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On Some Inventory Management Problems in Uncertain Environments

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List of Papers

A. List of Published/Accepted Papers

1. An EPL model with reliability-dependent randomly imperfect production system over different uncertain finite time horizons, ***Journal of Intelligent and Fuzzy Systems (Accepted, 2016)***, IOS Press. (SCIE)

2. Quality and pricing decisions for substitutable items under imperfect production process over a random planning horizon, ***Hacettepe Journal of Mathematics and Statistics (Accepted, 2016)***, Hacettepe University. (SCIE)

3. An economic production lot size model for randomly imperfect production system with stock-dependent demand and rework, ***International Journal Operational Research (Accepted, 2015)***, Inderscience Enterprises Ltd.

4. EPL models for complementary and substitute items under imperfect production process with promotional cost and selling price dependent demands, ***OPSEARCH (Published, 2015)***, SPRINGER.

B. List of Communicated Papers

1. Optimum ordering for two substitute items in a news-vendor management with promotional effort on demand using Rough Age based Genetic Algorithm
   Communicated to ***International Journal of Industrial Engineering-Theory, Application and Practice***, Univ. Cincinnati Industrial Engineering.

2. Green logistics under imperfect production system: A Rough age based Multi-Objective Genetic Algorithm approach
   Communicated to ***Computers & Industrial Engineering***, ELSEVIER.

3. EPL models with fuzzy imperfect production system including carbon emission : A fuzzy differential equation approach
   Communicated to ***Journal of Intelligent Manufacturing***, SPRINGER.

4. A learning effected imperfect production inventory model for several markets with fuzzy trade credit period and inflation.
   Communicated to ***International journal of Uncertainty, Fuzziness and Knowledge based system***, World Scientific.

5. A fuzzy imperfect EPL model with dynamic demand under bi-level trade credit policy.
   Communicated to ***Applied Mathematical Modelling***, ELSEVIER.

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Part-I : Introduction and Solution Methodologies

Chapter-1

1.1 Introduction

The control and maintenance of inventory, i.e., over-stocking and under-stocking, is a problem common to all organizations in any sector of the economy. Inventory problems in deterministic environment have been studied by several researchers since early twentieth century. The earliest simple Economic Order Quantity (EOQ) model has been developed by Ford Harris [19] of Westinghouse Corporation, USA. After few years R.H. Wilson [58] published same type of formula and it has been named as Harris-Wilson formula or Wilson's formula. Since then, lot of research work have been reported by several researchers of inventory control problems in different environments and the process still going on.

1.2 Historical Review on Inventory Models

Models with stock dependent demand: Mandal and Phaujder[37] considered linear form of stock-dependent demand, i.e. D = c + dq, where Mandal and Maiti [38] and Maiti and Maiti [32] and others took the demand as $D = dq^{\beta}$. Recently, Tyagi *et al.* [53] and Chakraborty *et al.* [7] investigated models for deteriorating items with stock-dependent demand in crisp and fuzzy environments respectively.

Models on imperfect production process: Salameh and Jaber [45] and Lin [27] studied the EOQ / EPQ model for the items with imperfect quality and proposed discount sales for them. Sana [47] presented an EPL model with random imperfect production process and defective units were repaired immediately when they were produced. Recently, Taleizadeh and Wee [52] extended a multi-product single machine manufacturing system with manufacturing capacity limitation and immediate reworking of imperfect products allowing partial back-ordering.

Models with complementary and substitute products: In marketing of several items, the demand of an item is affected by the other in the case of complementary and substitute items. Demand of each complementary item increases to some extent by the other such items where as in the case of substitute items, the demand of such an item may increase or decrease depending upon the choice of the customers [33, 36, 1].

Models with carbon emission: Hua *et al.* [22] examined both analytically and numerically the impact of carbon trade, carbon price and carbon cap on order decisions and total cost of an environmental inventory model based on the classical EOQ model. An EOQ model with a constraint on the emission of carbon is considered by Chen *et al.* [9]. He *et al.* [20] examined the production lot sizing issues of a firm under cap and trade and carbon tax regulations respectively. Xu *et al.* [55] studied the joint production and pricing decisions for products of a production firm under cap and trade and carbon tax regulations.

Models allowing credit period: The concept of trade credit was first introduced by Haley and Higgins [18]. Goyal [15] was the first who established an EOQ model with a constant demand rate under the condition of permissible delay in payments. Later, Ouyang *et al.* [41] considered two-level trade credit link to order quantity. Dye and Yang [13] considered issues of sustainability in the context of joint trade credit and inventory management in which the demand depends on the length of the credit period offered by the retailer to its customers.

Models with inflation and time value of money: At present, it is not possible to ignore the effect of inflation as the economy of any country changes rigorously due to high inflation. Considering this effect on inventory costs, first impetus was given by Buzacott [4]. In recent years, Chang [8], Jaggi *et al.* [23], Maiti [35, 34] Sana [46, 47] and Sarkar *et al.* [49, 50] and others presented inventory models in this direction.

Models with impreciseness: The first publication accommodating the uncertainty in non-stochastic sense was in 1965 by Prof. Zadeh [60]. After that extensive research work have been done in this area [12, 42, 28, 6]. Till now, Fuzzy Differential Equation (FDE) [3] and fuzzy integration [59]are little used to solve fuzzy inventory models [16], though the topics on fuzzy differential equations have been rapidly growing in the recent years. Wu [59] introduced the concept of fuzzy Riemann integral and its numerical integration.

Recently, new types of uncertain variables such as − rough, fuzzy-rough, random-rough, etc are used in the research field of science and technology including inventory control problems [29, 30, 54, 31]. Though some researchers are interested during last few years to deal with the above mentioned uncertainty, still there are lot of scope for the inventory practitioners to develop and solve real-life inventory models in rough, fuzzy-rough or random-rough environments.

1.3 Motivation and Objective of the Thesis

Motivation of the Thesis: In inventory control system, high level of inventory attracts more visibility and also may imply that the goods are popular and fresh. Thus the inventory problem with stock dependent demand is addressed by several researchers to reflect realistic circumstances. The problem of stock dependent demand has been studied in both empirical and theoretical papers. Empirical evidence of such demand of specific product has been provided in [56, 38, 32]. Moreover, the theoretical models are proposed for developing optimal policies in [37, 24, 53, 7]. Production disruption is a very familiar event in real life production process. This disruption may be defined in the form of defective units of the item. Most of the production inventory models are investigated with the assumptions of known proportion of defective units and fixed inspection state. Besides this, at present in case of economic condition throughout the world, one can't ignore the "psychic stock" of goods and period of uncertain out-of-control state.

After all these studies, some lacunas exist in the formulation of inventory models with stock dependent demand and imperfect production.

- In every manufacturing process, it is fact that environment is disturbed to some extent and for that, now-a-days, attention is paid not to pollute the environment. Till now, very few have introduced the environment protection cost (EPC) in EPL models, which again varies with the rate of production.
- Few investigators have considered the non-instantaneous out-of-control state (starting point of imperfect production) to be random during the production time, but none has imposed it as a chance constraint for the system.
- Defective production rate normally increases with the time elapsed from the out-of-control state and the production rate. Sana [47] considered this with constant demand and fully reworked defective units. None has investigated this phenomena in conjunction with stock-dependent demand and environmental protection cost.

Considering these facts, the author has formulated an **EPL model (Model-3.1) with stock dependent demand, random out-of-control state (including chance constraint) and EPC**.

In a production process, the defectiveness of the item depends on the reliability of the machine [48, 51]. For a long run process, learning knowledge [2, 14] is an important factor to a Decision Maker (DM). Moreover, it is generally observed that specially for electrical or electronic goods, the life time of an item is not infinite. It is finite, but not predictable, rather it fluctuates for different items. This phenomenon has inspired us to formulate **Model-3.2** which is constructed in terms of an **EPL model with reliability dependent random defective units in out-of-control state over different uncertain finite time horizons.**

In reality, for multi item inventory system, considerable savings may be achieved by the co-ordination of replenishment for a group of items. When inventory for a particular item has been exhausted, then the demand of that item is met by another substitutable item. Consequently the demand of a complementary item exogenously increase the demand of the relative item, such as mobile phone and sim card, tea and sugar, car and fuel etc. Kim and Bell [26], Maity and Maiti [36] and others researchers investigated the impact of the symmetrical and asymmetrical demands for substitutable and complementary items. But none has considered the following phenomena.

- A production-marketing system for complementary and substitute items under randomly imperfect production process and budget limitation.
- An investigation in conjunction with advertisement / promotional cost, selling price dependent demand and Environment Protection Cost (EPC).
- Commencement of imperfect units depends inversely on production rate.
- A part of the set up cost of the business system depends on the production rate.

These concepts motivated us to formulate an **EPL model for complementary and substitute items under imperfect production process with promotional cost and selling price dependent demands (Model-4.1).**

Several authors (Moon and Yun [40], Roy *et al.* [44], Guria *et al.* [17], Manna *et al.* [39]) suggested that for any kind of business process, planning horizon is neither infinite nor finite (fixed or deterministic). They established the concept of random planning horizon for single/multi-item EOQ or EPQ model. But none conceived the idea of random planning horizon for substitutable items. Besides that quality is an important criteria for substitutable items. Moreover, following facts are to be considered in an inventory control system.

- Learning effect on the set up cost of the system and maintenance of the machinery system is a realistic phenomena.
- Product substitution depends on the joint effect of price and quality or on the basis of either price or quality.
- Quality improvement cost which is a function of quality of an item, is a part of Unit Production Cost (UPC).
- Development of an efficient heuristic algorithm (Fuzzy Age based Genetic Algorithm (FAGA)) is necessary to solve such a complex inventory problem in uncertain environment.

Keeping all these in mind, we construct the **Model-4.2 (Quality and pricing decisions for substitutable items under imperfect production process over a random planning horizon).**

In certain situations, the inventory process is terminated after a small duration of time which implies that the entire inventory on hand has no value after that duration. In order to control this, news-boy problem is very common in our daily life. Here the DM makes the decisions about the level of inventory for a very short period. This is the case, for example with the products such as daily news paper, magazine, X-mass cards, etc. Several extensions of the news-boy model have been reported in the literature [43, 10, 57] with the assumptions- substitutable item, random demand, budget constraint, etc. But, till now none has considered a news-boy problem with

- Promotional effects on the random demands to boost the demand.
- Substitute items- one substitute the other when the latter is out of stock.
- Budget constraints on purchasing or promotional or both of these costs.
- A heuristic optimization method named as "Rough Age based Genetic Algorithm (RAGA)" for approximate single objective optimal solutions.

Therefore there is a strong motivation to construct the **Model-4.3** entitled **"Optimum ordering for two substitute items in a news-vendor management with promotional effort on demand using Rough Age based Genetic Algorithm."**

Some developed countries (UK, USA, Germany, etc.) have made a climate policy called "carbon-tariffs policy" on the production firms. This policy has a significant impact on the social welfare. For this reason, the production inventory management system is urged to consider the fact of carbon emission during the production of an item.

Uncertainty is the only certain phenomena in the world of uncertainty. Now-a-days, in the competitive market, the DM of the management system can't take the risk of considering fixed information data. Again the impreciseness (fuzziness) of the parameters / variables is defuzzified in different ways. During the last two decades, a lot of literatures are available in this context [60, 12, 42, 61, 29, 3, 59, **?**]. All these motivated us to consider the **Model-5.1** and **Model-5.2**. The other reasons for adaptation of **Model-5.1** are

- A production system producing imperfect quantities after the passage of sometime from the occurrence of production and emitting carbon from the beginning is considered and introduced under all the available carbon rules and regulations of the worldwide countries.
- A heuristic optimization method has been introduced– Rough age based Multi-Objective Genetic Algorithm (RMOGA) for multi-objective optimizations in fuzzy environment is presented.

Although there are lot of research works in fuzzy environment, but most of them were not formulated following Fuzzy Differential Equation (FDE) approach. Hence the cost function and sales revenue have become expressions as Fuzzy Riemann Integral (FRI). Moreover, the concept of Intuitionistic Fuzzy Set (IFS) can be seen as an alternative approach to define a fuzzy set instead of normal fuzzy set. Therefore, we consider an **"EPL models with fuzzy imperfect production system including carbon emission: A fuzzy differential equation approach" (Model-5.2)** under the strong motivation of the following.

- As the defective production starts after sometime from the commencement of production and is uncertain, this production time is taken as fuzzy and time dependent fuzzy defective production rate is considered.
- An EPL system is characterised by FDE and solved as Multi-Objective Optimization Problem using α-cut.
- For optimization, a new technique, Intuitionistic Fuzzy Optimization Technique (IFOT) is introduced.

In practical world, to make the co-operative relationship more attractive to boost the demand of the items, a wholesaler / retailer offers several concessions to their customers such as credit period, free transportation, etc. Again a manufacturer produces the items under a defective production process and sells items in different markets. Generally, it is observed that each market has different selling seasons. He *et al.* [21] considered an EOQ model with several markets. Das *et al.* [11] applied this conception in a supply chain model. But still, there are some gaps in the literature, like

- An EPL model for several markets along with the availability of trade credit period. This credit period may be uncertain in non-stochastic sense.
- Model with inflation and transportation cost which is reduced from cycle to cycle with the help of learning inspection.

These gaps motivated us to develop the **Model-6.1** named as **"A learning effected imperfect production inventory model for several markets with fuzzy trade credit period and inflation."**

Again in supply chain management system, there is a scope of bi-level trade credit. However, in practical situation as there is high rate of interest after the credit period, it is more profitable if all the payment is made as early as possible. These ideas influenced us to formulate **"A fuzzy imperfect EPL model with dynamic demand under bi-level trade credit policy" (Model-6.2)** with new realistic assumptions on credit period and its payment.

Objective of the Thesis: The main objectives of the thesis are

(i) To develop some inventory / production inventory model(s) for single and / or integrated system i.e. supply chain management system in different types of uncertain environments with few innovative and realistic assumptions which are not considered so far.

(ii) To modify some existing probabilistic or fuzzy programming method and to develop solution techniques (Expectation, FDE, Possibility, Necessity, FAGA, RMOGA, TLBO, IFOT, etc.) as per the requirements of the model described in (i). The models are solved by these methods.

(iii) To convert the uncertain models into the corresponding deterministic single or multi-objective problems by using different appropriate techniques.

(iv) To show different effects or relations of the models' parameters and decision variables through some numerical examples and to perform their sensitivity analyses.

Chapter-2

Solution Methodologies

2.1 Mathematical Prerequisite :

- Crisp, Fuzzy, Random, Fuzzy-Random, Rough, Fuzzy-Rough Set Theory
- Intuitionistic Fuzzy Set
- Fuzzy Differential Equation (FDE)
- Fuzzy Riemann Integration (FRI)

2.2 Solution techniques/methods used :

- Generalized Reduced Gradient Technique (GRG)
- Intuitionistic Fuzzy Optimization Technique (IFOT)
- Fuzzy Age-based Genetic Algorithm (FAGA)
- Rough Age-based Genetic Algorithm (RAGA)
- Rough age based Multi-Objective Genetic Algorithm (RMOGA)
- Teaching and Learning Based Optimization (TLBO)

Part-II: Inventory Problems in Uncertain Environment

Chapter-3

Inventory Problems with Stock dependent Demand

in Random Environment

Model-3.1: An EPL model for randomly imperfect production system with stock-dependent demand and rework

In this model, we consider a single item, imperfect EPL model with stock-dependent demand and partial rework. In real life EPL models, defective production commences from the out-of-control state, after the passage of some time from production commencement. Its occurrence is random and imposed here through a chance constraint. The set-up cost is partly production dependent. Unit production cost (UPC) is also production dependent and a part of it is taken as environment protection cost (EPC). Defective rate is also assumed to be random and production dependent. The model is formulated as an average cost minimization problem subject to a chance constraint and solved using a non-linear optimization technique- GRG method through LINGO 11.0 software. Several special cases are derived and more specifically, the present investigation derives the expressions of Sana [47] and Khouja and Mehrez [25]. Numerical experiments are performed to illustrate the general and particular models. Some sensitivity analyses are presented against few model parameters.

Model-3.2: An EPL model with reliability-dependent randomly imperfect production system over different uncertain finite time horizons

Imperfect EPL models are considered over different types of uncertain finite time horizons with stock-dependent demand, reliability dependent defective rate and random out-of-control state. Generally, in EPL models, defective production starts after the passage of some time from production commencement. So occurrence of defective production is random and imposed here through a chance constraint. Reliability of a machinery system affects on the defective rate and production cost to produce an item. Here UPC depends on reliability and production rate and part of it is taken against the EPC. Both linear and non-linear production dependent forms of quality are considered. The problems are formulated as total cost minimization problems with crisp, random, fuzzy, fuzzy-random, rough and fuzzy-rough constraints and solved using GRG method through LINGO 11.0 software. Several special cases are derived and numerical experiments are performed to illustrate the general and particular models.

Chapter-4

Inventory Problems on Complementary and Substitute Products in Random Environment

Model-4.1: EPL models for complementary and substitute items under imperfect production process with promotional cost and selling price dependent demands

This model considers a multi-item, imperfect EPL model with advertisement/promotional cost and selling price dependent demand and partially rework under a budget constraint. The items are either complementary or substitute to each other. In the EPL models, defective production commences from the out-of-control state, after the passage of some time from production commencement. Its occurrence is random after the lapse of certain time and it is imposed here through a chance constraint. The set-up cost is partly production dependent. UPC is also production dependent and a part of it is taken as EPC. Defective rate is also assumed to be random and production dependent. The model is formulated as an average profit maximization problem subject to chance constraints and a budget constraint and solved using the GRG method. Several special cases are derived and numerical experiments are performed to illustrate the general and particular models. Some sensitivity analyses are presented against few model parameters.

Model-4.2: Quality and pricing decisions for substitutable items under imperfect production process over a random planning horizon

This investigation determines the optimum qualities and prices of two substitute products for a manufacturer cum retailer in an imperfect production process over a random planning horizon for maximum profit. In this EPL process, items are produced simultaneously, defective production commences during the "out-of-control" state after the passage of some time from the commencement of production and the defective units are partially reworked. The items are substitutable to each other depending on their prices and qualities jointly or separately. UPC depends directly on raw-material, labour and quality improvement costs and inversely to the production rate. A part of it is spent against environment protection. Here learning effect is introduced in the set-up and maintenance costs. For the whole process, the planning horizon is random with normal distribution, which is treated as a chance constraint. A Fuzzy Age based Genetic Algorithm (FAGA) is introduced for the solution of a single objective problem. The above mentioned models are formulated as profit maximization problems subject to a chance constraint and solved using FAGA. The models are demonstrated numerically and the optimum results are presented graphically.

Model-4.3: Optimum ordering for two substitute items in a news-vendor management with promotional effort on demand using Rough Age based Genetic Algorithm

This model presents a single period news-vendor management system for two substitute items. Here substitution is made only when one item is exhausted and then the substitute item is sold at a different price. Here, the News-Vendor problem deals with stochastic uniform demand and a promotional effort to boost the random demand. Different scenarios are formulated and combined to find the expected profit of the system. The profit function is optimized with constraints on purchasing and/ or promotional costs. A Rough-Age based GA (RAGA) is developed for single objective optimization and applied to identify the optimal strategies of the proposed model. Finally, real-life experiments are performed to illustrate the models. Some sensitivity analyses are also presented to stabilize the numerical experiments.

Chapter-5

Inventory Problems with Carbon Emission in Fuzzy Environment

Model-5.1: Green logistics under imperfect production system: A Rough age based Multi-Objective Genetic Algorithm approach

Imperfect EPL models are considered with time dependent defective rate. Here, defective production starts after the passage of some time from the production commencement. Produced defective units are partially reworked and sold as fresh units. Under the environmental regulation, a cost (carbon tax) is charged by the government to mitigate global warming by reducing carbon emission (CE). Management also uses carbon trading when upper limit of carbon emission is given by the government. This cost brings a contradiction to production manager. For more profit, if more production is decided, then CE and tax due to that are more. The models are formulated as profit maximization problems and solved using Rough age based Multi-Objective Genetic Algorithm (RMOGA). Numerical experiments and graphical presentation are performed to illustrate the models. An algorithm with example for the firm management to achieve the maximum profit is also presented.

Model-5.2: EPL models with fuzzy imperfect production system including carbon emission : A fuzzy differential equation approach

This model outlines the production policies for maximum profit of a firm producing imperfect economic lot size with time-dependent fuzzy defective rate under the respective country's carbon emission rules. In this investigation, two criteria in production process are considered : (i) Generally in EPL models, defective production starts after the passage of some time from production commencement. So the starting time of producing defective units is normally uncertain and imprecise. Here it is taken as fuzzy. Thus produced defective units are fuzzy, partially reworked instantly and sold as fresh units. As a result, the inventory level at any time becomes fuzzy and the relation between the production, demand and inventory level becomes a Fuzzy Differential Equation (FDE). (ii) Under the environmental regulation, a cost (say carbon tax) is charged by the government to mitigate global warming by reducing CE. Firm management also uses carbon trading when upper limit of carbon emission is fixed by the government. This cost brings a contradiction to production management. For more profit, if more production is decided, then CE and tax due to that increase. To avoid the carbon penalty, total production may be reduced but in that case, profit will be less. Considering the above two real-life criteria, some production policies are outlined. Here models are formulated as profit maximization problems using FDE, the corresponding inventory and environmental costs are calculated using fuzzy Riemann-integration. α-cuts of average profits are obtained and the reduced multi-objective crisp problems are solved using Intuitionistic Fuzzy Optimization Technique (IFOT). Numerical experiments and graphical presentation are performed to illustrate the models. Considering different carbon regulations, an algorithm for a firm management in a country is presented to achieve the maximum profit. Real-life production problems for the firms in Annex I and developing countries are solved.

Chapter-6

Inventory Problems with Trade Credit Policy in Fuzzy Environment

Model-6.1: A learning effected imperfect production inventory model for several markets with fuzzy trade credit period and inflation.

This model consists of joint relationship among supplier, manufacturer-cum-retailer and multiple markets in which manufacturer-cum-retailer gets a facility of fuzzy credit period for purchasing of raw materials from supplier. The manufacturer produces the finished goods along with defective units at a constant rate. Here the finished product is transported to different markets in different seasons, with a transportation cost that depends on the amount of transportation and learning ratio. Also, the demand of the item is different in each market. Further, the optimal operation policy that maximizes total profit of the integrated system is derived under a constant rate of inflation. But due to impreciseness in trade credit period, profit function seems fuzzy in nature, thereby determining the optimal imprecise values of decision variables. Equivalent crisp profit function is obtained by applying fuzzy expectation method. The closed form solutions of the objective and it's concavity properties have been derived to obtain maximum profit. Finally, the models are illustrated with certain numerical data and graphical solutions are provided with sensitivity analysis with respect to model's parameters.

Model-6.2: A fuzzy imperfect EPL model with dynamic demand under bi-level trade credit policy

An imperfect EPL model with fuzzy dynamic demand is developed in a fuzzy production process under bi-level trade credit policy. Supplier offers a delay period (M) to the manufacturer-cum-retailer for payment of raw-material cost. Due to this facility, manufacturer-cum-retailer also offers a trade credit period (N) to the customers to boost the demand. During trade credit period of customers, demand of the item increases with time at a decreasing rate. Different parameters of demand are assumed as fuzzy. Depending upon the values of M and N, twelve scenarios are depicted. In each scenario, the model is represented through FDE whose solution is obtained using Chalco-Cano [5] technique. Thus average profit function is imprecise in nature and its α-cut values are maximized for making optimal decision of production run time. All scenarios are illustrated with numerical examples. Further more, an alternative approach of payment for the remaining inventory after the credit period M has also been proposed in the present study. In the new approach, retailer clears all dues before the end of business cycle whenever it is feasible. It has been explained with the help of numerical examples and the outcomes are compared against the above traditional approach.

Part-III : Summary and Future extension

Chapter-7

Summary of the Thesis: In this thesis, total nine virgin uncertain inventory /production-inventory models, of which five in random and four in fuzzy environments are formulated and solved.

- The models are developed for different types of demands like stock dependent demand, price and quality dependent demand, promotional effort and advertisement dependent demand, credit period dependent dynamic demand and news-boy type probabilistic demand.
- The models are formulated with in-control and out-control states, effects of learning and forgetting, carbon emission, advanced payment, trade credit policy, inflation of money and many more criteria which are visible in recent management system.
- The models are simplified (may be converted from fuzzy to crisp, random to non-random, etc.) by using methods of Fuzzy Differential Equation (FDE), Possibility, Necessity, Credibility, Trust measures, method of chance constraint, etc.
- Different techniques are developed/presented to transform the imprecise parameters/objectives to corresponding deterministic ones. For the solution of single and multi-objective with/without constraints, different optimization techniques such as Generalised Reduced Gradient method (GRG), Genetic Algorithm (GA), Fuzzy Age based GA (FAGA), Multi-objective GA (MOGA), Rough Age based GA (RAGA), Teaching and Learning Based Optimization (TLBO), Intuitionistic Fuzzy Optimization Technique (IFOT), etc. are developed / presented and used.
- The models are illustrated with appropriate numerical examples and the optimum results are presented graphically. Moreover, the obtained results are discussed with respect to managerial insights.
- In some cases, sensitivity analyses are made with respect to some model parameters and presented graphically and in tabular forms to look deep into the models.

Future Extension:

- Each model presented in the thesis has an impact to its future extensions. The models can be formulated in other type of uncertain environments, such as- fuzzy-rough, rough-fuzzy, random-fuzzy, etc.
- The models also can be extended with different types of uncertain parameters and/or variables like bi-fuzzy, type-2 fuzzy, rough, bi-rough etc.
- Different types of optimization techniques (Particle Swarm Optimization (PSO), Ant Colony Optimization (ACO), Geometric Programming (GP), etc) can also be applied to the models.
- Moreover, other recent development criteria like risk management, just-in-time inventory control, different forms of supply-chain, etc. also can be incorporated to the models.
- In the models, intension expression of customer to purchase the goods knowing the selling price at that time may be incorporated.
- Carbon emission due to transportation can be included in the models (presented in this dissertation) along with CE due to production.

Therefore, there is a huge scope to extend the research works presented in this thesis.

Part-IV: Bibliography

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