linear cost still exists in so many cases of managerial decision making problem such as planning of many complex resource allocation systems in the areas of industrial production, storing of foods, in which demands are of multi-choice type in practical situation. The contents of this chapter may be a source of producing better results in such kind of complex decision making situations.

(ii) The application of the utility function approach is an important area in a new research field of MOTP problems under uncertain environments.

(iii) The study of Conic Scalarization approach may be applied for a better result on the fuzzy transportation problem. Furthermore, the notion of multi-choice parameters can also be used in real-world supply chain management problems. In that context, the number of variables increases in GP or RMCGP but in Conic Scalarization approach with less number of variables, the proposed method allows for a better solution satisfying all the goals; consequently, the decision maker can take a proper decision under a multi-choice environment of multi-objective transportation problem.

(iv) The study of non-linearity in a TP through cost parameter can be extended to supply and demand parameters of the transportation problem.

(v) The time-cost minimizing concept of proposed thesis may be used to solve time-cost trade off transportation problem in multi-objective environment when the parameters incorporate the uncertain type of data.

(vi) The approach of Two-stage TP can be addressed with different types of data, such as fuzzy, stochastic, etc., and tested several examples such as route selection problem with consideration of real-life Two-Stage TP, technology selection problem, plant location selection problem, etc., using our proposed methodology. In addition to the aforementioned, one can employ our study for selecting the optimal goals in inventory optimization, supply chain management, and others.

(vii) The study of MMTP can be broadly analyzed in different real-life uncertain environments.

(viii) The concept of IOIT can be extended in different ground of inventory and transportation problem.

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List of Publications

1. Maity, G., Roy, S.K., (2014) Solving multi-choice multi-objective transportation problem: a utility function approach, *Journal of Uncertainty Analysis and Applications,* Springer Open, Vol. **2**, No. 11, doi:10.1186/2195-5468-2-11.

2. Maity, G., Roy, S.K., (2016) Solving a multi-objective transportation problem with nonlinear cost and multi-choice demand, *International Journal of Management Science and Engineering Management,* Taylor & Francis, ESCI, Vol. **11**, No. 1, pp. 62-70.

3. Maity, G., Roy, S.K., (2016) Solving multi-objective transportation problem with interval goal using utility function approach, *International Journal of Operational Research*, Inderscience, Scopus, Vol. **27**, No. 4, pp. 513-529.

4. Maity, G., Roy, S.K., Verdegay, J.L., (2016) Multi-objective transportation problem with cost reliability under uncertain environment, *International Journal of Computational Intelligence Systems (IJCIS)*, Atlantis Press and Taylor & Francis, SCIE, IF: 0.391, Vol. **9**, No. 5, pp. 839-849.

5. Roy, S.K., Maity, G., Weber, G.W., Alparslan Gök, S.Z., (2016) Conic Scalarization approach to solve multi-choice multi-objective transportation problem with interval goal, *Annals of Operations Research,* Springer, SCI, IF:1.406, DOI 10.1007/s10479-016-2283-4.

6. Maity, G., Roy, S.K., (2016) Multi-objective transportation problem using fuzzy decision variable through multi-choice programming, *International Journal of Operations Research and Information Systems*, IGI Global, Info-SCI, Vol 8, No. 2.

7. Maity, G., Roy, S.K., (2016) Multi-item multi-choice integrated optimization in inventory transportation problem with stochastic supply, *International Journal of Operational Research,* Inderscience, Scopus, Forthcoming.

8. Roy, S.K., Maity, G., (2016) Minimizing cost and time through single objective function in multi-choice interval valued transportation problem, *Journal of Intelligent & Fuzzy Systems*, IOS Press, SCIE, IF: 1.004.

9. Roy, S.K., Maity, G., Weber, G.W., Multi-objective two-stage grey transportation problem using utility function with goals, submitted after revision in *Central European Journal of Operations Research*, Springer, SCI, IF: 0.978.

The list of Communicated Papers in Journals

10. Maity, G., Roy, S.K., Transportation problem under fuzzy decision variable, Communicated to International Journal.

11. Maity, G., Roy, S.K., A new approach for solving transportation problem using multi-modal systems, Communicated to International Journal.

List of presented papers in conferences/seminars

1 Presented a paper entitled **Transportation problem with non-linear cost and multi-choice demand under multi-objective environment** in the international conference on "Facets of Uncertainties and Applications", ORSI Kolkata Chapter and Dept. of Applied Mathematics, Kolkata, West Bengal, 5-7 Dec, 2013.

2 Presented a paper entitled **Utility function approach to solve multi-objective transportation problem** in the National seminar "NSRAMA", Vidyasagar University, Midnapore, West Bengal, 25-26 Feb, 2014.

3 Presented a paper entitled **Solving multi-objective transportation problem: a utility function approach** in the international conference on "ICMES", Chitkara University, Himachal Pradesh, 20-22 March, 2014.

4 Presented a paper entitled **Multi-choice Transportation Problem under Fuzzy Decision Variable** in the international conference on "Futuristic Trends in Computational analysis and Knowledge management", IEEE conference, Amity University, Greater Noida, Uttar Pradesh, 25-27 Feb., 2015.

5 Presented a paper entitled **Multi-choice Multi-objective Optimization and Its Application to Transportation Problem using Utility Function Approach** in the 2nd international conference on "Recent Trends in Mathematics and Its Applications", Vidyasagar University, Midnapore, West Bengal, 18-19 March, 2015.

6 Presented a paper entitled **Cost and Time Minimizing Transportation Problem using Single Objective Function** in the 1st international conference on "Frontiers in Mathematics", Gauhati University, Guwahati, 26-28 March, 2015.

7 Presented a paper entitled **Solving Multi-item Multi-choice Integrated Optimization Problem in Inventory Transportation under Uncertain Supply** in "National Conference on Optimization and

Computational Mathematics", Vidyasagar University, Midnapore, West Bengal, 1-2 March, 2016.

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