Synopsis of the proposed thesis entitled

Studies On Imperfect Production Inventory System Under Different Environments

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List of Published Papers

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2. An EPQ model with promotional demand in random planning horizon: population varying genetic algorithm approach. ***Journal of Intelligent Manufacturing, 27(1) (2016) 1-17***, **Springer**.

3. Three-layer supply chain in an imperfect production inventory model with two storage facilities under fuzzy rough environment. ***Journal of Uncertainty Analysis and Applications, 2(1) (2014) 1-31***, **Springer**.

4. Multi-item EPQ model with shortages, rework and learning effect on imperfect production over fuzzy-random planning horizon. ***Journal of Management Analytics, (2016), DOI 10.1080/23270012.2016.1217755***, **Taylor & Francis**.

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5. A deteriorating manufacturing system considering inspection errors with discount and warranty period dependent demand. *Communicated.*

6. A fuzzy imperfect production inventory model based on fuzzy differential and fuzzy integral method. *Communicated.*

7. Two layers supply chain in an imperfect production inventory model with two storage facilities under reliability consideration. *Communicated.*

8. GA approach for controlling GHG emission from industrial waste in two plant production and reproduction inventory model with interval valued fuzzy pollution parameters. *Communicated.*

9. Two layers supply chain imperfect production inventory model with fuzzy credit period, time and production rate dependent imperfectness. *Communicated.*

1 General Introduction

Inventory Control System in Operations Research:
Operations Research (OR) was introduced during the Second World-War. Basically, it is the collection of modern methods on the problems arising in the management of large systems of men, machines, materials and money related to industry, business and defence. During this war, the military management in England called upon a team of scientist to study the strategic and tactical problems related to air and land defence of the country with very limited military resources, it was necessary to decide upon the most effective utilization of them, e.g., the efficient ocean transport, effective bombing, etc. As the team was dealing with research on (military) operations, the work of this team of scientists was named as 'Operational Research' in England.

After the end of war, the success of military teams attracted the 'Industrial managers', who were seeking solutions to their complex executive-type problems. In this field, the first mathematical technique called the Simplex Method of linear programming was developed in 1947 by American mathematician, George B. Dantzig (Dantzig (1963). Since then, new techniques and applications have been developed through the efforts and cooperations of interested individuals in academic institutions and industries both. According to Churchman et al. (1957), OR is defined as the application of scientific methods, techniques and tools to decision making problems (DMP) involving the operations of systems so as to provide these in the control of the operations with optimum solutions to the problem.

Today, the impact of OR can be felt in many areas. Apart from military and business applications, the OR activities include transportation system, library, hospital, city planning, financial institution, etc. For example, in real life, it is observed that a small retailer knows roughly the demand of his/her customer in a month or a week and accordingly places order on the wholesaler to meet the demand of his/her customer. But, this is not the case with a manager of a big departmental store or a big retailer, because in such cases the stocking depends on several factors, such as demand, time of ordering, time lag between the order and actual receipt, deterioration, amelioration, time value of money, inflation, trade credit period etc. and the impreciseness of these factors. So, the problem for managers/retailers is to have a compromise between over-stocking and under-stocking. The study of such type of problem is known by the term '*Inventory Control'*.

Inventory:
In broad sense, *inventory* is defined as an idle resource of an enterprize / company / manufacturing firm. It can be defined as a stock of physical goods, commodities or other economic resources which are used to meet the customer's demand or requirement of production. The inventories or stock of goods are classified into the following forms.

Imperfect Production:
In real world manufacturing system, it is seen that every produced item may not be 100% perfect due to different factors involved in the system such as machine, ray-materials, labour etc. In any production system, it is seen that initially the production process is in an in-control state, because every factors in the system are fresh and the items produced perfect quality items. Generally, increasing production-run-time increases the probability

of components of machine failure and impatience of labor staff, and thus accelerates the deterioration of the quality of the product. However, the production process starts in an in-control state by producing perfect items, and then it may become out-of-control state by producing mixture of perfect and imperfect item due to deterioration of machinery system as well as other factors. The rate of imperfectness may be constant or variable and also considered as crisp, stochastic or fuzzy in nature.

Different Environments:

The parameters, like inventory cost (viz., unit production cost, holding cost, set-up cost, shortage cost, transportation cost, advertisement cost, etc.), demand, available resources, etc. involved in the inventory system may be deterministic (crisp/precise) or some of these may be non-deterministic (i.e., stochastic or imprecise or both stochastic and imprecise). Depending on the nature of such parameters, the environment in which inventory models are developed can be classified as follows:

Crisp Environment: When all the system parameters and the resources, etc. are deterministic and precisely defined, the environment is known as crisp environment.

Stochastic Environment: In this environment, it may happen that the demand or any factor of a commodity in the society is uncertain, not precisely known, but some past data about it is available. From the available records, the probability distribution of demand or any other factor of the commodity can be determined and with that distribution the inventory control problem can be analyzed and solved.

Fuzzy Environment: In this system, some parameters and /or resources are fuzzy in nature. For example, when management launches a new product then they have no knowledge about demand and the other factors related to the product. Then management needs to collect the demand and the others information from experts. If the expert's opinion is imprecise then demand or other factors related to the expert opinion to be taken as a fuzzy and the corresponding environment is known as fuzzy environment.

Fuzzy-Stochastic Environment: It is an environment, which is the combination of both stochastic and fuzzy environments. Here, some parameters are fuzzy and some others are random. Some constraints / resources may be imprecise.

2 Historical Literature Reviews on Imperfect Production Inventory System in Different Environments

In developing the models, researchers have adopted many of the usual assumptions in imperfect production inventory, viz. (i) finite/infinite planning horizon, (ii) constant/varying production rate, (iii) constant/varying defective rate, screening rate and reworked rate, (iv) constant/varying demand rate, (v) allowing shortage or without shortage, (vi) constant/known distribution production time, (viii) single/multi-item production inventory system, (ix) single/multi-retailer, (x) inventory problems with delay in payment, etc. The available models can be grouped in the following ways:

(i) Imperfect Production Inventory Models in Crisp Environment

(ii) Imperfect Production Inventory Models in Fuzzy Environment

(iii) Imperfect Production Inventory Models in Stochastic Environment

(iv) Imperfect Production Inventory Models in Fuzzy Stochastic Environment

2.1 Historical Literature Reviews on Imperfect Production Inventory Models in Crisp Environment

In this environment, many parameters in imperfect production inventory system such as demand, production, production run time, inventory costs etc. are crisp nature. The earliest analysis of an inventory system was developed by Ford Harris [25] of Westinghouse Corporation, USA, in 1915. He derived the classical lot size formula. The same formula was also developed independently by R. H. Wilson [52], after few years and it has been named as Harris-Wilson model or Wilson's model.

In the existing literature of Economic Production Quantity (EPQ) models, most of the works are based on imperfect units. Lin et al. [39] formulated an integrated production inventory models with imperfect production process. Hsu and Hsu [27] developed an integrated vendor-buyer model with defective items which are treated as a single batch and returned to the vendor after a 100% screening process. For inventory problems, many studies considered the items produced as perfect in their models. However, imperfect items are produced due to non-ideal production processes. Wee et al. [50] have investigated the effect of imperfect items on the EOQ model. Taleizadeh et al. [49] presented an EPQ model with rework process for a single stage production with one machine.

Now from the literature survey on imperfect production inventory models, it is seen that there exist two classes of the models on the basis of inspection methods to sort out the defective units from the perfect one. In one class of research, it has been seen that over the period of time the produced items deteriorate in manufacturing system. In this field, the different researchers (Lee and Rosenblatt [43], Kim et al. [34], Jaber et al. [29], Lin et al. [39]) examined an inspection method on the produced items based on deteriorating production process.

In Harris and Wilson [52] lot-size model, demand was assumed as constant, but, in reality, it depends on several factors like time, initial/ on-hand displayed stock-level, selling price of an item, advertisement etc. Silver and Meal [47] published a lot size model taking time varying demand. After that, the research on the models with time dependent demand rate have been gradually studied by several researchers such as Donaldson [19], Goswami and Choudhuri [23] and others. According to the market research, it is observed that time-to-time advertisement of an item also changes its demand. For this reason, Cho [13] developed an optimal production and advertising policies in crisp environment. In this area, Bhunia and Maiti [6] and others developed different types of inventory or production models considering different real life situations.

2.2 Historical Literature Reviews on Imperfect Production Inventory Models in Stochastic Environment

In this environment, many parameters in imperfect production inventory system such as demand, production, production run time, inventory costs etc. are random in nature and specified by probability distributions.

In the literature, few Economic Production Lot-size (EPL) models are available for imperfect units in stochastic environment. Rosenblatt and Lee [43] studied the effects of an imperfect production process on the optimal production run time by assuming that time to out-of-control state is exponentially distributed. Hayek and Salameh [28] derived an optimal operating policy for the finite production model under the effect of reworking of imperfect quality items. They assumed that all defective units are repairable and allowed back-orders. Chiu [12] extended the work of Hayek and Salameh [28] and examined an EPQ model with defective items reworking the repairable units immediately. Sana [45] presented an EPL model with random imperfect production process and defective units were repaired immediately when they were produced.

It has been acknowledged that the displayed inventory has an effect on sales for many retail products-especially for style marchandise (Datta and Pal [17], Levin et al. [37]). This means that the demand rates of these items may be dependent on displayed stock level.

It is observed that, the demand rate of an item is influenced by the selling price of an item, as, whenever the selling price of an item increases, the demand for that decreases and vice- verse. Generally, this type of demand is seen for different finished goods. Several authors like Guria et al. [24], Bhunia and Maiti [6], Abad and Jaggi [2], Karimi-Nasab et al. [32] have investigated this type of inventory models.

Now-a-days, in the third world countries, with the introduction of open market system and advent of multi-nationals, there is a stiff competition amongst the companies to capture the maximum possible market. In the literature of discounted inventory problems, Chung and Lin [14] considered inventory replenishment models for deteriorating items in account of time discounting. Abad [1] determined an optimal policy for selling price and lot size when suppliers offer all unit quantity discount. Weng [51] developed a channel coordination model with quantity discounts.

Inventory models with two warehouse facilities, one is existing storage maintained by the own management named as own warehouse (OW) and another is hired on rental basis named as rented warehouse (RW), have been discussed by Hartely [26], Bhunia and Maiti [6], Benkherouf [5], Kar et al. [33], Dey et al. [18] and others.

The initial attempt to analyze the effect of inflation and the time value of money on inventory control systems was made by Buzacott [7] in 1975. He dealt with an EOQ model with inflation subject to different types of pricing policies. Later on, Chandra and Bahner [10] showed the effects of inflation and the time-value of money on some inventory systems. Several authors then extended these works to make the more realistic inventory models under inflation and the time-value of money. Among these works, one can refer the work of Chen [11], Dey et al. [18] and others.

In reality, the green house effect and global warming have gained much attention due to strong and more frequent extreme change of climate. In every developing countries, there is a scope and regulation for measuring and maintaining such carbon-emission. Benjaafar et al. [4] first presented a model that illustrated how carbon emission can be incorporated to a decision-making problem. Dye and Yang [22] studied a deteriorating inventory system under various carbon emissions policies, like carbon taxes, carbon subsides etc.

2.3 Historical Literature Reviews on Imperfect Production Inventory Models in Fuzzy Environments

In this environment, one or more parameters in imperfect production inventory system such as demand, production, production run time, inventory costs etc. are fuzzy in nature.

Introduction to fuzzy set theory and basic ideas of fuzziness was described by Zimmermann [57]. Bellman and Zadeh [3] first introduced fuzzy set theory in decision making process. After that, Zimmermann [57], Zadeh [56], Dubois and Prade [20] developed fuzzy models for single-period inventory problem.

The uncertain nature of demands, production inventory costs, damageable amounts, etc. results the imprecise rate of production. For this reason, imprecise production inventory model is vary much visible in imprecise decision making problem. Lee and Yao [36] developed an economic production quantity (EPQ) model with fuzzy demand quantity and fuzzy production quantity. After that, Lin and Yao [38] considered fuzzy EPQ model.

In inventory control problem, the assumption or consideration of market demand i.e., decay rate of the item is the fundamental issue of the system. In modern age, this rate of demand fluctuates day by day due to many reasons. In this context, inventory model with fuzzy demand was considered by Lee and Yao [36]. After that, Kao and Hsu [31], Dutta et al. [21] developed a single-period inventory model with fuzzy demand.

In spite of different parameters a production inventory system may be imprecise for some other phenomenons. Some times, the nature of fuzziness may be allowable for an imprecise constrains or in many events. Thus, a production inventory system may yield fuzzy differential equations or different inventory costs are expressions of fuzzy integral. Seikkla [46], Chalco-Cano and Roman-Flores [9] solved fuzzy differential equation problem. Wu [53] gives an approach of fuzzy Riemann integral and its numerical integration. As like stochastic environment, for the solution of this type of problems Liu and Iwamura [40] proposed a 'here and now' approach, i.e., the chance constraint programming approach in which a minimum probability level for satisfying each of the constraint is specified. Similarly, possibilistic constraints also may be defined by Zadeh [56], Dubois and Prade [20] and others. Due to imprecise parameters, the objective function (i.e., profit function) becomes fuzzy in nature. Since optimization of a fuzzy objective is not well defined, one can optimize the optimistic/pessimistic returns of the objectives with some degree of possibility/necessity according to requirement as proposed by Liu and Iwamura [40].

In the present competitive business world a permissible delay in payment which is termed as trade credit period in paying for purchasing cost, is very common business practice. It influences the demand of order and reduces the holding cost. Singh et al. [48] developed a two warehouse inventory model in crisp and fuzzy environments respectively with permissible delay in payment. Recently, Das et al. [16] proposed an integrated production inventory model under interactive fuzzy credit period for deteriorating item with several markets.

2.4 Historical Literature Reviews on Imperfect Production Inventory Models in Fuzzy-Stochastic Environment

In this environment, the parameter(s) involved in imperfect production inventory system such as demand, production, production run time, inventory costs etc. are fuzzy-random in nature and specified by imprecise probability distributions.

Introduction of fuzzy random variables and its applications is not much old. Kwakernaak [35] first introduced fuzzy random variables. After that, several researchers Liu [41] and others developed linear and non-linear programming methods in fuzzy stochastic environment with fuzzy stochastic or fuzzy and stochastic data. Petrovic et al. [42] considered the news boy problem with fuzzy demand and fuzzy inventory costs. They considered two fuzzy models one with (i) imprecisely described discrete demand and other with (ii) imprecisely estimated unit holding and unit shortage costs. Yao et al. [55] described a single period inventory management in fuzzy stochastic environment.

Recently, researchers have focused on the situations in which inventory parameters are random as well as imprecise. Models developed in such situations are known as fuzzy stochastic inventory models. In such mixed environment, very few models have been developed. Roy et al. [44] formulated an inventory model for a deteriorating item under fuzzy inflation and time discounting over a random planning horizon, which extend for a EPQ model with random planning horizon by Jana et al. [30]. Das and Maiti [15] developed production inventory model by considering one constraint in fuzzy environment and the other in random environment.

3 Motivation and Objective of the Thesis

In any manufacturing system, the production of defective units is a natural phenomenon occurring from the different difficulties such as raw material, labor experience, machine component, production rate etc. which are arise in a long-run production process. So, it is necessary to inspect each item after production to check whether the item is perfect or not before selling. In the literature, there is a large number of articles on the imperfect production inventory models. But very few researchers have developed the imperfect production inventory models by considering the reworking of defective units and re-manufacturing of returned items. Since in a competitive business world, advertisement, price-discount and warranty period etc. are important factors in creating a market demand in the society, it is essential to study to control inventory in the imperfect production inventory system. Moreover many business people use showrooms to attract the customer by display of stock of the units in the showroom to influence the demand. **Therefore, one objective of our research works is to analyze the effect of the reworking of defective items in the imperfect production inventory model with various demand rate.**

Recently, it is seen that, with the increased competitions in the global business, the manufacturing companies are forced to work closely in partnership with their suppliers, retailers and manufacturers. In supply chain management, the establishment of a long-term cooperative relationship between vendor/ manufacturer and buyer as an integrated Supply Chain Model (SCM) is beneficial for two parties with regard to costs/profits to get the tensionless stable sources of supply and demand as well as smooth running of business to gain optimum profit from each other. For that reason, the coordination in the supply chain is very important; otherwise, each stage of the supply chain acts independently without considering the impacts of the remaining stages in the supply chain and lack of coordination results in less profit than what could be achieved through coordination. **So, in our research works, one of the objectives is to co-ordinate the members of supply chain on imperfect production inventory models in different environment.**

Traditional inventory models generally hypothesize that a retailer pays his/her supplier at the time of purchase. But in real world, it is observed that a supplier often offers a retailer a delayed payment period known as the trade credit period to settle the account. Offering such a credit period, the supplier entices the buyer to increase the size of their order and hence reduce on-hand stock level. From the retailer's view-point, during the credit period he/she can sell the items and continue to accumulate the sales revenue and earn interest from it before payment to be made. Hence, the trade credit has an important role for the decision on the rate of supply and indirectly on the order quantity. So in an integrated inventory system, the offer of a credit period plays an important role to achieve the optimum profits. **Therefore, another objective of our research works is to analyze the effect of the credit period(s) in the supply chain model.**

Many researchers have worked on imperfect production inventory model with the considerations of various types of demand rate. In the present time, the advertisement has an important role in increasing the demand of a commodity. In the same time, from the market survey it is also observed that for the coming of other new brands of the same product, there is a declination of demand rate. In such circumstances, a manufacturer wants to produce the optimum amount of item per unit time to get the maximum profit from his/her business where he/she has capability to collect sufficient raw materials, labours, machines and other related resources to produce the item. Henceforth, the production rate should be considered to be a variable. In this case, the defective rate of the produced items must be dependent on the production rate. Due to the existence of defective production, the manufacturer decides to sell the items of the perfect quality after sorting the inventory. So during the production period, the screening process has been carried out simultaneously. These concepts have motivated us to develop an imperfect production inventory model with production rate dependent defective rate and advertisement dependent demand which is discussed in **Chapter 3**. Also in this chapter, the depreciation rate in demand function has been considered in which the screening rate is less than or equal to the production rate, but greater than or equal to the demand rate.

From the literature survey, it is seen that traditional inventory models generally hypothesize that the imperfect units are disposed. But in real world, these units are reworked and transformed into a good ones. Moreover, the time horizon of commonly used EPQ model are either infinite or finite. But in the reality, there may occur an uncertainty in the time horizon. Again the demand of the goods is not dependent on exactly one parameter in the production inventory system. Basically there are different parameters on which demand may vary. This sense motivated us to develop a model "An EPQ model with promotional demand in random planning horizon: population varying genetic algorithm approach" in **Chapter 4**, where the demand of the items has three basic parts such as (i) minimum requirement of the goods, (ii) demand enhancement due to lower selling price and (iii) demand increased through the motivation of the customer by advertisement.

Most of research articles on the EPQ models with imperfect production process considered the inspection process for searching the defective items in which the inspection process has been as error free. But in reality, this assumption is not true in business world. Practically, the inspection process is not error free due to different types of factors related to machine and human in the system. In practice, the manufacturer usually offers a warranty for all selling items for a specific warranty period with a view to increasing the selling rate and reliability of the product. Now it is seen that if any purchased product fails to work properly within its warranty period, then the servicing center replaces it with a new item or repairs the product by replacing one or more parts. Therefore, the retailers as well as users are motivated by the displays, advertisements, selling price discount and warranty period to procure the products such as mobiles, computers etc. This phenomena has inspired and motivated us to formulate and analyze the model "A deteriorating manufacturing system considering inspection errors with discount and warranty period dependent demand" by considering warranty period dependent warranty cost which is illustrated in **Chapter 5.**

Again, in a production system, it is seen that initially the production process is in an in-control state, because all factors associated with the system are fresh. But due to continuous running of the system these factors gradually lose their perfectness and thus reliability of the production process decreases. If the production rate increases, the occurrence of out-control state comes very first and hence it produces more non conforming items than earlier. So in such situations a development cost is required to control the occurrence of out-control state. Keeping the above sense in mind, we are motivated to formulate a model in **Chapter 6** where reliability of the production system is continuously maintained by imposing time dependent development cost to reduce the imperfectness of the product during production. Also we assumed that defective rate depends on production rate and the time length of out-control state.

It is observed that a large pile of goods in showroom in a super market will lead the customer to buy more and generate higher demand. But due to limited space facility of showroom, sometimes one or more warehouse(s) is hired on rental basis nearer the showroom. There is a good number of research papers (cf. Bhunia and Maiti [6]) published by several researchers on this ware house facility. This influences us to formulate a three layer supply chain model with two warehouse facility. All these help us to form a "Three-layer supply chain in an imperfect production inventory model with two storage facilities under fuzzy rough environment" which is discussed in **Chapter 7.**

Normally, the rate of defectiveness is not a constant due to many factors. So it should be taken as uncertain quantity. Also in practical business world, sometimes it is seen that the demand of a retailer changes due to various factors according to his/her business policy. So in nature, it is vague and imprecise. These situation have motivated us to develop a model "A fuzzy imperfect production inventory model based on fuzzy differential and fuzzy integral method" in **Chapter 8** where we consider demand of the product, defective rate and the time at which the production system shifts from in-control to out-control state are as fuzzy.

A classical logistic system gives a forward flow, i.e., the material and related information flow until the final products are delivered to the customer. But reverse logistic manages backward process, i.e., the used and reusable parts are returned from the customers to the producer. Environmental consciousness forces companies to initiate such product recovery systems with their disposal such as metal, glass, paper etc. In this way natural resources can be saved for the future generations. These concepts have motivated us to develop, the joint decision-making problem for the two plant production and reproduction inventory model with reworking of defective units over a finite planning horizon. Also our object maximized the expected total profit and minimized expected GHG emission in the imperfect production inventory model **Chapter 9.**

In business world, there are many parameters or components which are not fixed in reality. This situation of uncertainty has motivated us to form an imperfect production inventory model in fuzzy-random environment. Also the concept of learning from our daily mistakes expertises us to introduce the concept of learning effect on the imperfect production model to reduce the rate of defective units including a extra cost for this expertise, which is shown in the model "Multi-item EPQ model with learning effect on imperfect production over fuzzy-random planning horizon" in **Chapter 10**.

In the supply chain, there exists at least one process (production, repairing etc), through which the environment becomes polluted. Therefore, in the context of global business management, an extra cost should be introduced for the emission of carbon to keep the environment fresh. In spite of this carbon emission in a production, trade credit has an intrinsic connection with the demand in supply chain management. These lacunas of supply chain motivated us to form a model entitled "Two layers supply chain imperfect production inventory model with fuzzy credit period, imperfectness depend on time and production rate" in **Chapter 11.**

4 Organization of the Thesis

In the proposed thesis, some real-life Imperfect Production Inventory problems in crisp, stochastic, fuzzy and fuzzy stochastic environments are considered and analyzed. The proposed thesis has been divided into seven parts and twelve chapters as follows:

Part I: General Introduction, Basic Concepts and Solution Methodologies

- **Chapter 1:** General Introduction
- **Chapter 2:** Basic Concept and Solution Methodology

Part II: Studies on Imperfect Production Inventory System in Crisp Environment

- **Chapter 3:** Imperfect production inventory model with production rate dependent defective rate and advertisement dependent demand

Part III: Studies on Imperfect Production Inventory Systems in Stochastic Environment

- **Chapter 4:** An EPQ model with promotional demand in random planning horizon: population varying genetic algorithm approach
- **Chapter 5:** A deteriorating manufacturing system considering inspection errors with discount and warranty period dependent demand
- **Chapter 6:** Two layers supply chain in an imperfect production inventory model with two storage facilities under reliability consideration

Part IV: Studies on Imperfect Production Inventory Systems in Fuzzy Environment

- **Chapter 7:** Three-layer supply chain in an imperfect production inventory model with two storage facilities under fuzzy rough environment
- **Chapter 8:** A fuzzy imperfect production inventory model based on fuzzy differential and fuzzy integral method
- **Chapter 9:** GA approach for controlling GHG emission from industrial waste in two plant production and reproduction inventory model with interval valued fuzzy pollution parameters

Part V: Studies on Imperfect Production Inventory System in Fuzzy Stochastic Environment

- **Chapter 10:** Multi-item EPQ model with learning effect on imperfect production over fuzzy-random planning horizon
- **Chapter 11:** Two layers supply chain imperfect production inventory model with fuzzy credit period, time and production rate dependent imperfectness

Part VI: Summary and Extension of the Thesis

- **Chapter 12:** Summary and Future Research Work

Part VII: Appendices, Bibliography and Index

Part I

(General Introduction, Basic Concepts and Solution Methodologies)

The Part I is divided into two chapters - Chapter 1 and Chapter 2.

Chapter 1: General Introduction

This chapter contains an introduction giving an overview of the preliminary studies along with historical reviews on integrated production inventory control system with reworked of imperfect product in crisp, fuzzy and stochastic environments.

Chapter 2: Basic Concepts and Solution Methodologies

In this chapter, Generalized Reduced Gradient (GRG) technique, Genetic Algorithm (GA), Population Varying Genetic Algorithm (PVGA), Multi-Objective Genetic Algorithm(MOGA), Fuzzy Simulation Based Genetic Algorithm (FSGA), Possibility/ Necessity/ Credibility representation, Solution of Fuzzy Differential Equation (FDE) and Fuzzy Programming Technique(FPT) have been studied which are used to solve and develop the models described in the thesis.

Part II

(Studies on Imperfect Production Inventory System in Crisp Environment)

The Part II contains Chapter 3, in which an imperfect production inventory model is derived, solved and discussed in crisp environment.

Chapter 3: Imperfect production inventory model with production rate dependent defective rate and advertisement dependent demand

In this chapter, an economic production quantity (EPQ) model with imperfect production system and advertisement dependent demand has been presented in crisp environment. The advertisement rate has been assumed to be a function of time which has been increased with respect to time at a decreasing rate i.e., it grows exponentially with respect to time but rate of growth gradually decreases. Here, the rate of producing defective has been followed to be a function of production rate. Also, the produced units have been inspected in order to screen the defective but the screening rate is less than or equal to the production rate and greater than the demand rate. For the developed EPQ model, the total profit has been maximized to obtain the optimum production rate and production run time in the system. Here, an algorithm has been developed for finding the optimal profit of the imperfect production inventory model. Finally, different numerical examples have been considered to illustrate the feasibility of the model taking different special cases in the system and then some sensitivity analysis have been carried out to get the impact of some parameters on the objective function of the model.

Part III

(Studies on Imperfect Production Inventory System in Stochastic Environment)

The Part III contains Chapter 4, 5, & 6, in which different imperfect production inventory models have been derived in stochastic environment and solved then.

Chapter 4: An EPQ model with promotional demand in random planning horizon: population varying genetic algorithm approach

One of the Economic Production Quantity (EPQ) problems that have been of interest to researchers is the production with reworking of the imperfect items including waste most disposal form and vending the units. In this chapter, an imperfect production inventory model is developed over a finite random planning horizon (which is assumed to follow the exponential distribution with known parameters) with the assumption that the decay rate of the items is satisfied from three different points of view: (i) minimum demands of the customer's requirement, (ii) demands to be enhanced for lower selling price and (iii) demands of the customers who are motivated by the advertisement. The model has been illustrated with a numerical example, whose parametric inputs are estimated from market survey. Here the model is optimized by using a population varying genetic algorithm.

Chapter 5: A deteriorating manufacturing system considering inspection errors with discount and warranty period dependent demand

This chapter deals with selling price-discount and warranty period dependent demand in an imperfect production inventory model under the consideration of inspection errors and time dependent development cost. Normally, due to long-run, a production process deteriorates with time and here we assume that the process shifts from in-control to out-of-control state at any random time. A time dependent development cost has been constructed to increase the reliability of the production system i.e., to decrease the deterioration of the system during the production run. As a result, the less amount of items are rejected. Here, two types of inspection errors such as Type-I error and Type-II error, have been considered during the period of product inspection process. In Type-I error, an inspector may choose falsely a defective item as non-defective and in Type-II error an inspector may choose falsely a non-defective item as defective. Due to these phenomena, the inspection process would consist of the following costs: cost of inspection, cost of inspection errors. The purpose of this chapter is to investigate the effects of time dependent development cost on the defective items, selling price-discount and warranty policy on the market demand and finally optimize the expected average profit under consideration of such inspection costs in infinite time horizon. Some numerical examples along with graphical illustrations and sensitivity analysis are provided to test the feasibility of the model.

Chapter 6: Two layers supply chain in an imperfect production inventory model with two storage facilities under reliability consideration

This chapter focuses on an imperfect production inventory model considering production system reliability as well as development cost to improve the system reliability and reworking of imperfect items in the environment of two layer supply chain management. Here we consider, the production system may be shifted from "in-control" state to an "out-of-control" after a time which is a random variable and distributed exponentially with mean $\frac{1}{\lambda}$, where λ is the system reliability depending on the production rate. A development cost is incurred to improve the reliability of the production system. The rate of defectiveness of the imperfect quality items (which produce in the "out-of-control" state) is also assumed as random and depends upon the production rate and time length of the "out-of-control" state. A portion of the imperfect quality items is transformed into perfect quality items after some necessary rework. Another portion of imperfect quality items, termed as ‘less perfect quality items’, is sold at a reduced price to the retailer and the portion which can-not be either transformed to the perfect quality items or sold at a reduce price, is being rejected. For such rejection of some items, a disposal cost per unit of rejected items is incurred to minimize the environmental pollution. Here, a retailer purchases both perfect and imperfect quality items from manufacturer to sale the items to the customers through his/her respective showrooms of finite capacities. A secondary warehouse of infinite capacity is hired by the retailer on rental basis to store the excess quantity of perfect quality items. Finally, average profit of the integrated model has been maximized by optimizing the production rate as well as defective rate of the production system and some numerical examples have been given to illustrate the feasibility of the model.

Part IV

(Studies on Imperfect Production Inventory System in Fuzzy Environment)

The Part IV contains Chapter 7, 8, & 9, in which different imperfect production inventory models have been developed in fuzzy environment and then optimized.

Chapter 7: Three-layer supply chain in an imperfect production inventory model with two storage facilities under fuzzy rough environment

This chapter focuses on an imperfect production inventory model considering product reliability and reworking of imperfect items in three layers supply chain under fuzzy rough environment. In the model, the supplier receives the raw materials, all are not of perfect quality, in a lot and delivers the items of superior quality to the manufacturer and the inferior quality items are sold at a reduced price in a single batch by the end of the cent percent screening process. Manufacturer produces a mixture of perfect and imperfect quality items. A portion of the imperfect items is transformed into perfect quality items after rework. Another portion of imperfect items, termed as 'less perfect quality items', is sold at a reduced price to the retailer and the portion which can not be either transformed to the perfect quality items or sold at a reduce price, is being rejected. Here retailer purchases both the perfect and imperfect quality items from the manufacturer to sale the items to the customers through his/her respective showrooms of finite capacities. A secondary warehouse of infinite capacity is hired by the retailer on rental basis to store the excess quantity of perfect quality items. This model considers the impact of business strategies such as optimal order size of raw materials, production rate and unit production cost in different sectors in a collaborating marketing system that can be used in the industry like textile, footwear, electronics goods etc. An analytical method has been used to optimize the production rate and raw material order size for maximization of the average profit of the integrated model. Finally, a numerical example is given to illustrate the model.

Chapter 8: A fuzzy imperfect production inventory model based on fuzzy differential and fuzzy integral method

This chapter considers a fuzzy economic production quantity (FEPQ) model with interactive fuzzy demands. At the beginning of a production process, the system is assumed to be in a controlled state i.e., under this stage only perfect items are produced. But after some time to be considered as fuzzy here, the manufacturing production process shift to an 'out-control' state i.e., during this stage the system produces both of perfect and imperfect items simultaneously. Here the defective rate of production system has been considered also as fuzzy. Here the screening process of produced items has been considered during production period. Finally some numerical examples have been illustrated to study the practical feasibility of the production inventory model along with sensitivity analysis of some parameters.

Chapter 9: GA approach for controlling GHG emission from industrial waste in two plant production and reproduction inventory model with interval valued fuzzy pollution parameters

This chapter investigates an imperfect production − reproduction inventory model for two types of quality items (item-I and item-II) produce in two different plants (plant-I and plant-II) in the same premises under single management system over a known-finite time horizon with consideration of environment pollution control through industrial waste management. Both the production plant-I and plant-II produces mixture of perfect and defective units. Some of the defective units are rework and non-reworkable defective units are continuously transferred to the raw material processing unit. Used units are collected from the customers and deposited in the raw material processing unit as raw materials for plant II. Treatment of industrial waste from both the plants and raw material processing unit is considered to protect the environment from water pollution and Green House Gas (GHG) emission as industrial waste is become a serious environmental issue. Two conflicting objectives are integrally considered of which one is maximization of the total profit out of two plants and other is minimization of Green House Gas (GHG) emission from industrial waste over the finite time horizon .

Part V

(Studies on Imperfect Production Inventory System in Fuzzy-Stochastic Environment)

The Part II contains Chapter 10 & 11, in which a imperfect production inventory model have been developed, solved and discussed in fuzzy-stochastic environment.

Chapter 10: Multi-item EPQ model with learning effect on imperfect production over fuzzy-random planning horizon

Uncertainty is certain in the world of uncertainty. This study revisits an economic production quantity (EPQ) model with shortages for stock-dependent demand of the items with reworking and disposing of the imperfect ones over a random planning horizon under the joint effect of inflation and time value of money, where the expected time length is imprecise in nature. Transmission of learning effect has been incorporated to reduce the defective production. The total expected profit over the random planning horizon is maximized subject to the imprecise space constraint. The possibility, necessity and credibility measures have been introduced to defuzzify the model. The simulation-based genetic algorithm is used to make decision for the above EPQ model in different measures of uncertainty. The model is illustrated through an example. Sensitivity analysis shows the impacts of different parameters on the objective function in the model.

Chapter 11: Two layers supply chain imperfect production inventory model with fuzzy credit period, time and production rate dependent imperfectness

This chapter focused on an integrated production inventory model with rework of the imperfect units and stock dependent demands of the customer from several retailers. There is an opportunity to build model to measure the amount of carbon emissions during the time of production and the corresponding rate of carbon emission parameters are random which follows Beta distribution. Here, the rate of imperfectness is assumed to be a function of time and production rate. One portion of produces imperfect units is transformed into perfect quality items after some necessary rework. In this chapter, a manufacturer-retailer-customer chain system is developed in which the retailer gets an upstream trade credit period ($\widetilde{M}$) from the manufacturer and retailers offers a down stream trade credit period (N_i) to customers to stimulate demand as well as sales and reduce inventory. We employ the sequential optimization structure of the extensive problem under different scenarios of trade-credit periods. The model has been developed as a profit maximization problem with respect to the manufacturer and retailers. The production time and expected profit has been optimized using develop algorithm and non-linear optimization technique Generalized Reduced Gradient method (LINGO). Finally, several numerical examples and sensitivity analysis are provided to illustrate the utilization of our model.

Part VI
(Summary and Extension of the Thesis)

Chapter 12: Summary and future research work

At the end, a summary of the thesis, its limitation and the scope of future research work have been given.

Part VII
(Appendices, Bibliography and Index)

This part of the thesis contains Appendices, Bibliography and Index.

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